WILLIAM ROWAN HAMILTON (August 4, 1805 – September 2, 1865)

by Heinz Klaus Strick, Germany

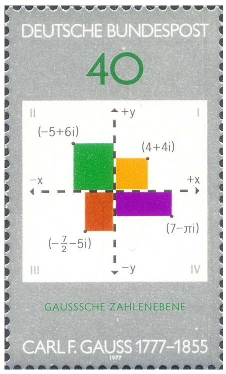
William Rowan Hamilton grew up as the son of a lawyer in Dublin. He learnt Latin, Greek and Hebrew at the age of 5 and later other languages including German, Sanskrit and Malay. For mathematics he only began to take an interest at the age of 12, read the *Elements* of Euclid in Latin at 13 and *Elements d' Algèbre* by Alexis-Claude Clairaut in French. At the age of 15 he studied the works of Isaac Newton (1643 – 1727) and Pierre Simon Laplace (1749 – 1827.

At 17 he discovered a mistake in Laplace's *Mécanique Céleste* (Celestial Mechanics), which is what caused the Irish astronomer John Brinkley to conclude: "*This young man, I do not say will be, but is, the first mathematician of his age*."

At the age of 18 Hamilton entered *Trinity College* in Dublin. His first publication at the *Royal Irish Academy* followed a year later and his first groundbreaking research on optics in 1826. When he was taking his intermediate examinations at the university, one of his examiners persuaded him to apply for the post of *Royal Astronomer* at Dunsink observatory.

Although he was not particularly interested in this field, he applied and prevailed against all competitors. This carried the title *Royal Astronomer* of Ireland. However, he wrote only one scientific paper on astronomy in his life, namely on the theory of the moon.

His contributions to physics, on the other hand, significantly developed the teaching of theoretical mechanics and optics. His contributions to mechanics have been occasionally equated in the literature with Newton's achievements. They are among the foundations of quantum physics.



In 1831 Carl Friedrich Gauss published a work on biquadratic residues – these are remainders which can occur when biquadratics (= 4th powers) are divided by an integer. In this work he introduced the term *complex numbers* for the numbers of the form $a + b \cdot \mathrm{i}$. He also points out that such numbers could be understood as points $(a \mid b)$ in a 2-dimensional coordinate system – i.e. as geometrical objects.

This is now known as the *Argand diagram* or in the German-speaking world as the *Gauss plane*.

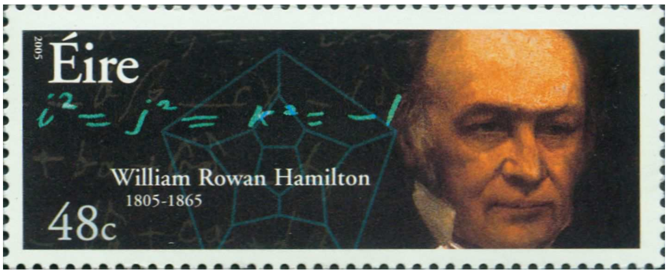
In 1832 Hamilton went one step further. He regarded complex numbers as algebraic objects: as pairs of numbers $(a\,;\,b)$, for which there are two arithmetic operations, an addition and a multiplication, as defined below:

$(a\,;\,b) \oplus (c\,;\,d) = (a + c\,;\,b + d)$ and $(a\,;\,b) \otimes (c\,;\,d) = (a \cdot c - b \cdot d\,;\,a \cdot d + b \cdot c)$

Under these two operations, the set of complex numbers is closed, i.e. the operations do not lead out of the set, and the associative and commutative laws apply to both operations as well as the distributive law. Furthermore, neutral elements exist with respect to both operations, and for each element (not equal to zero) there is a multiplicative inverse element.

Hamilton worked intensively on the question of whether similar operations could be defined in 3-dimensional space. But no matter what approach he chose for a "multiplication" of number triples $(a\,;\,b;\,c)$, it led to a contradiction. (In 1898 it was proved by Adolf Hurwitz that such a multiplication is only possible for the dimensions 1, 2, 4, 8.)

However, dealing with 4-dimensional objects (a ; b; c; d), Hamilton had the vital idea during a walk in 1843.



If the property of *commutativity* does not have to be satisfied, then a multiplication is possible:

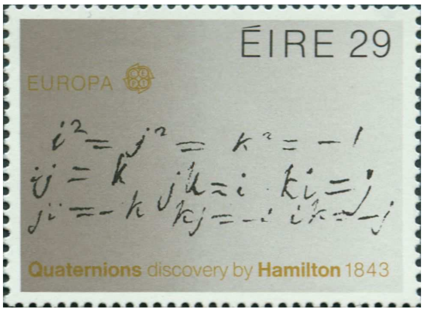
$(a; b; c; d) \otimes (e; f; g; h) =$

$(a \cdot e - b \cdot f - c \cdot g - d \cdot h; a \cdot f + b \cdot e + c \cdot h - d \cdot g; a \cdot g - b \cdot h + c \cdot e + d \cdot f; a \cdot h + b \cdot g - c \cdot f + d \cdot e)$

The addition is the same as for complex numbers – defined componentwise:

$(a; b; c; d) \oplus (e; f; g; h) = (a + e; b + f; c + g; d + h)$

If the element $(a; b; c; d)$ is denoted by $a + b \cdot i + c \cdot j + d \cdot k$, then the following relations apply between the "numbers" i, j, k: $i^2 = j^2 = k^2 = i \cdot j \cdot k = -1$; $i \cdot j = k = -j \cdot i$.



⊗	*e*	***i***	***j***	***k***
e	*1*	*i*	*j*	*k*
i	*i*	*-1*	*k*	*-j*
j	*j*	*-k*	*-1*	*i*
k	*k*	*j*	*-i*	*-1*

HAMILTON carved these relations into a stone of a bridge in Dublin during the above-mentioned walk. Today a plaque commemorates the event (see also the Irish stamp).

He called these 4-dimensional objects *quaternions* (Latin: four-numbers) and with their discovery opened a new mathematical field of research: vector algebra. For example, he himself discovered that the multiplication of quaternions can be understood as the rotation of vectors in space.

1835 – WILLIAM ROWAN HAMILTON was only 30 years old – he was knighted and from then on he was allowed to call himself SIR WILLIAM HAMILTON.

In 1832, based on theoretical considerations, he had predicted a phenomenon in the refraction of light on biaxial crystals, which was experimentally confirmed shortly afterwards. This gave him as much recognition in the world of science as his work *On a General Method in Dynamics* (now known as *Hamilton's Theory*).

He was appointed a member of the *Royal Society*, President of the *Royal Irish Academy* and a Corresponding Member of the *Academy of St Petersburg*.

His private life, however, was less "successful". At the age of 19, he had fallen in love with CATHERINE, whom he could not marry, however, because he did not have a secure income as a student.

Out of love-sickness he sought refuge in literature, read poetry in Persian and Arabic, and wrote poetry himself. He showed his poems to a friend, the poet WILLIAM WORDSWORTH, who had difficulty in convincing HAMILTON that the poems were less successful than his mathematical and physical treatises.

HAMILTON finally married HELEN BAYLEY, who lived close to the Observatory. However, his marriage, which produced three children, was not a very happy one. His wife was often ill, he was always very worried, and this affected his creativity.

A quiet, secluded family life, which he had promised her before the marriage, was in contrast to the social obligations resulting from his growing fame and the offices he was offered. Added to this there were periods during which HAMILTON restlessly worked on his projects and withdrew to his study for days on end.

His wife's repeated absences due to illness were as much a topic of conversation in Dublin society as HAMILTON's alleged excessive consumption of alcohol. After an incident at a reception of the *Geological Society of Ireland*, HAMILTON refrained from drinking alcohol for a long time.

When he met his childhood love CATHERINE one day, he realised how unhappy she was in her marriage, and his love for her awakened anew. HAMILTON was therefore pleased to be able to help CATHERINE's eldest son as he prepared for his university exams in 1848.

CATHERINE's letter of thanks was followed by a lively correspondence between the two until she realised that these letters were a violation of the strict moral rules of the time. She felt guilty and informed her husband about her contact with HAMILTON. After a failed suicide attempt on her part, she separated from her husband.

In the following years, HAMILTON and CATHERINE had frequent correspondence with each other. At the end of 1853, HAMILTON received a parcel containing a pencil box with the inscription

From one who you must never forget, nor think unkindly of, and who would have died more contented if we had once more met.

Dismayed, HAMILTON rushed to CATHERINE, who was terminally ill. They both admitted that they never stopped loving each other. He handed her the first freshly printed copy of his *Lectures on Quaternions*.

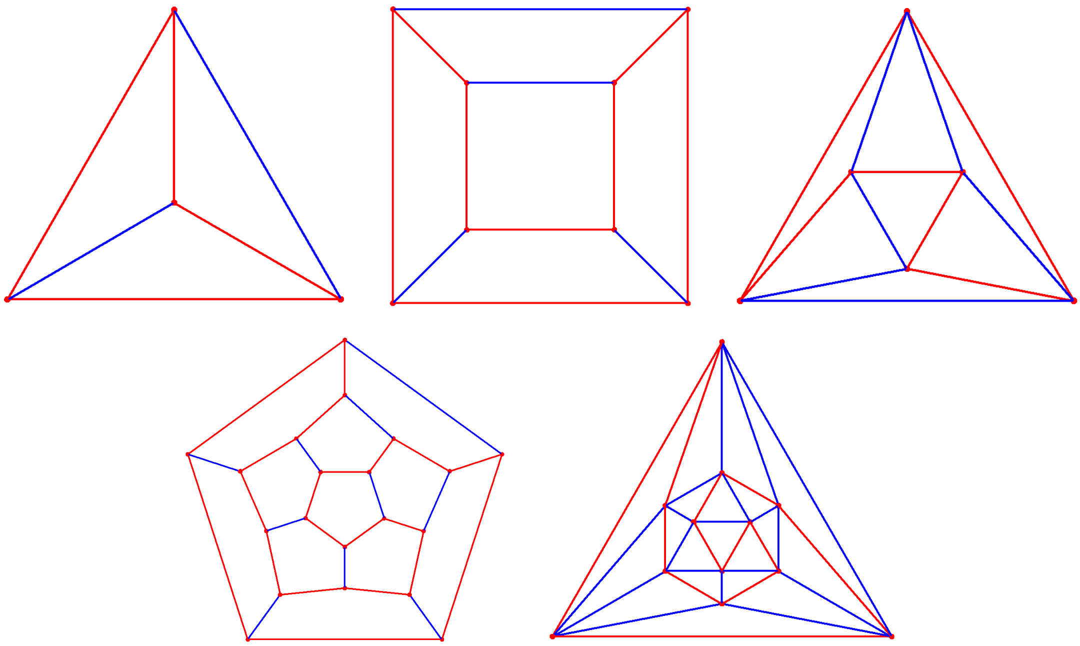
After her death, HAMILTON threw himself back into his work. He was aware that the readability of the *Lectures* must be improved. For seven years, he worked on a new work: *Elements of Quaternions*, with the *Elements* of EUCLID as a model. When he died in 1865, the work comprised 800 pages and was still not finished; one of his sons added the missing pages for printing.

Shortly before his death he received the news that the *National Academy of Sciences* of the USA had appointed him their first foreign member.

The name HAMILTON is associated above all with advances in theoretical physics and vector algebra. However, he also introduced important ideas to another field: graph theory. He developed a "game" based on the problem:

- *How many closed paths are there along the edges of a regular dodecahedron such that each corner point is traversed exactly once?*

The figures below show that HAMILTONian paths are possible on all PLATONic solids. Finding such paths in any graph is generally a computationally difficult problem.



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