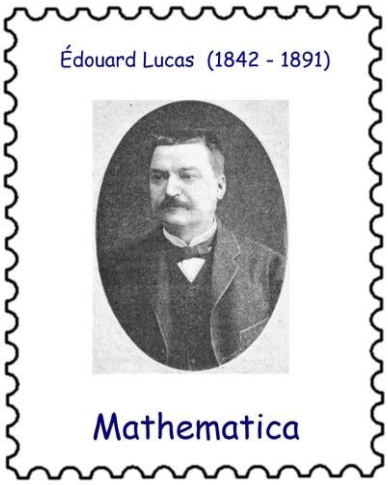
ÉDOUARD LUCAS (April 4, 1842 – October 3, 1891)

by HEINZ KLAUS STRICK, Germany



A more curious cause of death can hardly be imagined: During a banquet of the *Société Mathématique de France*, a waiter dropped a plate on the floor. A splinter from the plate hit ÉDOUARD LUCAS on the cheek: the cut became infected and a few days later the respected mathematician died as a result of the infection.

FRANÇOIS ÉDOUARD ANATOLE LUCAS came from a humble background; his father worked in Amiens as a cooper, i.e. a craftsman who made barrels out of wood.

After successfully passing the entrance examination and thanks to a scholarship from the municipality, ÉDOUARD was able to attend the *École Normale Supérieure* in Paris.

Afterwards, he first worked at the Paris Observatory, but does not get on well with its director URBAN LE VERRIER.

After the Franco-German War, he became a mathematics teacher, first in Moulins, then at two of the most prestigious "big" schools in Paris, the *Lycée Charlemagne* and the *Lycée Saint-Louis*, the only public secondary school in France to offer preparatory education classes for the *Grandes Écoles*.

In 1876 LUCAS caused a sensation when he claimed to have proved that the 39-digit *MERSENNE number* $2^{127}-1$, i.e. a number of the type 2^p-1 with a prime number *p* as its exponent, is itself a prime number. In addition, he contradicted MERSENNE's assertion that the number $2^{67}-1$ was a prime number, but without being able to give a concrete factorisation of the number.

EUCLID had already dealt with numbers of the type $M_n = 2^n-1$ and proved the theorem:

- *If M_n is a prime number, then $2^{n-1}\cdot(2^n-1)$ is a perfect number.*

MARIN MERSENNE had claimed that
$M_2=3,\ M_3=7,\ M_5=31,\ M_7=127,\ M_{13}=8191,\ M_{17}=131071,\ M_{19}=524287$ as well as
$\mathrm{M}_{31}\approx 2.1\times 10^9,\ \mathrm{M}_{67}\approx 1.5\times 10^{20},\ \mathrm{M}_{127}\approx 1.7\times 10^{38}$ and $M_{257}\approx 2.3\times 10^{77}$ are prime numbers.

In his work *Théorie des Fonctions Numériques Simplement Périodiques*, LUCAS described a test procedure for *MERSENNE primes*, which would be simplified in 1930 by the American mathematician DERRICK HENRY LEHMER and is called the LUCAS-LEHMER test for *MERSENNE numbers* in honour of both of them.

In this test procedure, a sequence S_n of numbers is considered, which are defined recursively by: $S_{n+1} = S_n^{\,2} - 2$ with initial value $S_2 = 4$.

➢ *A* M*ERSENNE number* $M_p = 2^p - 1$ *is a prime number if and only if* M_p *divides the element* S_p *of the sequence without remainder.*

Examples: $M_3 = 7$ divides $S_3 = 14$, $M_4 = 15$ doesn't divide $S_4 = 194$, $M_5 = 31$ divides $S_5 = 37634$.

In addition, LUCAS developed a *prime number test* for arbitrary natural numbers in 1876, which he published in an improved form in 1891 in his book on number theory (*Théorie des nombres*).

The basic idea of the test came from PIERRE DE FERMAT, who had discovered a property of prime numbers in 1640:

➢ *If p is a prime number and a is an integer that is not a multiple of p, then the number* $a^p - a$ *is divisible by p without remainder* (FERMAT's little theorem).

(*Note*: Instead of $a^p - a$ one can also consider $a^{p-1} - 1$.)

The *converse* of this theorem does not generally apply. However, there are only a few composite numbers (so-called *FERMAT pseudo-primes*) for which this is true.

➢ *If for all numbers a with 1 < a < n and gcd(a, n) = 1 it can be shown that* $a^{n-1} - 1$ *is divisible by n, then n is a prime number* with very high probability.

Exceptions are the so-called CARMICHAEL numbers, named after the American mathematician ROBERT CARMICHAEL; the smallest CARMICHAEL number is $561 = 3 \times 11 \times 17$.

LUCAS first proved the following theorem in 1876:

➢ *A natural number n > 2 is a prime number if and only if there is a natural number a with 1 < a < n for which both the condition "*$a^{n-1} - 1$ *is divisible by n" is fulfilled and the condition "*$a^m - 1$ *is not divisible by n" applies to all natural numbers m with 0 < m < n – 1.*

This time-consuming examination of all natural numbers *m* between 2 and *n* – 2 can be restricted to the integer divisors *m* of *n* – 1, as LUCAS proved in 1891. LEHMER was able to simplify this test procedure in 1951.

ÉDOUARD LUCAS also dealt with the generalisation of the sequence of FIBONACCI numbers, i.e. the recursively defined sequence of numbers $F_0 = 0,\ F_1 = 1,\ F_2 = F_0 + F_1 = 1,\ F_3 = F_1 + F_2 = 2,\ \ldots,$ whose *n*th element is defined as the sum of the two predecessors.

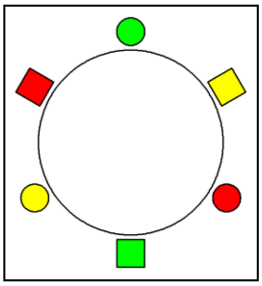
He then examined the (today so called) *LUCAS sequence* with initial values $L_0 = 2$ and $L_1 = 1$ and the corresponding recursion rule $L_n = L_{n-1} + L_{n-2}$.

There is a wealth of relations between the two sequences, e.g. $L_n = F_{n-1} + F_{n+1}$. And as with the sequence of FIBONACCI numbers, the sequence of quotients of successive entries converges to the *Golden Ratio* $\Phi = 1.6180339\ \ldots$:

$\frac{1}{2} = 0.5,\ \frac{3}{1} = 3,\ \frac{4}{3} = 1.333...,\ \frac{7}{4} = 1.75,\ \frac{11}{7} = 1.571428...,\ \frac{18}{11} = 1.6363...,\ \frac{29}{18} = 1.6111...,$ etc.

For L_n, too, there is an explicit representation of the sequence members with the help of Φ:

$$L_n = \Phi^n + (1-\Phi)^n = \Phi^n + (-\Phi)^{-n} = \left(\tfrac{1+\sqrt{5}}{2}\right)^n + \left(\tfrac{1-\sqrt{5}}{2}\right)^n .$$



In his *Théorie des nombres* one finds, among other things, a combinatoric question, the *Problème des ménages*: the question is about the number of ways in which a group of 3 (or *n*) married couples can be seated at a table so that women and men sit alternately and no one is placed next to their own partner.

Solution: For three couples there are $2 \times 3! = 12$ possibilities, but then it gets complicated: for four pairs there are 96, for five pairs 3120 possibilities ...

ÉDOUARD LUCAS became famous above all for a four-volume work published between 1882 and 1894 (posthumously): *Récréations mathématiques*.

This work contained an extensive collection of exercises in recreational mathematics that can be traced back to BACHET DE MÉZIRIAC.

Volume I began with various problems of crossing a river, including the classical problem with a wolf, a goat and a cabbage, and the problem of three (four, *n*) couples in which the jealous men take care that the women are not alone with strange men.

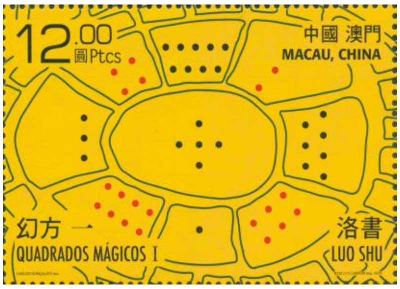
Further followed EULER's Königsberg bridge problem, paths in labyrinths, the 8-queens problem, solitaire, *Baguenaudier* (Chinese rings), *Taquin* (the 14-15 puzzle).

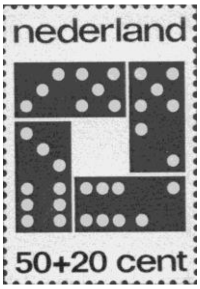
In Volume II he dealt with the game of draughts, with domino and Nine Men's Morris variations, with simple parquetry as well as various games of patience and at the end with HAMILTONian paths.

Members of the *Société Mathématique de France* compiled volumes III and IV from his estate.

A treatise on finger arithmetic in different cultures was followed by some explanations of calculating machines (including the binary abacus and the NAPIER rods) as well as numerous variants of siege games.

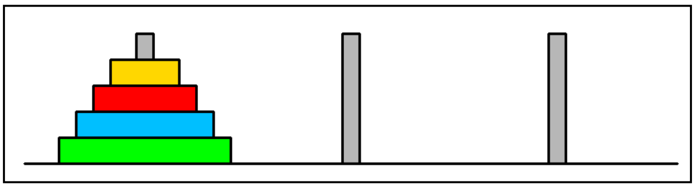
Volume IV dealt with calendar questions, with figurate numbers and magic squares as well as with the colouring of maps.



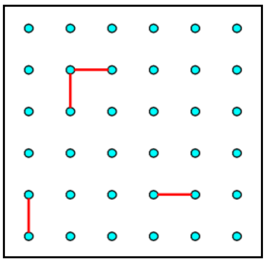


Volume III contained the probably most famous of all ÉDOUARD LUCAS's tasks: the problem of the Towers of Hanoi (actually: Tower of Hanoi), which he had already published as a game under the pseudonym N. CLAUS DE SIAM (an anagram for LUCAS D'AMIENS), supposedly a professor at the Collège of LI-SOU-STIAN (an anagram for SAINT-LOUIS).

The game was about a tower of 64 golden discs stacked one on top of the other in order of size, which the monks of a Brahma temple in Benares had to move one by one to form a new tower in such a way that only smaller discs lie on top of larger ones; an auxiliary tower may be formed during the rearrangement. To complete the task, the monks would need at least $2^{64} - 1 \approx 1.8 \times 10^{19}$ steps.
For the stack of four coloured discs shown, you would have to move a disc 15 times.



LUCAS also created the game *La Pipopipette*, published in 1895 in the collection *L'Arithmétique Amusante*:



There are 6×6 pegs equally spaced on a board. Two players take turns setting "bridges" between two (vertically or horizontally) adjacent pins; the aim of the game is to frame as many square areas as possible.

One variation of the game is also known in German as *Käsekästchen* – this is a crank: the French word *caisse* (box) sounds in German like *Käse* (cheese) – and in English it is called "Dots and Boxes").

First published 2016 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/edouard-lucas-1842-1891/1420169

Translated 2021 by John O'Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at europablocks@web.de with the note: "Mathstamps".