MARIUS SOPHUS LIE (December 17, 1842 – February 18, 1899)

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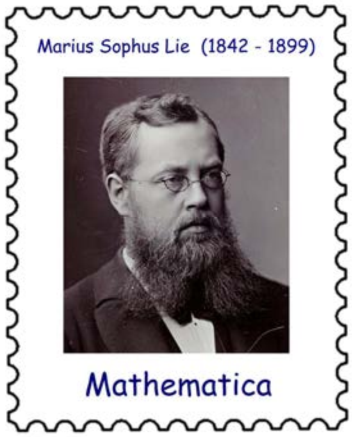

In the English-language Wikipedia list of terms that are named after the mathematician LIE, there are 73 entries starting with a minor planet LIE up to algebraic structures that begin with the *n*-dimensional torus surface (the two-dimensional torus is also popularly known as a *donut*). Mathematics students often encounter the so-called *Lie groups* in their studies; outside the mathematical community, however, the name of this Norwegian scholar is hardly known.

MARIUS SOPHUS LIE was born the youngest of six children of a Protestant pastor in Nordfjordeide, a place on the west coast of Norway, between Bergen and Ålesund. From 1857 he attended a private Latin school in Christiania (the former name of Oslo) with the aim of pursuing a military career. Since he had only poor eyesight, he has to give up this wish.

At 17, he moved to the university. He attended lectures that prepared him for a future job as a teacher, including by the mathematicians PETER LUDWIG MEJDELL SYLOW and CARL BJERKNES (father of the famous physicist and meteorologist VILHELM BJERKNES). In SYLOW's lectures, LIE learnt about the relationship between *equation theory* and *group structure* (as part of an introduction to *GALOIS theory*) and especially the importance of the contributions of his fellow countryman NIELS HENRIK ABEL.

After the final exam in 1865, he began teaching as a private tutor - initially without concrete plans for the future. This activity did not satisfy him; he had an academic career in mind - fluctuating between astronomy, physics, botany and zoology. Finally, his interest focused on mathematics.

Christiania University Library's loan records can still be used to trace how intensively LIE worked on various mathematical topics. In particular, he was fascinated by the innovative geometric ideas of JEAN-VICTOR PONCELET and JULIUS PLÜCKER. PLÜCKER had developed the idea of looking at straight lines (or spheres) that pass through points or contain points, instead of the points in space themselves. Curves could be "replaced" by tangents and curved surfaces by tangent planes.

In 1869, LIE published a first paper (at his own expense, since Christiania's *Academy of Sciences* was not prepared to support the revolutionary approaches contained in the article). But when the submission of the paper *Repräsentation der Imaginären der Plangeometrie* (Representation of the imaginaries of plane geometry) to *Crelle's Journal* was successful, he even received a travel grant to be able to get in touch personally with mathematicians in Berlin and Göttingen.

ERNST EDUARD KUMMER granted LIE the opportunity to present the new ideas in his seminar, and he was full of praise for it. His encounter with FELIX KLEIN, who as PLÜCKER's former assistant was particularly open to LIE's ideas, was even more enjoyable. An intensive friendship developed between the two from their resulting collaboration.

(drawings (c) Andreas Strick)

Together they travelled to Paris to exchange ideas with Gaston Darboux, Michel Chasles and Camille Jordan. In particular, Jordan, whose recently published essay *Traité des substitutions et des équations algébriques* (Treatise on substitutions and algebraic equations) was considered one of the fundamental texts in group theory, confirmed Lie's belief that he was on the right path.

After the outbreak of the Franco-German war in July 1870, Klein, as a Prussian citizen, had to leave the country as quickly as possible. Lie believed that he was not at risk because of his Norwegian citizenship, but as German troops approached the French capital, he decided to leave. On the way to Italy, he was arrested in Fontainebleau as a supposedly German spy – his notes with German-language terms that were incomprehensible to non-mathematicians were assumed to be encrypted messages. Only after a month was Darboux able to organise his release.

In 1871 Lie returned to Christiania after a short stay in Berlin, where he wrote his dissertation *On a class of geometric transformations* and was appointed to a chair specially set up for him. Together with Sylow, more than forty years after Abel's death, he published the collected works of this brilliant mathematician, who died early.

He married in 1874; a happy marriage which produced three children.

Over the next few years, Lie's research would make tremendous progress. In this context, he developed a method of determining further solutions from special solutions of differential equations. Although his results were important, he received little feedback. This was partly because his train of thought was not always easy to understand, and partly because in Christiania he was cut off from the centres of mathematics in Europe. His friend Felix Klein, who took over a professorship in Leipzig in 1880, noticed how much Lie was suffering and – with Lie's consent – sent Friedrich Engel, one of the Leipzig doctoral students, to Christiania. This 9-month collaboration developed into a collegial friendship that lasted for many years, and ultimately led to the collaborative publication of the 3-volume work *Theory of the Transformation* Groups (1888 to 1893).

In 1886 Lie accepted a call to the University of Leipzig (as the successor to Klein, who moved to Göttingen). Here Lie finally found immediate recognition. His lectures were attended by an above-average number of students from different countries. During his 12 years in Leipzig he had 26 doctoral students.

The unit circle S^1 in the complex number plane is a simple example of a so-called *Lie group*: The product of two complex numbers with modulus 1 is again a complex number with modulus 1. There is also an inverse element for every element of S^1, which also is in S^1. The commutative and associative law are satisfied. The product of two elements from S^1 can be understood as a rotation in $\mathbb{R}^2$.

Such rotations can also be described using 2×2 matrices. The set of rotation matrices also has a group structure; it is called a special orthogonal group SO(2). The operations are differentiable and therefore the set of 2×2 rotation matrices represents a *differentiable sub-manifold* of the space $\mathbb{R}^4$.

In the 1890s, LIE's mental and physical condition deteriorated. He worked too much, suffered from insomnia, was quickly irritated about everyday life and often reacted insensitively. His state of health can only be temporarily stabilised by staying in a sanatorium. During this time, his relationship with KLEIN also changed dramatically. KLEIN had published a paper in which he described the genesis of his *Erlangen program*. LIE, who was undoubtedly instrumental in the development of the *program*, considered his role in this account to be undervalued and got into a dispute with KLEIN. In the foreword to the third volume of the theory of transformation groups, LIE even attacked his former friend FELIX KLEIN. With his attack, however, he damaged his own reputation more than that of KLEIN.

Despite growing international recognition (admission to various academies), LIE felt uncomfortable abroad; he missed the loneliness of the wild landscapes of Norway, which he roamed as a hiker. His physical health was also deteriorating.

In 1898 he returned to his chair in Christiania. Full of optimism, he announced his plans for future publications. He tried to resume teaching, but his health deteriorated only a few weeks after he returned. MARIUS SOPHUS LIE died of pernicious anaemia, a form of anaemia due to a lack of vitamin B12.

In the 1920s and 1930s, LIE's *Collected works* by FRIEDRICH ENGEL and POUL HEEGAARD was published in six volumes.

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https://www.spektrum.de/wissen/marius-sophus-lie-1842-1899/1520575

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