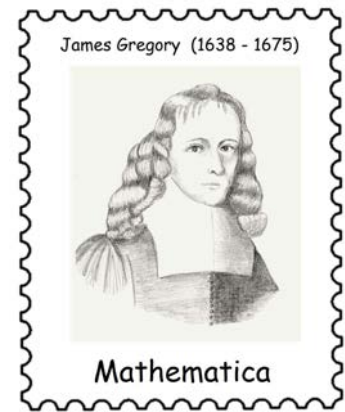


JAMES GREGORY (November 1638 – October 1675)

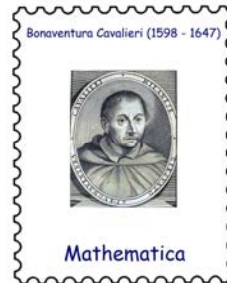
by HEINZ KLAUS STRICK, Germany

The Scottish mathematician JAMES GREGORY (sometimes also written GREGORIE) owed his talent for mathematics to his mother rather than his father, who worked as a pastor at Drumoak (near Aberdeen). His mother's brother was one of FRANÇOIS VIÈTE's students and, after his death, the editor of his writings. GREGORY's mother taught the boy geometry, and he had no problems working through the *Elements* of EUCLID. After attending Grammar School, he moved to a college in Aberdeen.

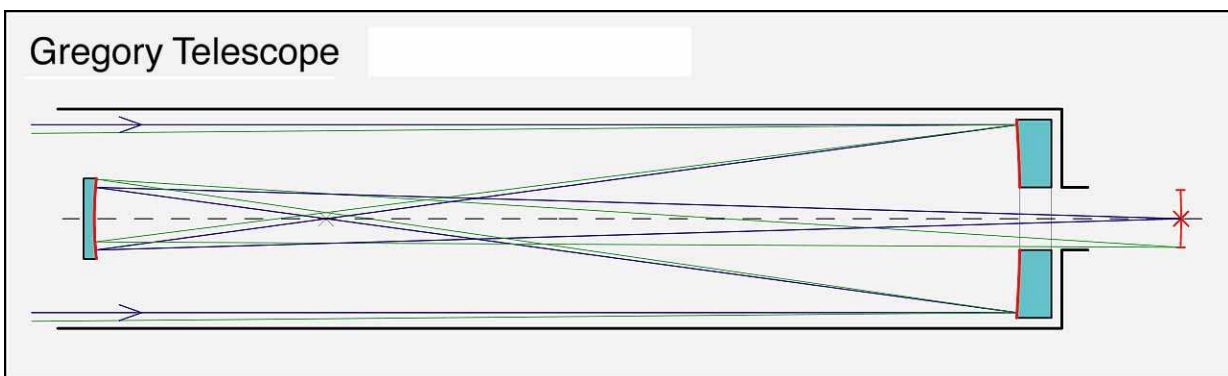
(drawing: © Andreas Strick)



Encouraged by his 10 years older brother DAVID, JAMES became engaged in the construction of telescopes. After the telescopes built by GALILEO GALILEI (1608) and JOHANNES KEPLER (1611) which used lenses, BONAVENTURA CAVALIERI (1632) and MARIN MERSENNE (1636) – inspired by the writings of IBN-AL-HAYTHAM (ALHAZEN) – developed the theory of telescopes that used the principle of reflection to observe the planets and the starry sky.



JAMES GREGORY was so fascinated by this idea that he designed his own telescope of this type (it is still called a *Gregory telescope* today) and presented his invention in a book *Optica Promota* (Progress in optics). The book also described a method of using the transit of Venus or Mercury across the Sun to determine the distance from Earth to the Sun. This was later accomplished by EDMOND HALLEY.

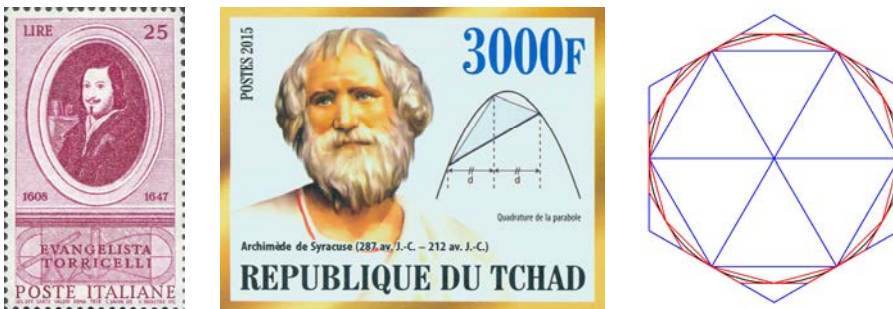


GREGORY himself was unable to build such an observation instrument because he lacked the necessary knowledge to grind mirrors and lenses. In 1663 he went to London and tried unsuccessfully to find a suitable craftsman. It was only 10 years later that the practically gifted polymath ROBERT HOOKE took up GREGORY's idea and built such an instrument according to his specifications.



In London, GREGORY became friends with JOHN COLLINS, a librarian and accountant who was very interested in mathematics. In 1664 he travelled to Flanders to give CHRISTIAAN HUYGENS a copy of his book, but he missed him there, followed him to Paris, but did not find him there either. He travelled to Rome and finally to Padua.

There he met the Jesuit, STEFANO DEGLI ANGELI, who was a student of CAVALIERI and EVANGELISTA TORRICELLI, and who had further developed the method of the calculus to determine areas and volumes as well as the focal points of figures and bodies. The fruitful collaboration resulted in two GREGORY's publications.



The first work was entitled *Vera circuli et hyperbolae quadratura* (True Quadrature of Circles and Hyperbolas, 1667). To determine the area of a circle or a hyperbola, he examined (generalizing ARCHIMEDES' method) sequences of inscribed or circumscribed polygons, which increasingly approached the curve under consideration and whose areas formed nesting intervals giving a common limit value. GREGORY was the first to speak of *sequences converging*.

Using the terms of such episodes he tried to prove that the number π cannot be represented by means of an algebraic formula, that is, π cannot be calculated directly (i.e. without a limiting process) by applying the four basic arithmetic operations and square roots. The second work *Geometriae pars universalis* (The universal role of geometry, 1668) already contained the most important ideas of differential and integral calculus, including the connection between tangents and determination of areas.

GREGORY returned to London in 1668 and hoped to find positive feedback from HUYGENS, to whom he has sent a copy of the *Vera quadratura* from Italy. Instead, HUYGENS published a review in a journal in which he considered the transcendence of the number π . He described it as incorrect, pointed out actual mistakes in the writing, but above all – wrongly – indicated that some of the work was not done by GREGORY. Despite this insult, GREGORY continued to work on problems of analysis and published *Exercitationes Geometricae* (Geometrical Exercises, 1668), as a polemical answer to HUYGENS' allegations. The work contained – without revealing the derivation – the series developments of trigonometric functions:

$$\sin(x) = \frac{1}{1!} \cdot x^1 - \frac{1}{3!} \cdot x^3 + \frac{1}{5!} \cdot x^5 - \frac{1}{7!} \cdot x^7 + \dots; \quad \cos(x) = \frac{1}{0!} \cdot x^0 - \frac{1}{2!} \cdot x^2 + \frac{1}{4!} \cdot x^4 - \frac{1}{6!} \cdot x^6 + \dots$$

$$\tan(x) = \frac{1}{1} \cdot x^1 + \frac{1}{3} \cdot x^3 + \frac{2}{15} \cdot x^5 + \frac{17}{315} \cdot x^7 + \dots; \quad \arcsin(x) = \frac{1}{1} x^1 + \frac{1}{2} \cdot \frac{1}{3} x^3 + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} x^5 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} x^7 + \dots$$

GREGORY also stated that (as it is called) a *primitive* for $\sec(x)$ is $\ln(\sec(x) + \tan(x))$ which was important for the calculation of nautical tables.

He was accepted into the *Royal Society* in the summer of that year, and thanks to the support of a member from Scotland, a chair was established for him in St Andrews: the Regius Chair of Mathematics.

GREGORY moved back to his distant Scottish homeland. He married a young widow and had three children with her. He gave two public lectures a week in a building adjacent to the university campus, but with little encouragement. To his dismay, hardly anyone at the university seemed to be interested in the topics he was dealing with. He relied on correspondence with JOHN COLLINS, who (not always impartially) endeavoured to assume a role similar to that of MARIN MERSENNE as a science broker. He informed GREGORY about ISAAC BARROW's lectures on optics, geometry and mathematics. The elaboration of these lectures by the holder of the *Lucasian chair* at Cambridge University were partly written by COLLINS himself and partly by ISAAC NEWTON and other students. Realising that his student was more talented than himself, BARROW relinquished his chair in 1669 in favour of NEWTON.

In a letter to COLLINS from 1671, GREGORY said that he had discovered how the value of a (arbitrarily differentiable) function near a point x_0 could be calculated from the function value and the values of the derivatives at this point – 40 years before BROOK TAYLOR. COLLINS replied that NEWTON was also working on such a theorem. After the unpleasant experiences with HUYGENS, GREGORY decided to wait until NEWTON's publication before publishing his own findings.

Another letter contained his insight that π could also be determined using a series expansion:

$$\arctan(1) = \frac{1}{4} \cdot \pi = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

as a special case of the series development $\arctan(x) = x - \frac{1}{3} \cdot x^3 + \frac{1}{5} \cdot x^5 - \frac{1}{7} \cdot x^7 + \dots$

In 1672, NEWTON presented the *Royal Society* with the mirror telescope he had developed, which essentially only differed from the GREGORY model in that the observer looked into the instrument from the side. NEWTON claimed that he had no knowledge of GREGORY's design, but GREGORY's work was found in NEWTON's library (with the corners bent over on important pages).



GREGORY was still afraid to publish his diverse new insights. This also applied to his remarkable discovery that the phenomenon of breaking down light into colours could also be caused by diffraction – something he observed by holding a bird's feather in a sunbeam.

At the beginning of the 1670s, GREGORY intensified his astronomical observations. By measurements during a lunar eclipse, he was able to determine the exact longitude of his St Andrews observation site. With regard to the establishment of an observatory, the university supported it only to a limited extent. So he moved to the University of Edinburgh in 1674. He suffered a stroke a year later while observing Jupiter's moons with his students. He died a few days later. He was not yet 37 years old.

Only through the evaluation of his manuscripts, some of which took place centuries later, did it become clear how many topics this mathematician had dealt with. And like other scientists of the time, some discoveries had to be made multiple times before they became common knowledge.

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<https://www.spektrum.de/wissen/james-gregory-1638-1675-schottischer-pionier-der-infinitesimalrechnung/1361523>

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