

## GOTTHOLD EISENSTEIN (April 4, 1823 – October 11, 1852)

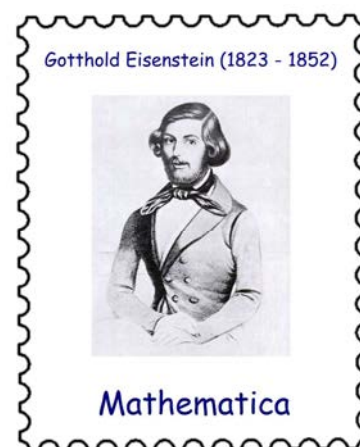
by HEINZ KLAUS STRICK, Germany

The mathematics historian MORITZ CANTOR reported that in 1877, shortly before his death, CARL FRIEDRICH GAUSS said that there had only been three epoch-making mathematicians:

ARCHIMEDES, NEWTON and EISENSTEIN.

We cannot be sure that GAUSS really said this, but it can be concluded from many other statements of the *Princeps Mathematicorum* that he had a high opinion of EISENSTEIN.

This brilliant mathematician died when he was only 29 years old and one can only guess what achievements he could have accomplished if he had not died so early.



Even as a child, GOTTHOLD EISENSTEIN was often ill. The oldest son of a Berlin merchant barely survived a meningitis infection, but remained susceptible to illness for the rest of his short life. All five siblings born after him died at an early age, four of them from meningitis. His consistently poor physical condition prevented him from playing in the streets with children of the same age. His mother anxiously looked after his well-being, teaching him the letters of the alphabet when he was only two years old. The arithmetic lessons in the first years of school bored him, because he could not understand why, for example, multiplication had to be practised for days. In contrast, he liked anything that challenged his logical thinking. Because of his state of health, he was sent to the countryside for a few months, which, however, set him back in his school development.

At the age of ten, EISENSTEIN moved to a boarding school in rural Charlottenburg (which only became part of Berlin from 1920). The military drill at school and the Spartan way of life did not contribute to improving his health, but rather led to the first signs of depression. Later EISENSTEIN remembered in retrospect and in gratitude one of his teachers, because he knew how to awaken his interest in mathematics.

In 1837, EISENSTEIN transferred to the Friedrich-Wilhelm-Gymnasium in Berlin; his mathematics teacher SCHELLBACH recognised his special talent; he encouraged the 14-year-old and persuaded him to study the works of EULER and LAGRANGE on differential and integral calculus independently. EISENSTEIN then moved to the Friedrich-Werder-Gymnasium and at the same time attended lectures by PETER GUSTAV LEJENUE DIRICHLET at Berlin University.



(drawing © Andreas Strick)

However, before he could complete his schooling, the family moved to England. His father, who had had little success with his professional activities in Berlin so far, wanted to try his luck in England. In their vain search for a suitable job, the three also travelled through Wales and Ireland.

GOTTHOLD EISENSTEIN delved into a French translation of the GAUSS's *Disquisitiones arithmeticae* and began his own investigations.

In Dublin he met WILLIAM R HAMILTON, who asked him to submit a treatise written by him to the Berlin Academy: *On ABEL's proof of the impossibility of solving general equations higher than the fourth degree*.



In June 1843 EISENSTEIN returned to Berlin with his mother and applied for the right to take the school leaving examination as an external student. For admission, he wrote an extensive *curriculum vitae*, in which he described, among other things, how his occupation with the writings of GAUSS, JACOBI and DIRICHLET led to his desire to study mathematics.

In September he passed the examination. One of his examiners was his former maths teacher SCHELLBACH, who recorded this in his school leaving certificate:

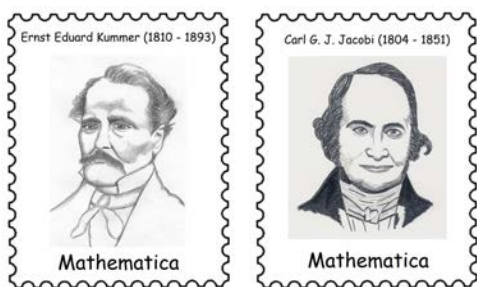
*His knowledge of mathematics goes far beyond the scope of the secondary school curriculum. His talent and zeal lead one to expect that some day he will make an important contribution to the development and expansion of science.*

From October onwards, EISENSTEIN attended lectures at the University of Berlin. In January 1844 he submitted HAMILTON's treatise to the *Academy of Sciences* together with his own paper: *The cubic forms with two variables*. This was immediately published in CRELLE's Journal (*Journal für die reine und angewandte Mathematik*). Other treatises by EISENSTEIN followed; in 1844 alone, a total of 25 contributions were printed in CRELLE's Journal.

Through AUGUST LEOPOLD CRELLE, a first contact was also established with ALEXANDER VON HUMBOLDT, who from then on was constantly concerned with the personal well-being and financial security of the highly talented young scientist. EISENSTEIN travelled to Göttingen with a letter of recommendation from HUMBOLDT to GAUSS.

GAUSS was delighted to meet the 21-year-old personally, as he had already read some of his writings before his arrival.





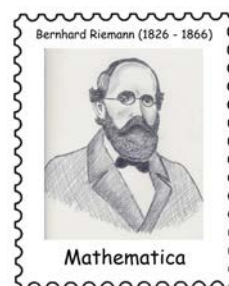
At the beginning of February 1845, ERNST EDUARD KUMMER and CARL GUSTAV JAKOB JACOBI proposed to the University of Breslau that EISENSTEIN be awarded an honorary doctorate. In the same month the 3rd semester student was awarded a doctorate as *Dr. phil. honoris causa* !

In the summer of 1845 EISENSTEIN's physical and mental health deteriorated dramatically. When he was accused of plagiarism by JACOBI at the beginning of 1846, things got even worse for EISENSTEIN: One year after his own proposal to award EISENSTEIN a doctorate, the influential JACOBI informed the Prussian Minister of Education that he considered EISENSTEIN to be not very suitable for an academic career because of his *scientific recklessness*. (In a paper on elliptical functions, EISENSTEIN had not explicitly emphasised which findings came from JACOBI, in contrast to other papers in which he had described in detail the advanced work of GAUSS.)

GAUSS, who continued to regard EISENSTEIN's talent as a special one, *which nature gave to only a few in every century*, tried to correct EISENSTEIN's damaged reputation in the academic world.

At the end of 1846 HUMBOLDT used all his influence to free EISENSTEIN from the threat of military service.

In May 1847 EISENSTEIN habilitated at the University of Berlin, which under the current regulations would not have been possible until 1849 at the earliest. As a private lecturer, he began lecturing in the summer semester and one of his first students was BERNHARD RIEMANN.



In September, his *Mathematical treatises especially in the field of higher arithmetic and elliptic functions*, was published and GAUSS wrote a foreword for it.

In March 1848 EISENSTEIN by chance was involved in a street fight during the so called *Spring Revolutions*. After his arrest he was maltreated in the *Spandau Citadel*. Although he was released the following day, the state support granted to him was reduced from 500 to 300 thalers a year (from 1850 on, it was increased to 400 thalers again due to the efforts of HUMBOLDT) because of false reports that he was a follower of the Republicans.

EISENSTEIN's state of health deteriorated and he also suffered from the fact that he could not earn his own living and depended on the goodwill of others. On some days he had to lecture from his sickbed (which was possible due to the low number of students in the mathematics department).

His numerous new publications were becoming increasingly erratic in their thought processes - from today's perspective, one might suspect that he felt that he did not have much time left.

In January 1851 he was elected - on the proposal of DIRICHLET and JACOBI (!) – as a member of the *Berlin Academy*. In fact, the election had no effect, because three candidates were elected, but only two posts were vacant. He was not appointed until one year later. At GAUSS's suggestion, he was elected as a corresponding member of the *Göttingen Academy* (together with KUMMER).

In the summer semester of 1851 he was unable to give his lectures. A cure (his third) was unsuccessful. In July 1852 he suffered a haemorrhage due to tuberculosis.

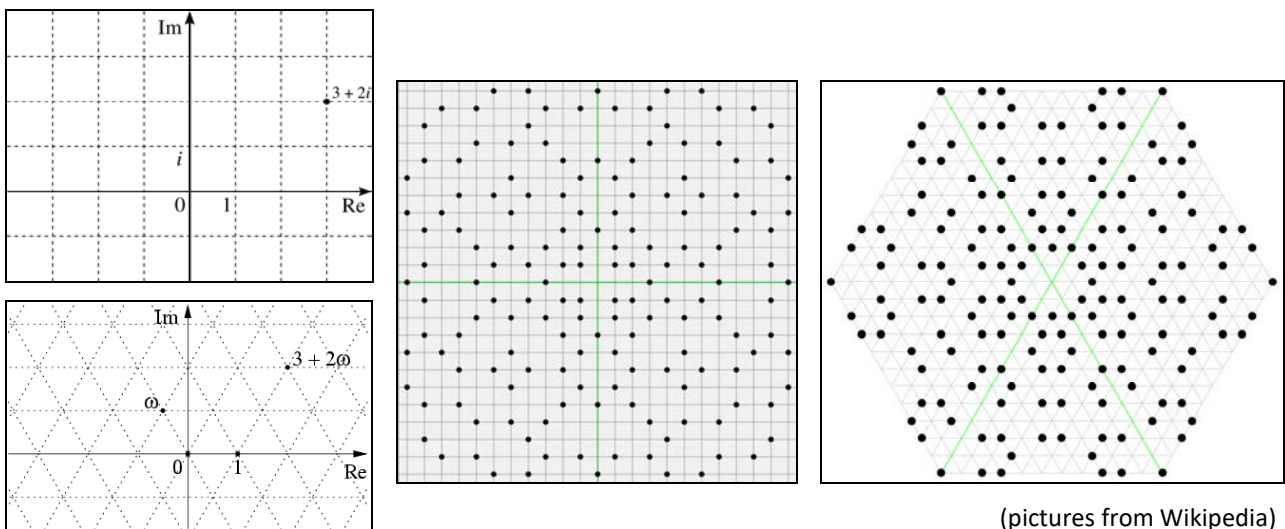
HUMBOLDT tried to obtain a grant for a one-year cure in Sicily. However, EISENSTEIN was no longer able to make the journey and he died on 11 October, alone and abandoned.

EISENSTEIN's first publications resulted from his preoccupation with the *Disquisitiones arithmeticae*. He succeeded in generalising some of the discoveries of his great model GAUSS or in proving presumed generalisations, for example the generalisation from the quadratic to the cubic and biquadratic reciprocity laws or findings in the investigation of cubic and biquadratic forms.

The name EISENSTEIN is associated with various concepts of mathematics:

- *EISENSTEIN numbers* are complex numbers of the form  $E = a + b \cdot \omega$ , where  $a, b \in \mathbb{Z}$  and  $\omega = -\frac{1}{2} + \frac{1}{2} \cdot \sqrt{3} \cdot i$ . ( $\omega$  is one of the solutions of the so-called circle division equation  $z^6 = 1$ .)

In GAUSS's number plane (see fig. top left) these numbers form a triangular grid (see fig. bottom left). In higher algebra the term *prime number* in the set of natural numbers can be generalised to the term *prime elements* for other sets of numbers. The GAUSSIAN integers are complex numbers of the form  $G = a + b \cdot i$  with  $a, b \in \mathbb{Z}$ . In the middle figure some prime elements of these are marked. In the figure on the right are some prime elements of the EISENSTEIN number.



- The *EISENSTEIN criterion* in algebra allows statements regarding the decomposability of polynomials with integer coefficients  $a_0, a_1, \dots, a_{n-1}, a_n$ :  
 For a polynomial  $P(x) = a_n x^n + \dots + a_1 x + a_0$  one checks whether there is a prime number  $p$  with the following properties:
  - (1)  $p$  is a divisor of the coefficients  $a_0, a_1, \dots, a_{n-1}$ ,
  - (2)  $p^2$  is not a divisor of the coefficient  $a_0$ ,
  - (3)  $p$  is not a divisor of the coefficient  $a_n$ .

If these three conditions are met, the polynomial cannot be broken down into factors with rational coefficients. (If one of the conditions is not fulfilled, no statement can be made without further investigation.)

*Example:* The polynomial  $P(x) = x^3 - 3x^2 - 6x + 12$  has no rational zero.

LEOPOLD KRONECKER, with whom EISENSTEIN became friends during his short stay at the University of Berlin, continued his approach to elliptic functions after EISENSTEIN's death with the help of (what are now called) *EISENSTEIN series*.

First published 2017 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

<https://www.spektrum.de/wissen/gotthold-eisenstein-1823-1852/1515229>

Translated 2020 by John O'Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at [europablocks@web.de](mailto:europablocks@web.de) with the note: "Mathstamps".

