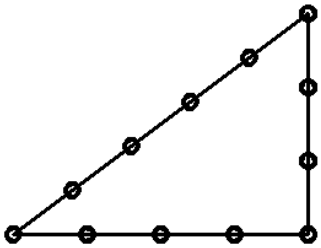
PYTHAGORAS OF SAMOS (about 580 BC – about 500 BC)

by HEINZ KLAUS STRICK, Germany

(Almost) everyone knows the theorem of PYTHAGORAS, but actually very little is known about the person whose name this famous theorem bears. No writings by him have survived and other sources from this period are also missing. Accounts of his life were not written until decades after his death – by HERODOTUS (484 – 424 BC) and ARISTOTELES (384 – 322 BC), among others – and most of the legends arose centuries later. In many accounts, PYTHAGORAS was described primarily as a philosopher and religious prophet, rather than as a mathematician.

Often, the traditions only speak of the *Pythagoreans*, i.e. the "school" of PYTHAGORAS, so that one does not know exactly what comes from PYTHAGORAS himself and what from his pupils. Whether PYTHAGORAS was the son of a gem-cutter or a merchant is just as unclear as the time and duration of his journeys to Phoenicia, Egypt and Mesopotamia. He probably got to know THALES OF MILETUS (624 – 547 BC) personally since the island of Samos lies directly off the coast of Asia Minor and THALES' pupil ANAXIMANDER (611 – 546 BC) was possibly his teacher.

It is unclear whether the so-called *Harpedonaptai* (surveyors who used stretched ropes) in Egypt and Mesopotamia were already using the theorem named after him more than 1000 years before PYTHAGORAS.



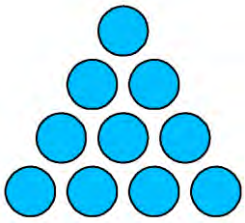
However, we can assume that PYTHAGORAS brought the knowledge of this rule, which originated in practice, from there to Greece. It is to his credit that he *freed mathematics from these practical applications* and then mathematics was practised *in order to come closer to the divine* (B. L. VAN DER WAERDEN).

Probably around 530 (520?) BC, PYTHAGORAS settled in the Greek colony of Kroton in southern Italy and founded a secret society there. The sect members believed in the immortality of the soul. One could only escape the fate of transmigration and the eternal cycle of rebirth if one occupied oneself with the mysteries of numbers and harmony. Their cosmos was ordered by numbers and in this sense mathematics was part of their religion.

Among his disciples, the Pythagoreans, PYTHAGORAS was considered the perfect sage; the term *philosopher* goes back to PYTHAGORAS as *a person who loves wisdom*. The disciples lived according to strict religious rules. Their ascetic, vegetarian way of life was cause for ridicule by some contemporaries.

The Pythagoreans occupied themselves with astronomy and astrology, with arithmetic and musical theory. According to the teachings of their leader, the path to the transcendent was only possible through a preoccupation with the properties of (natural) numbers, because: *Everything is number*.

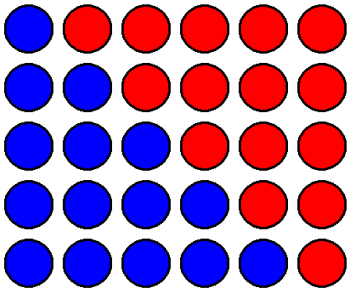
One was not an actual number, but the starting point of all numbers.

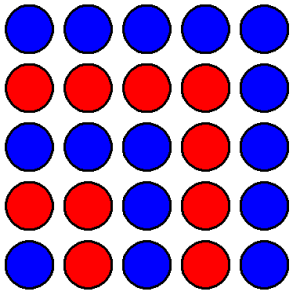


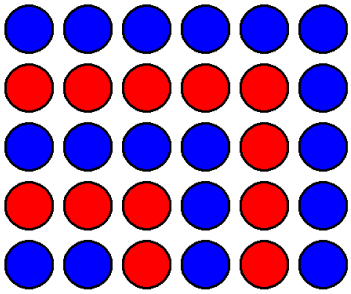
The number 10 was considered a sacred number since it is the sum of the first four numbers and the basis of our number system and moreover, it can be represented in the form of an equilateral triangle (*tetraktys* = fourness).

The numbers 1, 2, 3 and 4 also played a decisive role in musical harmony: if the length of a string is shortened from its original length to half, i.e. changed in a ratio of 2:1, the new note is an octave higher, if shortened in a ratio of 3:2 or 4:3, it is a fifth or fourth higher.

For arithmetic, the Pythagoreans used dark and light stones placed in geometric figures. In this way, formulas for sums could be read off, e.g. for the sum of successive natural numbers or successive odd or even numbers – in the following last two patterns, the stones were placed in the form of so-called *Gnomons* (L-shapes):





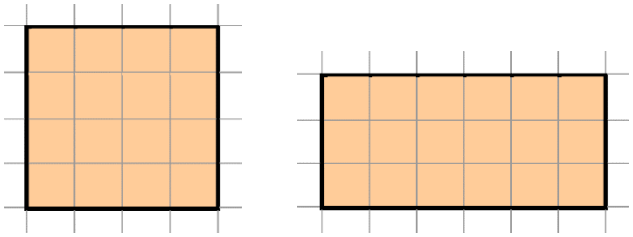


$1 + 2 + \cdots + n = \frac{1}{2} \cdot n \cdot (n+1)$ $\quad 1 + 3 + \cdots + (2n-1) = n^2$ $\quad 2 + 4 + \ldots + 2n = n \cdot (n+1)$

For the Pythagoreans, each number had its own mystical personality. Even numbers were feminine, odd numbers were masculine. The number 5, as the sum of the smallest numbers, the smallest even and odd number, was a symbol for marriage.

The number 6 is equal to the sum of its divisors – a *perfect* number. The Pythagoreans knew the general rule: If the sum $1 + 2 + 2^2 + \cdots + 2^n$ is a prime number *p*, then $2^n \cdot p$ is a perfect number. For the proof, one needs the summation formula for the geometric series, which was already known to the Babylonians.

The number 16 is not only a square number but it also represents a square of the length 4 with *circumference* 16 (length units) *and area* 16 (area units). Similarly, the number 18 represents the special rectangle with side lengths 3 and 6 (*perimeter* = 18, *area* = 18).

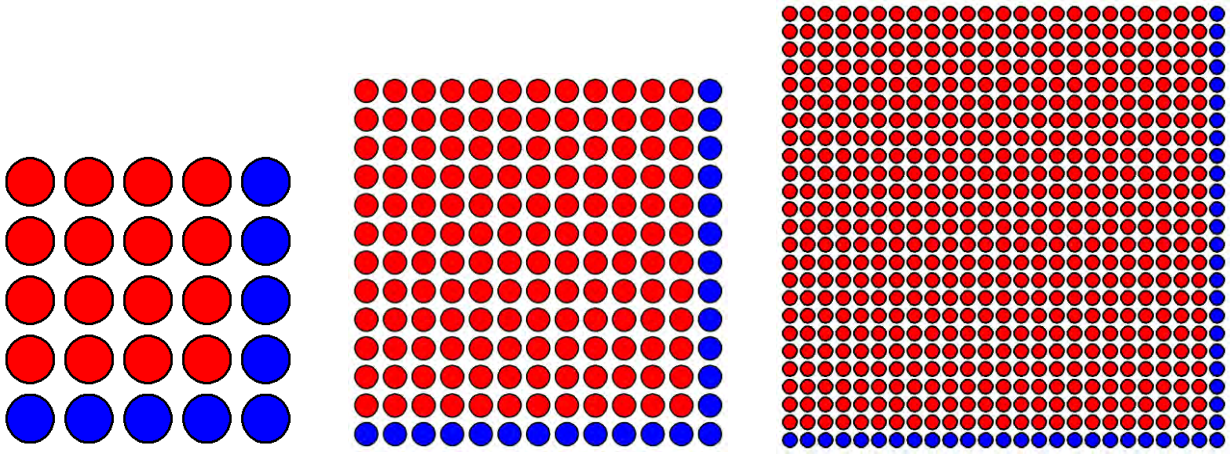


For the Pythagoreans, the fact that a right-angled triangle could be expressed by the numerical ratios 3, 4 and 5 indicated divine providence. We do not know whether PYTHAGORAS was able to "prove" the theorem named after him.

However, the following rule for special *Pythagorean number triples* was known to him:

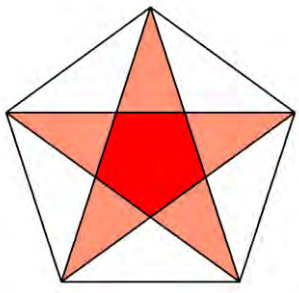
The triple (x, y, z) with $x = m,\ y = \frac{1}{2} \cdot (m^2 - 1),\ z = \frac{1}{2} \cdot (m^2 + 1)$, where *m* is an odd number, satisfies the equation $x^2 + y^2 = z^2$.

For $m = 3$ you get $3^2 + 4^2 = 5^2$; $m = 5$ leads to $5^2 + 12^2 = 13^2$; $m = 7$ gives $7^2 + 24^2 = 25^2$ etc.



This can also be seen in patterns: If you put an L-shape with x^2 dark (blue) stones around a square of y^2 light (red) stones, you get a square with $x^2 + y^2 = z^2$ stones.

Of the regular solids PYTHAGORAS knew only the tetrahedron, hexahedron (cube) and dodecahedron. The star pentagon (*pentagram*) became the distinctive sign of the Pythagoreans. The diagonals of this symmetrical figure intersect each other in the *golden section*.



About 50 years after the death of PYTHAGORAS, it was proved – possibly by his pupil HIPPASUS – that this division led to incommensurable distances, i.e. number ratios which were irrational and not in the ratio of whole numbers – a shock for the Pythagoreans?

Among the students of PYTHAGORAS, mathematics developed from a mystical number theory into an exact science and many of their findings were incorporated into the *Elements* of EUCLID (365 – 300 BC).

Raffael: School of Athens (probably) EUCLID (left picture in each case), PYTHAGORAS (right picture)

In the course of a century, however, the association of the Pythagoreans disintegrated because of differing political views.

PYTHAGORAS himself was expelled from Kroton in 510 BC and settled in neighbouring Metapont and he probably died there around 500 BC.

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https://www.spektrum.de/wissen/pythagoras-von-samos-580-500-v-chr/943152

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