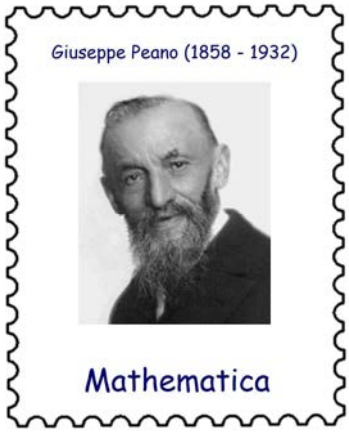
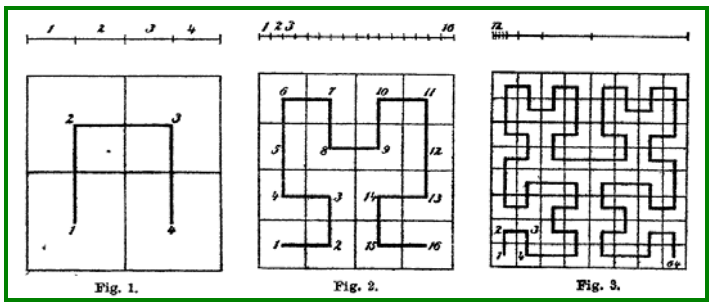
GIUSEPPE PEANO (August 27, 1858 –April 20, 1932)

by HEINZ KLAUS STRICK, Germany

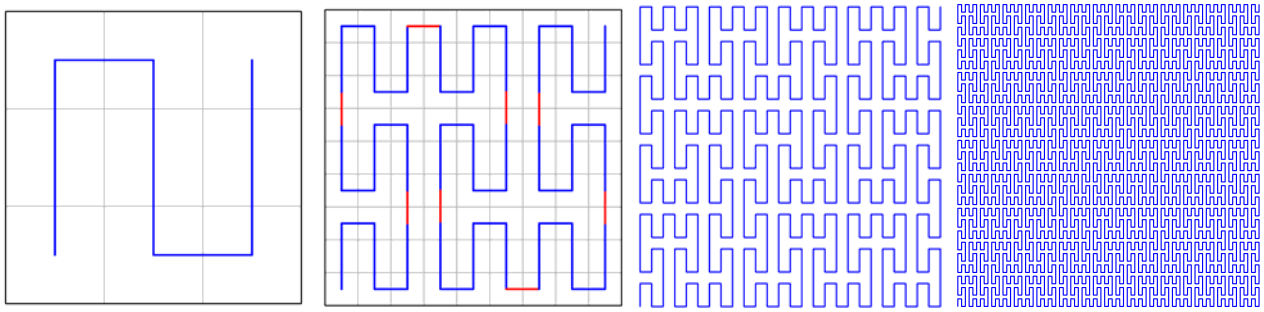


In 1891, DAVID HILBERT's essay *Über die stetige Abbildung einer Linie auf ein Flächenstück* (On the continuous mapping of a line on a piece of surface) appeared in the *Mathematische Annalen*. In this paper he described a method of how the unit square can be filled with an infinitely long line and illustrated it by sketches.



He referred to a contribution by the Italian mathematician GIUSEPPE PEANO, which had been printed in the same journal the year before: *Sur une courbe, qui remplit toute une aire plane* (On a curve which filles an entire area). PEANO had been the first to describe an example of a space-filling curve there, but had refrained from illustrating it. For in his contribution PEANO was only concerned with the formal proof that the recursively defined curve (now named after him) had the postulated property, and he had deliberately omitted an illustration so as not to "distract" the reader.

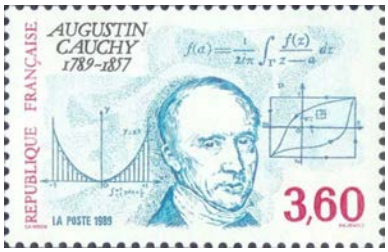
The first of the following figures shows the first-order PEANO curve in the unit square, which is divided into nine equal squares. The second-order PEANO curve is then composed of nine reduced first-order PEANO curves, correspondingly the third-order PEANO curve of nine second-order PEANO curves, and so on.



GIUSEPPE PEANO grew up near Cuneo, a small Piedmontese town where his parents worked as farm workers. When an uncle living in Turin noticed that GIUSEPPE is a very intelligent boy, he took the twelve-year-old into his home.

With his opportunities as a priest and legal scholar, the uncle was able to support him so that GIUSEPPE could successfully complete school (*Ginnasio* and *Liceo*) and start studying at the University of Turin at the age of 18. In particular, the first and second year professors, ENRICO D'OVIDIO (geometry and algebra) and ANGELO GENOCCHI (analysis), took care of the talented student who – in contrast to his fellow students – did not choose an engineering focus of the degree, but instead "pure" mathematics.

At the age of 22, PEANO obtained a doctorate in mathematics and was employed as an assistant, first to D'OVIDIO, then to GENOCCHI, whose lectures he gradually took over.

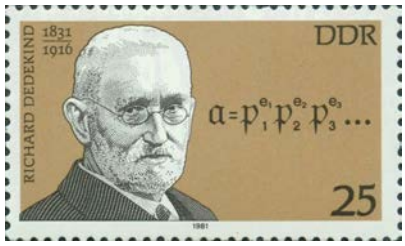


GENOCCHI was the first mathematics professor in Italy who – in the style of CAUCHY – paid special attention to the rigour of the arguments.

When the publication of his analysis lectures appeared in 1884 (*Calcolo differenziale e principii di calcolo integrale publicate con aggiunto dal Dr. Peano*) (Differential calculus and principles of integral calculus with additions by Dr Peano), he realized that it was not appropriate to mention PEANO only as someone who made a few additions ("aggiunto"); rather, the work was above all the merit of this "outstanding young man", who decisively shaped the texts he wrote through numerous comments and changes.

After his *habilitation* in 1886, PEANO also took on teaching positions at the military academy in Turin and published several articles on the solvability of differential equations.

In his book *Calcolo geometrico secondo l'Ausdehnungslehre di H. GRASSMANN, preceduto dalle operazioni della logica deduttiva* (Geometric calculation according to GRASSMANN's "Expansion theory", preceded by the operations of deductive logic) published in 1888, he was the first to deal with GRASSMANN's hitherto incomprehensible ideas on vector calculus and introduced the concept of vector space.



The book begins with a chapter on mathematical (deductive) logic. With this unusual opening he makes it clear that exact mathematics cannot do without a logical foundation.

In 1888 in the work *Was sind und was sollen die Zahlen?* (What are numbers and what should they be?) RICHARD DEDEKIND had tried to build the natural numbers by set-theoretic rigorous considerations.

The following year PEANO took up this approach and published his *Arithmetices principia: nova methodo* (The beginnings of arithmetic: a new method) - in Latin (!).

Among other things, it contains the famous *PEANO axioms* named after him, by which the set of natural numbers can be characterised.

Peano axioms for the natural numbers

(P1) 1 is a natural number.

(P2) Every natural number *n* has a natural number *n*' as a successor.

(P3) 1 is not the successor of a natural number.

(P4) Natural numbers with the same successor are the same.

(P5) If the set *X* contains 1 and for every natural number *n* also its successor *n*', then the natural numbers form a subset of X.

PEANO later replaced the number one in the axioms with the number zero as the smallest natural number.

When GENOCCHI died in 1889, PEANO was finally able to apply for his chair. However, since there were difficulties in establishing the selection committee, the appointment was delayed until December 1890. In the meantime, PEANO had attracted more attention by discovering the space-filling curve (see above).

In 1891 PEANO founded the journal *Rivista di Mathematica* for logic and the fundamentals of mathematics. He edited the submitted contributions with great precision, and he felt compelled to point out to authors if they had not checked the requirements of mathematical theorems carefully enough or if the evidence presented was incomplete.

In 1892 he started a new, ambitious project: *Formio Mathematico*, a universal collection of mathematical theorems and all methods - expressed with the symbols of formal logic. He was convinced that in future lectures professors would only have to explain to their students how to read the "formulas". While he was enthusiastic about his idea himself, it met with unanimous rejection by his colleagues.

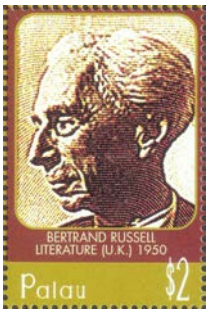
When the volume on analysis was finished, he used it himself in his lectures, and what his colleagues had feared happened: the students were hopelessly overwhelmed!

PEANO, who was always revered as a lecturer in the past, became an unpopular teacher.

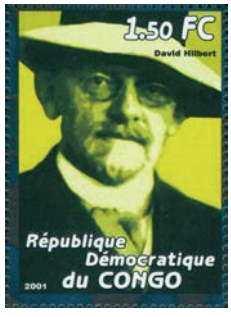
The military academy would have liked to fire him, and some of his university colleagues would also like to see him quit teaching or at least return to his previous style of teaching. But it was one of the privileges of university professors to give lectures as they saw fit, and PEANO continued his work undeterred.

At two international congresses for philosophy and mathematics, which were held one after the other in Paris in 1900, PEANO's contributions to formal logic were the focus of interest.

BERTRAND RUSSELL was full of admiration and later noted in his memoirs that no other lecturer made such a convincing impression on him as PEANO.



After HILBERT's lecture on the 20th Century Problems, PEANO temporarily toyed with the idea of solving the second problem, namely whether the "system of axioms of arithmetic" he described in 1889 was free of contradictions.



But by then PEANO was so fascinated by the idea of a universal language that he was almost exclusively occupied with this project. He developed a language *Latino sine flexione* (later called *Interlingua*), which mainly uses Latin words (supplemented by vocabulary from other languages), but without grammar. The fact that he wrote the last volume of his *Formulio Mathematico* in this artificial language certainly contributed to the fact that this work, which comprises more than 4,000 mathematical statements and formulas, received little attention.

The highly honored mathematician and logician died of heart failure in 1932.

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https://www.spektrum.de/wissen/giuseppe-peano-erfinder-einer-universellen-sprache-die-nur-er-versteht/1578188

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