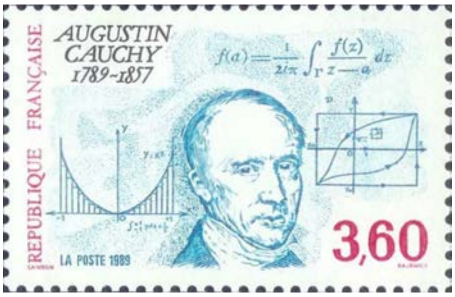
AUGUSTIN CAUCHY (August 8, 1789 – May 5, 1857)



by HEINZ KLAUS STRICK, Germany

Augustin LOUIS CAUCHY was born in Paris in troubled times – only a few weeks after the storming of the Bastille. His loyalist father, LOUIS FRANÇOIS, a police official until the revolution, soon lost his position and fled Paris with his family. He returned only after the end of the Reign of Terror. He continued his career under NAPOLEON and became general secretary of the Senate.

His acquaintance with Senator JOSEPH LOUIS LAGRANGE and the home secretary, PIERRE SIMON LAPLACE, led to a mutual friendship. These two friends recognized the unusual mathematical talent of the young AUGUSTIN. LAGRANGE recommended that the boy first attend a school that focused on the humanities, expressing the fear lest "He will become a great mathematician yet scarcely be able to write in his native language."

At the age of 15, AUGUSTIN CAUCHY entered the *École Polytechnique*, where he studied analysis under ANDRÉ MARIE AMPÈRE (1775–1836). Then at 18, he attended the *School for Bridges and Roads*. On completing his studies, he began work at the harbor of Cherbourg, which NAPOLEON was preparing for launching a flotilla to invade England.

In his scarce free time, he worked on mathematical problems, submitting his first scientific paper on a generalization of EULER's *theorem on polyhedra*. The enormous demands of work led to periods of depression. His health, which had never been good since his childhood, when he had suffered hunger, worsened. He applied unsuccessfully for positions at various academies in Paris and to university lectureships.

It was not until the end of NAPOLEON's reign that this rigid Catholic monarchist's path to a successful career became open. He became a lecturer, then a professor at the *École Polytechnique*. With an essay on wave theory he won a competition of the *Academy of Sciences*.

When Lazare Carnot (1753–1823) and Gaspard Monge (1746–1818) were dismissed from the *Academy* on political grounds, Cauchy was named (not elected) a member of that prestigious institution.

His mathematical publications had a great influence on the scientific world, but his brusque, arrogant, often insulting manner made collaboration with him difficult. For three years he allowed the groundbreaking papers on algebra submitted to the *Academy* by Niels Henrik Abel (1802–1829) to remain unopened, and ultimately rejected them (after Abel's untimely death).

Abel characterized Cauchy's dual nature thus: "Cauchy is mad, and there is nothing to be done about it, yet he is the only one among us who knows how mathematics should be done."

Cauchy gave lectures on analysis, defining in a stricter sense than heretofore the notion of limit and the conditions for the convergence of series, which made possible a precise definition of the integral. Already in 1814 he introduced functions of a complex variable, being the first to do so, and he gave conditions for their differentiability (the *Cauchy-Riemann differential equations*). He extended the concept of the integral over an interval in IR for paths in the complex plane, and derived theorems for computing integrals along such paths (*Cauchy integral theorem*).

Elaborations of his – pathbreaking – lectures were quickly translated into other languages, and they became standard works for decades.

His students were not enamoured of his efforts to make mathematical concepts precise. They were unaccustomed to such rigour, and considered his (frequently excursive) explanations too theoretical. Above all, his conservative political stance was alienating, as were his religious views, which he proclaimed with a missionary zeal.

The overthrow of the king in 1830 led Cauchy to emigrate; he refused to take an oath of loyalty to the new (citizen-)king Louis Philippe. In Turin, the king of Sardinia offered him a chair in theoretical physics. He then accepted a position in Prague as teacher of mathematics and physics to the grandson of the former king Charles X. For this – not very successful – service he was made a baron, a rank on which subsequently he placed great importance.

The extent to which he came into contact there with Bernard Bolzano, who also believed in a more rigorous approach to mathematical proof, is unclear. On his return to Paris he again became a member of the *Academy*. However, he was unable to attain any of the posts to which he applied.

The period from 1839 to 1848 was CAUCHY's most productive time. He published around 300 (out of a total oeuvre of 789) papers in the journal of the *Academy*, which led to a restriction on the number of articles that an individual could submit.

Following the *February Revolution* of 1848, the Bourbons, contrary to CAUCHY's hope, did not assume power, but rather NAPOLEON III. Nevertheless, CAUCHY obtained a professorship due to his undoubted academic competence, this time without the requirement of a loyalty oath.

His last years were marred by a priority dispute on which he dogmatically refused to admit that someone else could have had an idea before he did.

We have CAUCHY to thank for an abundance of mathematical theorems and criteria, for example the convergence criterion (which BOLZANO had found four years before him):

- **CAUCHY's convergence criterion:** A sequence $(a_n)_{n \in IN}$ is convergent if and only if for every $\varepsilon > 0$, there exists a number n_0 such that $|a_n - a_m| < \varepsilon$ for all $n, m \geq n_0$.

He proved the convergence of geometric series for $|q| < 1$ and from that derived the convergence criterion that today bears his name:

- If $(a_n)_{n \in IN}$ is a sequence of positive numbers and if there exists a number $0 < q < 1$ such that from a certain point on, $\frac{a_{n+1}}{a_n} \leq q < 1$ (**quotient criterion**) or $\sqrt[n]{a_n} \leq q < 1$ (**root criterion**), then the associated series (the sequence of partial sums) $\left(\sum_{k=0}^{n} a_k\right)_{n \in \mathrm{IN}}$ converges.

- **CAUCHY's condensation test:** The series $\left(\sum_{k=0}^{n} a_k\right)_{n \in \mathrm{IN}}$ converges if and only if the sum of $1 \cdot a_1, 2 \cdot a_2, 4 \cdot a_4, \ldots, 2k \cdot a_{2k}, \ldots$, i. e. $\left(\sum_{k=0}^{n} 2^k a_{2k}\right)_{n \in \mathrm{IN}}$ converges.

- **CAUCHY's product test:** If the series $\left(\sum_{k=0}^{n} a_k\right)_{n \in \mathrm{IN}}$ and $\left(\sum_{k=0}^{n} b_k\right)_{n \in \mathrm{IN}}$ are absolutely convergent to the limits A and B, then the series $\left(\sum_{k=0}^{n} (a_0 b_k + a_1 b_{k-1} + \ldots + a_k b_0)\right)_{n \in \mathrm{IN}}$ is absolutely convergent to the product A · B.

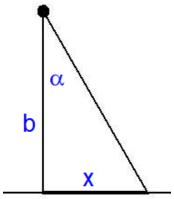
CAUCHY gave a rigorous proof of TAYLOR's theorem on the series development of multiply differentiable functions. He presented a method for determining the radius of convergence of such series as well as an estimate of the error incurred by truncating the series at the *n*th summand (*CAUCHY's remainder*).

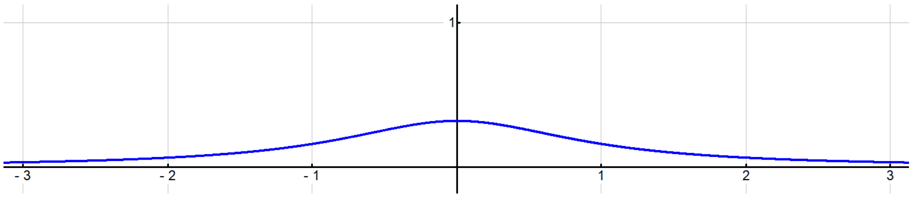
He showed that the geometric mean of *n* numbers is at most as great as the arithmetic mean
$\sqrt[n]{(a_1 \cdot a_2 \cdot \ldots \cdot a_n)} \leq \frac{1}{n} \cdot (a_1 + a_2 + \ldots + a_n)$

and he proved what is today known as the

- **CAUCHY-SCHWARZ inequality**: $(a_1 \cdot b_1 + a_2 \cdot b_2 + \ldots a_n \cdot b_n)^2 \leq (a_1{}^2 + a_2{}^2 + \ldots + a_n{}^2) \cdot (b_1{}^2 + b_2{}^2 + \ldots + b_n{}^2)$

Also bearing his name is the distribution of a random variable that describes the distance at which a ray intersects a horizontal line at distance $b = 1$ if the angle α is chosen at random between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$ (**Cauchy distribution**); the associated density function is $f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}$.





Moreover, he published a number of papers on the solution of functional equations:

What sorts of functions satisfy one of the following equations:

$f(x + y) = f(x) + f(y)$, $f(x + y) = f(x) \cdot f(y)$, $f(x \cdot y) = f(x) + f(y)$, $f(x \cdot y) = f(x) \cdot f(y)$?

as well as on questions in physics in such areas as the theory of elasticity and wave theory.

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https://www.spektrum.de/wissen/augustin-louis-cauchy-1789-1857/868068

Translated by David Kramer

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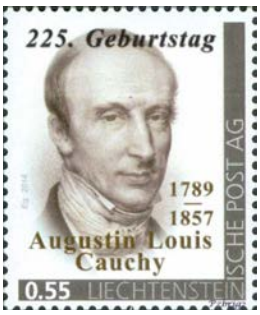
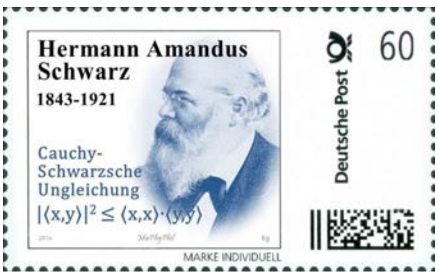
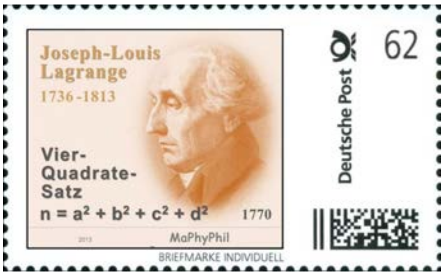
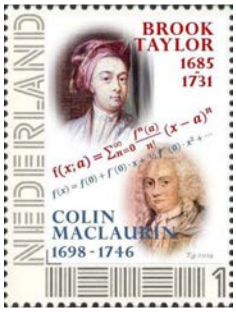
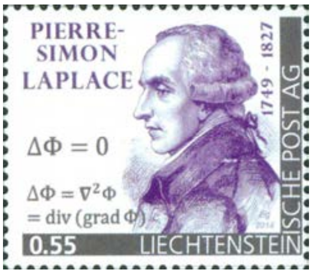
Here an important hint for philatelists who also like individual (not officially issued) stamps:











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