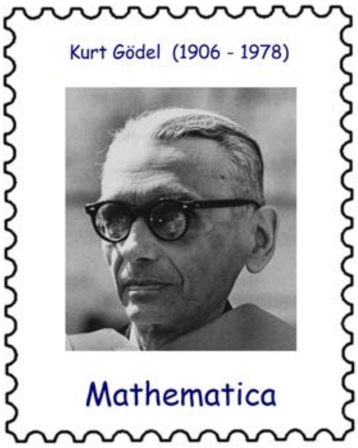
KURT GÖDEL (April 28, 1906 – January 14, 1978)

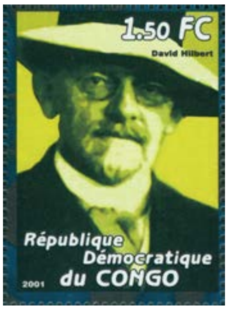
by HEINZ KLAUS STRICK, Germany



In August 1900, DAVID HILBERT, President of the *German Association of Mathematicians* (DMV), presented at the Paris International Congress a list of 23 unsolved problems in mathematics that he believed would influence research in the coming century. HILBERT was convinced that any mathematical problem could be solved:

> *There is the problem, look for the solution. You should be able to find it by pure reason; because there is no ambiguity in mathematics.*

Three decades after this memorable speech, HILBERT's optimism was dampened by KURT GÖDEL, a young doctoral student at the University of Vienna. In the second of the 23 problems, HILBERT had expressed the hope that contradiction-free axiomatization of arithmetic would soon succeed. The Italian mathematician GIUSEPPE PEANO had published an axiom system for arithmetic in 1889, the freedom from contradiction of which had not yet been clarified in 1900.



In 1928, HILBERT himself presented a logic calculus of formal closure at a congress in Bologna, which he assumed would fulfill the condition of freedom from contradictions. In 1930, however, KURT GÖDEL was able to write his treatise *Formally undecidable sentences of the Principia Mathematica and related systems* to show that the proof of the absence of contradictions in a theory cannot be done with the means of the theory alone.

The importance of this knowledge for the development of mathematics was taken up by the Portuguese Post Office in 2000, with a photograph of KURT GÖDEL along with HENRI POINCARÉ and ANDREI N KOLMOGOROV, on the millennium stamps issued with portraits of outstanding personalities who had a particularly strong influence on science in the 20th century.

KURT GÖDEL was born the son of a co-owner of a textile factory in Brünn (now Brno in the Czech Republic), the centre of the Moravian province in the Austro-Hungarian Empire, and had a happy childhood there. At the age of six he developed rheumatic fever after a streptococcal infection, but soon recovered. However, this condition had a lifelong effect on him. When KURT was eight years old, he began reading medical books about the symptoms that such an illness can have, and from then on he was hypochondriacally convinced that he was suffering from heart failure.

Before the First World War, the city of Brünn still had a predominantly German-speaking population. After 1918, with the founding of the Czechoslovak Republic, this changed, but the city remained bilingual. Like his brother RUDOLF, who was four years older, KURT GÖDEL attended German-speaking schools.

During his school days his curiosity in all sorts of areas earned him the nickname "The Lord Why". In 1923 he passed the secondary school exam with high marks. His achievements in Latin and mathematics in particular were impressive.

RUDOLF GÖDEL later reported that his brother did not make a single grammatical mistake in Latin throughout the school and that during his time there he had already dealt with mathematical topics that were only taught at university.

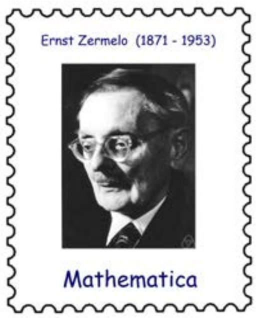
Kurt Gödel, who had now acquired Austrian citizenship, followed his brother to Vienna, where he studied medicine. After starting with the intention of studying physics he was so fascinated by mathematics lectures of Philipp Furtwängler and Hans Hahn that he changed his focus.

Gödel received important inspiration for his later work from the *Vienna Circle* (as it was later called), in whose discussions about the philosophical foundations of science he was allowed to participate as a student. This discussion group, which Hans Hahn and the philosopher Moritz Schlick had founded, also included personalities such as the later philosophy professor Rudolf Carnap and the mathematicians Richard von Mises and Karl Menger.

In 1929 Gödel did his doctorate with Hans Hahn with the thesis *The Completeness of the First Order Functional Calculus*. Here he showed that statements could be proven on the basis of an axiom system.

In the next step, Gödel then dealt with the approach of Bertrand Russell and Alfred North Whitehead, who in their three-volume work *Principia Mathematica* tried to derive mathematical sentences that were not axioms by logical reasoning. Gödel then came to the shocking conclusion that no matter how many axioms you add to the system, there will always be more true statements that cannot be proven inside the system. Gödel submitted his treatise (which is referred to today as *Gödel's incompleteness theorem*) as a *habilitation* thesis. This meant that from March 1933 he was entitled to give lectures at the University of Vienna as a private lecturer. Due to his sensational discovery, he was invited to give guest lectures at the newly founded *Institute for Advanced Study* in Princeton (New Jersey), but had to cut short his visit there due to health reasons. During several months of treatment in a sanatorium, Gödel recovered enough that he was able to return to Princeton in the autumn of 1935. However, the next breakdown occurred a month later. Because of his hypochondria, he regularly took pills for his supposed heart weakness, kept a constant record of his own body temperature and digestion and only consumed small amounts of food to protect his stomach. This in turn affected his physical strength.

Gödel hardly took notice of the political changes in Austria (the abolition of democratic structures by Federal Chancellor Engelbert Dollfuss – Austrofascism). However, when the philospher Moritz Schlick was murdered by one of his former doctoral students in 1936, Gödel was shocked and was unable to work for months. (In the trial, the assassin justified himself with ideological arguments. In the National Socialist press, the murderer was portrayed as the actual victim and after Austria's annexation, he was released on parole.)



Gödel lectured in Vienna in 1937 and in Göttingen in the summer semester of 1938. He spent the academic year 1938/39 again in Princeton and Indiana. Here he published *The Consistency of the Axiom of Choice and of the generalized Continuum-Hypothesis with the Axioms of Set Theory*. According to this, if the axiom system of Ernst Zermelo and Abraham Fraenkel supplemented by the axiom of choice is free of contradictions (which most mathematicians assume, but which cannot be proven inside the system according to the incompleteness theorem), then the extended system in which in addition, the continuum hypothesis is also accepted as an axiom, is itself without contradictions.

It was only in the 1960s that Paul Cohen was able to conclude that the continuum hypothesis cannot be deduced from the axioms of set theory, so that it (or the negation of the continuum hypothesis) can be added as an additional axiom. Cohen received the *Fields Medal* for his proof.

- **Continuum hypothesis** (after GEORG CANTOR, 1878): *There is no set whose cardinality lies between the cardinality of natural numbers and the cardinality of real numbers.*
- **Axiom of choice** (after ERNST ZERMELO, 1904): *For every set of non-empty sets there is a function by which an element can be selected from each of these sets.*

Shortly before GÖDEL went to the United States again, he married ADELE PORKERT. He had had a relationship with this woman who was six years older than him since the late 1920s. The relationship with a divorced woman who had made a living as a dancer in a nightclub had been disapproved by his parents, so he kept this secret for a long time. In the years until GODEL's death, his wife ADELE sacrificed herself to take care of her husband. When GÖDEL developed the delusion that someone was trying to poison him, she had to taste all his food before he would eat anything.

In contrast to other private lecturers, GÖDEL's right to give lectures at Vienna University was treated hesitantly by the new university administration after Austria's annexation to the German Reich – possibly because he was "suspected" of being Jewish because of his numerous Jewish acquaintances. Instead, he received a certificate of approval and, despite his poor constitution, he was classified as fit for the military.

Only then did he realize the seriousness of his situation. Almost at the last minute (through the mediation of JOHN VON NEUMANN) he received a visa for himself and his wife to go the USA and the Nazi authorities gave them permission to leave the country. After an arduous journey on the Trans-Siberian Railway and a passage from Japan to San Francisco, he arrived in Princeton in mid-March 1940, where he first received annual contracts as a lecturer, then a permanent position until 1953. Afterwards he held a professorship without lecturing until the end of his life.

In 1947 GÖDEL was granted American citizenship. At the hearing, in which he was supposed to demonstrate knowledge of the country and the constitution, he notified the judge responsible that there were contradictions in the American constitution – but thanks to the forbearance of the judge and the help of GÖDEL's sponsors (ALBERT EINSTEIN and OSKAR MORGENSTERN), the interview was decided in his favour.

A close friendship developed with EINSTEIN, until his death in 1955. EINSTEIN picked up GÖDEL for a long walk as often as possible, which helped to stabilise him.

On Einstein's 70th birthday in 1949, Gödel presented a previously unknown solution to Einstein's equations for the theory of gravitation. This Gödel universe was a rotating system with a period of 70 million years: every journey in space is a journey through time. After the death of Einstein and von Neumann (1957) Gödel withdrew more and more. His last paper appeared in 1958.

When Adele Gödel suffered a stroke in 1977 and was hospitalised for six months, Gödel lost all hold on reason. Since there was no longer anyone to taste his food, he ate almost nothing. After six months, he weighed only 30 kg and was admitted to hospital too late. A few days later he died of malnutrition. During his lifetime, Gödel was honoured with numerous honorary memberships, but he would not accept honours from his old homeland Austria.

Addendum:

Gödel's incompleteness theorem also had consequences with regard to an important issue in theoretical computer science: the so-called *decision problem* : Is there an algorithm by which it can be decided for each (sufficiently formalized) statement of mathematics whether it is true or false? If there is such an algorithm that can do this, the easiest way to prove its existence is to state it. But how can you prove that there is no such algorithm? And what should such a proof look like, which is itself an algorithm?

In 1936 Alan Turing's article *On Computable Numbers with an application to the decision problem* appeared, in which he described a simple abstract calculator (the so-called *Turing machine*), which follows a finite set of fixed rules.

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https://www.spektrum.de/wissen/kurt-goedel-1906-1978/1527579

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