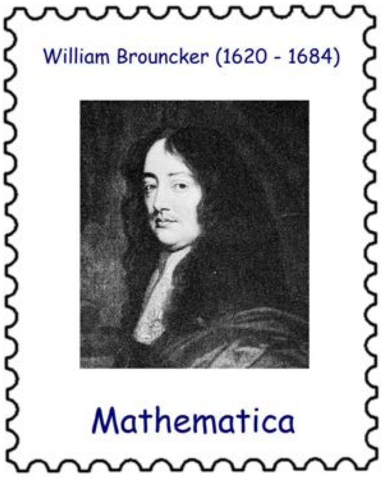
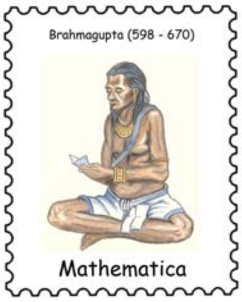
WILLIAM BROUNCKER (1620 – April 5, 1684)

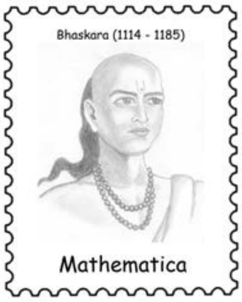
by HEINZ KLAUS STRICK, Germany

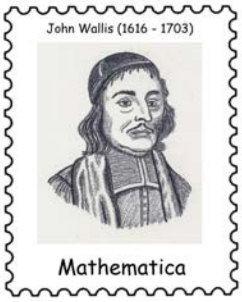


PIERRE DE FERMAT had always taken great pleasure in informing his numerous correspondents when he had solved another problem, and he was always eager to see whether his correspondents would be able to meet this challenge. In 1657 FERMAT succeeded in finding the smallest possible solution for equations of the type $n \cdot x^2 + 1 = y^2$ for natural numbers $n \leq 150$. (FERMAT did not know that equations of this type had already been treated previously by BRAHMAGUPTA and BHASKARA).

His Parisian correspondent FRÉNICLE DE BESSY passed the information on to the English mathematician JOHN WALLIS, who informed his friend WILLIAM BROUNCKER. A short time later BROUNCKER reported back to WALLIS that he had found a solution. WALLIS published BROUNCKER's method first as an appendix to his work *Commercium epistolicum* (1658) and later also in the second volume of his *Opera mathematica* (1693).







(drawings: © Andreas Strick)

At the beginning of the 1730s, LEONHARD EULER studied the works of WALLIS, but mistakenly attributed the method of solution to the English mathematician JOHN PELL. WALLIS had quoted works by PELL in his work, but on different matters. EULER did not notice the mistake; and so he wrote in 1771 in the second part of his *Complete Guide to Algebra*: "*For this purpose, a learned Englishman, named PELL, invented a very clever method, which we will explain here*."

For example, EULER found a solution to the equation $2 \cdot x^2 + 1 = y^2$ through the approach $2 \cdot x^2 + 1 = (x+p)^2$ i.e. $x = p + \sqrt{2p^2 - 1}$ which for the integer $p = 1$ gives $(x, y) = (2, 3)$ as a solution. For $3 \cdot x^2 + 1 = y^2$, the substitution $y = x + p$ or $y = 2x - p$ leads to the solution $(1, 2)$. For $5 \cdot x^2 + 1 = y^2$ one can put: $5 \cdot x^2 + 1 = (2x + p)^2$ so $x = 2p + \sqrt{5p^2 - 1}$ i.e., which for $p = 1$ leads to the integer solution, $(4, 9)$ etc.

JOSEPH-LOUIS LAGRANGE, who gave the final proof of the solvability of all equations of this type, referred to EULER, and therefore since then, they have always been called PELL's equations (in France: *équations de PELL-FERMAT*) in the specialist literature.

There are no documents which show exactly when WILLIAM BROUNCKER was born. On the basis of the known dates of his life, 1620 has been established as the presumed year of birth. It is known that he studied mathematics, languages and medicine at Oxford: mathematics being the arithmetic skills that merchants and craftsmen were expected to master.

William's father, who played an important role at the court of the King of England, saw Charles I, the absolutist ruler, encounter increasing difficulties in maintaining his rule.

For a while, the king even tried to rule without Parliament, but this was not possible in the long run, as the resources for the fight against the rebellious Scots could only be released by Parliament. In this financial emergency, William's father was able to buy the title of *Viscount* of the Irish province of *Castle Lyons* in 1645 (whereby he ruined himself financially – as malicious tongues claimed). However, he only held this new title for two months, and then he died. William, as the elder of two sons, inherited the title and became the *Second Viscount of Castle Lyons*.

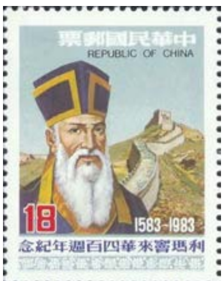
Six months later King Charles I abdicated and in 1649 he was executed and the *Lord Protector* Oliver Cromwell took over the reign.

William Brouncker, a royalist, lived in seclusion until the reintroduction of the monarchy in 1660 and devoted himself to a wide range of scientific studies. Although he had received his doctorate in medicine from the University of Oxford in 1647, he was mainly interested in mathematical questions.

In 1650 Brouncker's only book was published, but not under his name – the author was referred to as a *Person of Honour*. It was the English translation of a posthumously published work by René Descartes on musical theory, supplemented by his own extensive reflections on the construction of an appropriate scale of notes. Descartes wrote his treatise as early as 1618, but did not pursue the subject further.

Since ancient times, musicians and mathematicians had repeatedly dealt with the scales. In Pythagorean tuning, the 12 tones of the scale are defined by a series of pure fifths (frequency ratio $3:2$) - but 12 pure fifths do not correspond exactly to 7 octaves (frequency ratio $2:1$), because $(\frac{3}{2})^{12} \approx 129.7 \neq 2^7 = 128$.

Around 1605, Simon Stevin was probably the first European to realise that the semitone steps had to be defined using the constant factor $\sqrt[12]{2} \approx 1.0595$ of a geometric sequence in order to create an equal pitch. Whether Stevin discovered this himself or was inspired to do so by the Italian Jesuit missionary and mathematician Matteo Ricci (1552-1610) is not clear. In 1584 the Chinese scholar Zhu Zaiyu had calculated the semitone interval with 9-digit precision, and Ricci had mentioned this in a report.



MARIN MERSENNE, the head of the *Academia Parisiensis* and coordinator of the many letter contacts between European scholars, suggested $\sqrt[4]{\frac{2}{3-\sqrt{2}}} \approx 1.0597$ after reading DESCARTES' treatise as an approximate value for the constant factor $q = \sqrt[12]{2}$. BROUNCKER, in his supplementary treatise, defined the ratio by means of "artificial numbers", i.e. logarithms (invented by JOHN NAPIER in 1614). He also gave the factor $q = \sqrt[17]{\frac{2}{3-\sqrt{5}}} = \sqrt[17]{1+\varphi} \approx 1.0582$ for an interval of 17 semitones, since he assumed a connection with the golden number $\varphi = \frac{1}{2}\cdot(1+\sqrt{5})$.

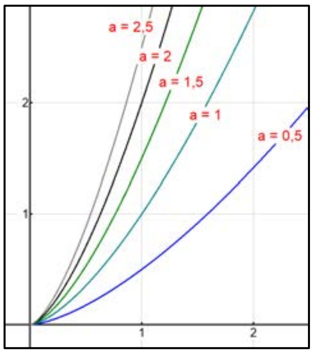
BROUNCKER established contact with JOHN WALLIS in the late 1640s. A group of scholars who deal with philosophical, scientific and medical issues met weekly at *Gresham College* in London. WALLIS, appointed professor of geometry at Oxford University under CROMWELL's government, published *De sectionibus conicis* in 1652, in which he described parabolas, ellipses and hyperbolas using coordinate equations. This work inspired BROUNCKER to take an approach which enabled him to determine the area under the graph of the displaced normal hyperbola.

He succeeded in showing that $\int_0^1 \frac{1}{1+x}dx = \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \ldots = (1-\frac{1}{2}) + (\frac{1}{3}-\frac{1}{4}) + (\frac{1}{5}-\frac{1}{6}) + \ldots = \ln(2)$

and 1655 from the product presentation found by WALLIS $\frac{4}{\pi} = \frac{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7\cdot\ldots}{2\cdot 4\cdot 4\cdot 6\cdot 6\cdot 6\cdot\ldots}$.

BROUNCKER developed a representation as a continued fraction: $\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \ldots}}}}$.

In 1659 BROUNCKER improved WILLIAM NEILE's method for determining the arc length of the algebraic curves, which are defined by $y^2 = a^2 \cdot x^3$ (so-called NEILE parabolas). The positive branches are shown in the graph on the right.



In 1660 BROUNCKER was elected to the *Constituent Parliament*, which voted for the reintroduction of the monarchy and elected the son of the executed king as CHARLES II. The latter knew BROUNCKER from the time when his father was in service of CHARLES I. The Viscount soon made a career at court, being appointed *Chancellor of Queen Anne* and *Keeper of the Great Seal*.

The Society for the Promoting of Physico-Mathematical Experimental Learning to promote experimental philosophy, founded at *Gresham College*, was confirmed by the King as the *Royal Society* in 1662 and BROUNCKER was elected its first President.

The unmarried scholar held this office with great commitment for many years. In 1677, when he attended meetings only irregularly, a motion was made to elect a new president, whereupon BROUNCKER left the meeting in indignation.

After his death, his universally unpopular brother HENRY inherited the assets and titles of nobility and after his death the title expired.

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https://www.spektrum.de/wissen/william-brouncker/1717448

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