HERMANN GRASSMANN (April 4, 1809 – September 26, 1877)

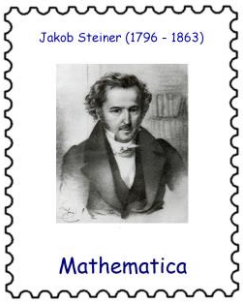
by HEINZ KLAUS STRICK, Germany

Today he is regarded as one of the most important mathematicians of the 19th century; during his lifetime, essentially only his contributions to the study of Sanskrit, the ancient Indian language, were recognised.

HERMANN GÜNTHER GRASSMANN was born in Stettin, the third of twelve children of a grammar school teacher of mathematics and physics. During his schooling he did not stand out for his exceptional achievements and at times his father even considered letting him learn a trade.

Towards the end of his school years, however, the young man blossomed. As the second best pupil in his year, HERMANN left school to study theology in Berlin (influenced by his maternal grandfather, who was a Protestant pastor). He also attended lectures in philosophy and the ancient languages – mathematics and physics played no part.

In the autumn of 1830 he returned home and he did not pursue possible thoughts of working as a pastor at first. Instead, he decided to pursue a profession as a teacher of mathematics and physics at the Gymnasium. For a year he prepared for the exams in Berlin – without ever having heard lectures in the subjects before. His level of knowledge did not seem to have satisfied the examination board – he only received a teaching permit for the lower classes and the option to repeat the examination.



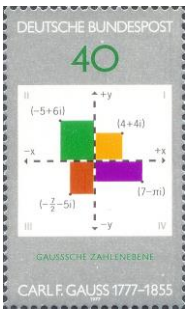
From spring 1832 onwards, he worked as an assistant teacher at the Gymnasium in Stettin. In 1834 he passed the first stage of theology examinations for prospective pastors at the Council of the Lutheran Church in Stettin, but then decided to take up a post in Berlin as a mathematics teacher at the *Gewerbeschule* (a kind of secondary school) previously held by JACOB STEINER. After a year he returned to work at another grammar school in Stettin and at the same time prepared for the repeat examination in Berlin.

In 1839 he passed the second stage of the theology examinations, and in 1840 the teacher's examination, which finally allowed him to teach mathematics, physics, chemistry and mineralogy in all classes of the grammar school. As part of the teacher's examination, he had to submit a written paper. The topic he was given was the presentation of a theory of the tides.

In his 200-page treatise *Theory of the tides*, he presented the basic principles as developed by PIERRE- SIMON LAPLACE in *Méchanique céleste* and JOSEPH-LOUIS LAGRANGE in *Méchanique analytique*, but he used a representation with the aid of vectors, which allowed a simpler approach. Although the written work was accepted by the examination committee, none of the members recognised the genius of the novel approach. Until then, the use of vectors had only been common in physics, e.g. for the description of forces and velocities, and also for vector fields such as electric and magnetic fields, but without an exact definition of the vector concept.

Furthermore, JEAN ROBERT ARGAND and CARL FRIEDRICH GAUSS had recognised that complex numbers could be understood as vectors and that the addition of these numbers could be illustrated by placing arrows next to each other.

GRASSMANN himself did not use the term *vector* because of its association with physical processes; instead he used the term *extensive quantity*.



From 1832 onwards, he developed his vector calculus, in which an *n*-dimensional space is spanned by a system of units (*basis vectors*) that are linearly independent and with the help of which one can represent any other vector by linear combination.

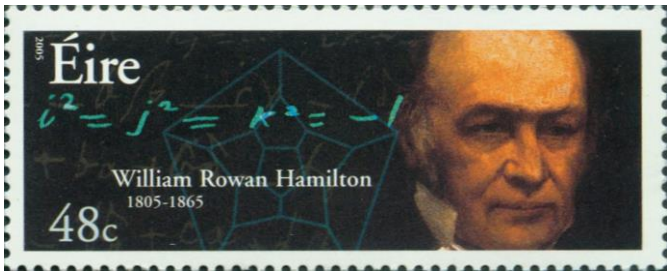
1844 saw the publication of his work *Die Wissenschaft der extensiven Größe oder Die Ausdehnungslehre, eine neue mathematische Disciplin* [The Science of Extensive Size or the Theory of Extension, a New Mathematical Discipline]. In this work, GRASSMANN developed an algebraic theory for geometric objects between which combinations could be defined (addition and various versions of multiplication).

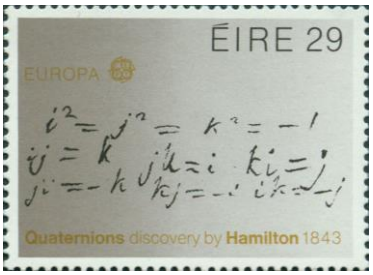
He formulated and proved a theorem which hardly differed from the theorem which today is called *STEINITZ's exchange theorem* – after the German mathematician ERNST STEINITZ (1871-1928). He also gave the procedure by which the *coordinates* of points could be calculated when the basis of the vector space was changed.

GRASSMANN also dealt with subspaces; he proved the dimensional formula for the union of two subspaces *U* and *V*, which today is expressed as follows:

$$\dim(U \cup V) = \dim(U) + \dim(V) - \dim(U \cap V).$$

The book found no buyers. This was not only due to the extensive philosophical prefatory notes but also because of the elaborate formulae. Above all, it was the unusual degree of abstractness of the treatise, a book of geometric objects, in which these only occurred in abstract form.

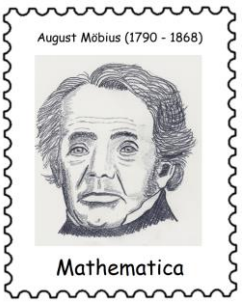




With the exception of WILLIAM ROWAN HAMILTON, who enthusiastically mentioned GRASSMANN's approach in his Lectures on Quaternions (1853), the book met with incomprehension. HAMILTON had worked intensively on the question of whether operations similar to those in the set of complex numbers (which can be understood as 2-dimensional objects) could also be defined in the 3-dimensional space.

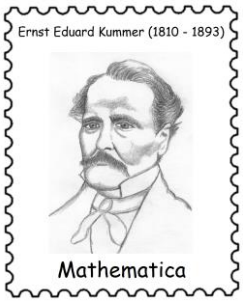
GRASSMANN's request to AUGUST MÖBIUS to review his book was rejected by the latter, as even he did not recognise the importance of the work.

Nevertheless, in 1846, as a member of a jury, he recommended that GRASSMANN be awarded a prize by the *Princely JABLONOWSKI Society* for his treatise *Geometrical Analysis Linked to the Geometric Characteristics Invented by LEIBNIZ* (possibly also because GRASSMANN made a reference to MÖBIUS's barycentric calculus).



(drawings © Andreas Strick)

In 1847 GRASSMANN, by then at the Friedrich-Wilhelms-Schule in Stettin, was appointed senior teacher. In the same month he addressed himself to the Prussian Ministry of Education with the request for a Transfer to the university.



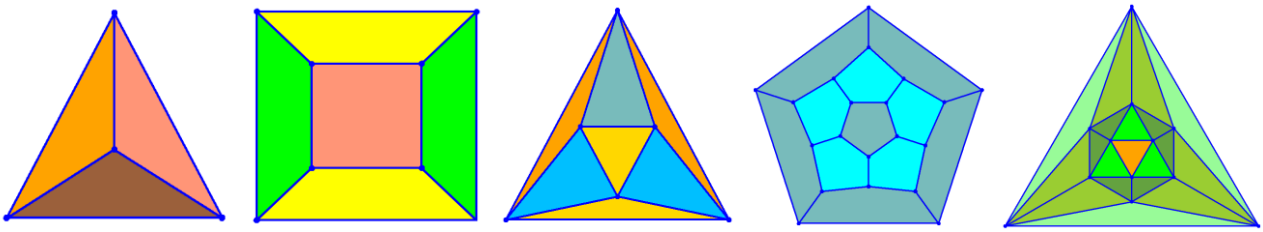
EDUARD KUMMER, who was asked to comment, appreciated the new approach of the applicant, but criticised the quality of the presentations. In addition, he stated that GRASSMANN's professional aptitude was probably limited. This virtually ruled out a university career in the future.

When GRASSMANN's father JUSTUS died in 1852, his son HERMANN was appointed his successor, so at least he had reached the rank of *Gymnasium professor*. But his theory of extensions had still not found recognition, and he decided to re-edit the work. In 1862, the heavily modified version *Die Ausdehnungslehre – Vollständig und in strenger Form bearbeitet* [The theory of extension – Completely and rigorously edited] was published – but with as little success as the 1844 version.

Disappointed that he was unable to convince the mathematicians of his time of the importance of his ideas, he immersed himself in linguistic studies. Years earlier, he had studied the structure of the language of the Goths and Sanskrit. He now resumed these studies and published several papers, including a comparison of grammars and a dictionary of the Vedic language, which led to an honorary membership in the *American Oriental Society* and an honorary doctorate from the University of Tübingen.

GRASSMANN died of heart failure at the age of 68. In the last years of his life, the first acknowledgements of his mathematical achievements were made, including his appointment as a corresponding member of the Göttingen *Society of Sciences and Humanities*, but it was only after his death that the profound approaches in GRASSMANN's theory of extension were properly understood.

Just one year after GRASSMANN's death, a biography was published with a respectful account of his scientific achievements, written by VICTOR SCHLEGEL, who had met GRASSMANN personally, as he also had worked temporarily as a teacher at the Stettin Gymnasium. (SCHLEGEL is known for the SCHLEGEL diagrams named after him, which are 2-dimensional projections of 3-dimensional bodies)



SCHLEGEL diagrams of the Platonic Solids

GRASSMANN's marriage in 1849 produced eleven children and three of his sons studied mathematics: two of them became teachers at "his" Stettin Gymnasium, and one became a professor at the University of Giessen.

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