RENÉ DESCARTES (March 31, 1596 – February 11, 1650)

by HEINZ KLAUS STRICK, Germany

The French postage stamp from 1937 pictured here displays in the fore-ground the mathematician and philosopher RENÉ DESCARTES in front of his most famous work, *Discours de la méthode*.

In producing this stamp, somebody blundered: The artist misremembered the title of the work as *Discours sur la méthode*, and it was several days before the error was discovered. Because a relatively large number of “bad” stamps (shown to the left) were sold, these misprints are worth no more today than the “good” stamps. A similar lack of knowledge plagued the designer of the Albanian stamp pictured to the right. The Latinized spelling of the name, RENATUS CARTESIUS, was incorrectly transcribed.

RENÉ DESCARTES’s father was a jurist at the supreme court in Brittany. RENÉ’s mother died when her son was only a year old, and until he was eight, the boy lived with his grandmother. He then attended a Jesuit boarding school in La Flèche. On account of his poor health, he was granted the special privilege of remaining in bed until 11 o’clock in the morning, a habit that he observed for the rest of his life.

One of his schoolmates was MARIN MERSENNE (1588-1648), with whom he maintained a lifelong friendship. After completing legal studies, he served for a time in the military, taking part in the first campaigns of the Thirty Years’ War.

Then he began a restless period of travel in Europe. He journeyed through Bohemia, Hungary, Germany, the Low Countries, Italy, and then back to France, making contact with the greatest intellects of his time. In 1628, he settled in the Dutch Republic – hoping to have there greater freedom of expression – and began work on a book that was to have the title *Traité du monde* (The World); however, he abandoned the project after learning about the problems faced by GALILEO with the Inquisition.

Eventually, his friends convinced DESCARTES to publish his philosophical ideas. Finally, there appeared in 1637 – at first anonymously – his *Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences* (Discourse on the method of rightly conducting one’s reason and of seeking truth in the sciences) with three appendices: *La Dioptrique* (On refraction), *Les Météores* (On meteors) and *La Géométrie* (Geometry).

In 1641 his *Meditationes* appeared, with the famous sentence *Cogito, ergo sum* (I think, therefore I am), first in Latin, and later in French as well. The full title, in English, reads *Meditations on first philosophy, in which the existence of God and the immortality of the soul are demonstrated*.

Descartes's method of philosophical reasoning included the following precepts:

- *Accept nothing as true that admits of doubt.*
- *Break difficult problems into smaller subproblems.*
- *Begin with the simple and progress to the difficult.*
- *Determine whether one's investigation is complete.*

He was convinced that all natural phenomena admit of a rational formulation and explanation. For example, he gives a correct explanation for the appearance of rainbows. Descartes was the first to formulate the *law of conservation of momentum*; he explained the origin of the solar system as resulting from a vortex of matter being set in motion by God, from which arose the Sun, the planets, and the comets. His theories, however, allow for no interactions without immediate material contact, which rules out the possibility of magnetism and gravitation.

Descartes's approach stands in contrast to the previously uncontested worldview of Aristotle. His physical theories, however, were largely speculative and were only gradually superseded by Newtonian physics, which was based on the scientific method of observation and scientific deduction.

In 1649, Descartes accepted an invitation from Queen Christina of Sweden to move to Stockholm, where he was forced to abandon his custom of remaining late in bed, since the queen expected him to appear at her five o'clock breakfast to discuss mathematical and philosophical problems.

Descartes did not survive his first Nordic winter; from his early-morning walks with the queen he came down with a lung infection, and he died soon thereafter.

With the work *La Géométrie*, a new branch of mathematics was born: *analytic geometry*. Descartes showed that algebraic equations can be solved with geometric constructions and that geometric objects can be described in terms of algebraic equations. Even though he did not employ the coordinate system that is called "Cartesian" in his honor, his methods revolutionized mathematics: Geometry and algebra support each other, and since Descartes, the one has been inseparable from the other.

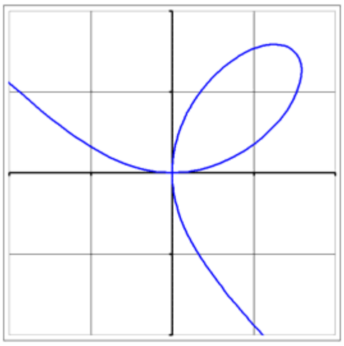
Descartes was the first mathematician to use consistently the signs + and –, as well as exponential notation and the square root sign $\sqrt{\ }$, and to use the last letters of the alphabet systematically to name variables.

In addition to the rectangular coordinate system, there is also a curve that bears his name: *the folium of Descartes*.

The equation of the curve in Cartesian coordinates is $x^3 + y^3 = 3ax$.

In parametric form, the equation becomes

$x = \frac{3at}{1+t^3}$ and $y = \frac{3at^2}{1+t^3}$.



The left branch of the curve arises as t runs from −1 to 0;
the loop is the graph for values of t between 0 and +∞;
and the right branch results as t ranges between −∞ and −1.

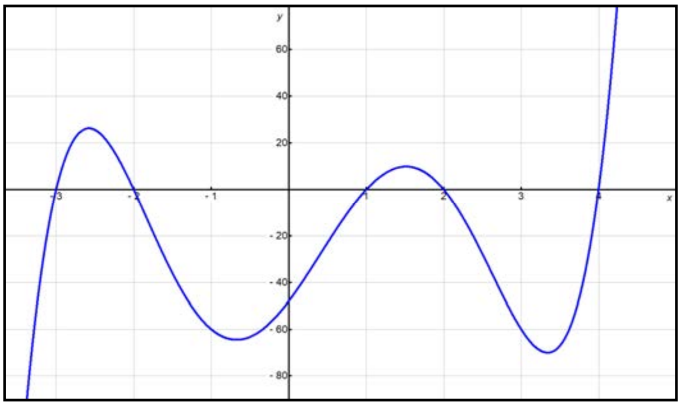
The parametric representation is not defined for $t = -1$.

(see also the Albanian stamp above)

DESCARTES also discovered a relationship between the number of real zeros of a polynomial and the signs of its coefficients:

- **Descartes' rule of signs:** The number of positive zeros is less than (by a multiple of 2) or equal to the number of changes in sign in the sequence of coefficients. If all the zeros are real, then equality holds.

As an example, consider the polynomial $f(x) = x^5 - 2x^4 - 15x^3 + 20x^2 + 44x - 48$



The sequence of coefficients

+1, −2, −15, +20, +44, −48

has three sign changes; hence by DESCARTES's sign rule, $f(x)$ has at most three positive zeros.

To find the negative zeros, one considers the polynomial $f(-x)$. In this case the sequence of coefficients of the polynomial

$f(-x) = -x^5 - 2x^4 + 15x^3 + 20x^2 - 44x - 48$

has two sign changes, and thus the original polynomial has at most two negative zeros.

DESCARTES also discovered

- **DESCARTES' four-circle theorem on APOLLONIAN circles:** What is the radius r_4 of a circle that is tangent to three mutually tangent circles of radii r_1, r_2, r_3?

DESCARTES found the simple relationship: $2 \cdot (k_1{}^2 + k_2{}^2 + k_3{}^2 + k_4{}^2) = (k_1 + k_2 + k_3 + k_4)^2$

where k_i (i = 1, 2, 3, 4) is the curvature of the ith circle (with a negative sign for k_4 if the fourth circle is externally tangent to the other three circles).

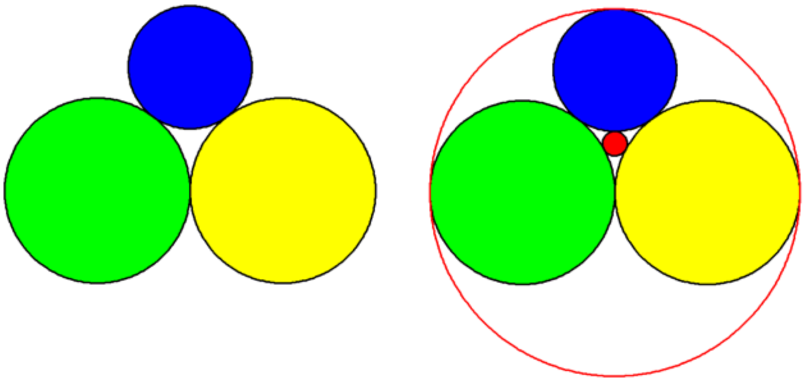
To determine $r_4 = \frac{1}{k_4}$, it suffices simply to solve a quadratic equation in the variable k_4!

Example: $r_1 = 15$ (*green*); $r_2 = 15$ (*yellow*); $r_3 = 10$ (*blue*)

$$\left[x - \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{10}\right)\right]^2 = 2 \cdot \left[\left(\frac{1}{15} + \frac{1}{15} + \frac{1}{10}\right)^2 - \left(\frac{1}{225} + \frac{1}{225} + \frac{1}{100}\right)\right]$$

$$\Leftrightarrow \left[x - \frac{7}{30}\right]^2 = 2 \cdot \left[\left(\frac{7}{30}\right)^2 - \frac{17}{900}\right] \Leftrightarrow \left(x - \frac{7}{30}\right)^2 = \frac{64}{900} \Leftrightarrow x = \frac{1}{2} \quad \vee \quad x = -\frac{1}{30}$$

This means: $r_4 = 2$ (interior circle, *red*) or $r_4 = 30$ (exterior circle, *red line*)



(Graphics of the *kissing circles* from Strick, H. K. (2017): Mathematik ist schön, Springer, Heidelberg)

First published 2006 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

www.spektrum.de/wissen/rene-descartes-1596-1650/862794

Translated by David Kramer

English version first published by the European Mathematical Society 2011
