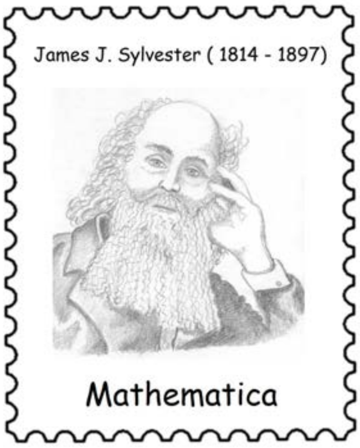
JAMES JOSEPH SYLVESTER (September 3, 1814 – March 15, 1897)

by HEINZ KLAUS STRICK, Germany



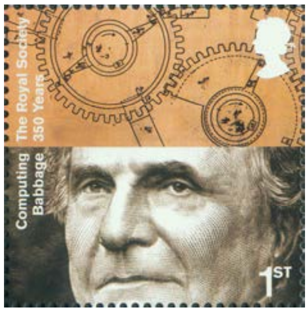
Actually, his name was JAMES JOSEPH, and he was the son of the Jewish merchant ABRAHAM JOSEPH from London. But when his eldest brother emigrated to the USA, the immigration authorities insisted on the rule that everyone must have three names, he added the last one. And because he liked the choice of his brother, JAMES JOSEPH also adopted the family name SYLVESTER.

JAMES first attended two schools in London before his father enrolled him at the newly founded *University College*, the first non-denominational university in England. His mathematics teacher was AUGUSTUS DE MORGAN, who quickly recognised the boy's mathematical talent.

But JAMES did not stay at the college for long, as when he threatened a classmate with a knife in a fight, his father withdrew him from school as a precaution. After successfully attending a college in Liverpool, SYLVESTER enrolled as a student at *St John's College* in Cambridge. Despite interruptions due to illness, he completed his studies of mathematics with the second-best exam of the year.

However, this degree was not confirmed by a graduation certificate, because as a devout Jew he refused to take an oath to accept the 39 Articles of the Anglican Church.

From 1838 to 1841 SYLVESTER was able to work as a physics teacher at University College. During this period he published 15 papers, on fluid dynamics and the solution of algebraic equations.



However, his preoccupation with physical problems did not particularly satisfy him. Therefore he was very relieved when he was able to obtain the longed-for university degrees as Bachelor and Master of Arts at *Trinity College* in Dublin. He then took up a chair in mathematics at the University of Virginia (USA), to which he was appointed, not least thanks to the recommendations of JOHN HERSCHEL, CHARLES BABBAGE and AUGUSTUS DE MORGAN ("No person in this country has a higher reputation as a mathematician than Mr. SYLVESTER").

In fact, even without the formal university degrees, SYLVESTER already enjoyed a high reputation among mathematicians in England. As early as 1839 he was appointed a fellow of the *Royal Society*.

However, he was not satisfied with the working conditions at the American university. When the university administration did not take serious steps to reprimand undisciplined students, he soon ended his teaching career. Back in England, he found work with an insurance company and also gave mathematical tuition (one of his students was FLORENCE NIGHTINGALE).

Then he decided to become a lawyer and was trained by the *London Bar Association*. Here he made friends with ARTHUR CAYLEY, who was also working as a lawyer – but their conversations always revolved around mathematical topics.

During his work as a lawyer, SYLVESTER also dealt with the problem of solving equations. In 1851, he discovered a criterion for cubic equations that allows statements to be made about the number and type of solutions. The characterisation was done by an expression, for which he coined the term *discriminant*.

The criterion for *quadratic equations* is as follows:

> A quadratic equation $ax^2 + bx + c = 0$ has two real solutions, if for the coefficients *a*, *b*, *c* the *discriminant* $\Delta = b^2 - 4ac > 0$. It only has a single real solution if $\Delta = 0$ and no real solution if $\Delta < 0$. Here the discriminant Δ consists of two summands, each with degree 2.

Properties of solutions of *cubic equations*:

> $ax^3 + bx^2 + cx + d = 0$ can be determined by the *discriminant*
> $\Delta = b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd$ a term whose five summands each have degree 4.
> If $\Delta > 0$ then the equation has three distinct real solutions; if $\Delta < 0$ then the equation has one real and two conjugate complex solutions; if $\Delta = 0$ then all solutions are real, but at least two of them match.

The discriminants for higher degree equations consist of an exponentially growing numbers of summands (4th degree: 16; 5th degree: 59; 6th degree: 246).

The terms can be obtained by calculating the determinants of the corresponding SYLVESTER *matrices*: The first *n* – 1 lines of these matrices consist of the coefficients of the polynomial and the next *n* lines come from the coefficients of the 1st derivation of the polynomial:

Second degree:

$$(-1)\cdot a\cdot\Delta_2 = \left|\begin{pmatrix} a & b & c \\ 2a & b & 0 \\ 0 & 2a & b \end{pmatrix}\right| = ab^2 + 4a^2c - 2ab^2 = 4a^2c + ab^2 = a\cdot\left(4ac - b^2\right)$$

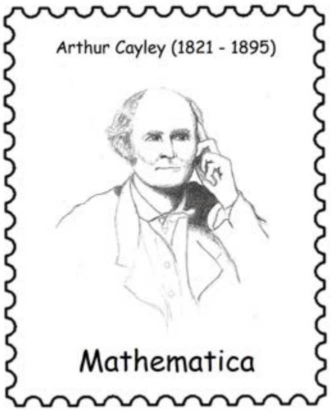
Third degree:

$$(-1)^3\cdot a\cdot\Delta_3 = \left|\begin{pmatrix} a & b & c & d & 0 \\ 0 & a & b & c & d \\ 3a & 2b & c & 0 & 0 \\ 0 & 3a & 2b & c & 0 \\ 0 & 0 & 3a & 2b & c \end{pmatrix}\right| = \ldots = a\cdot\left(b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd\right)$$

The expressions become simpler if the polynomials are *normalised* (i.e. *a* = 1) or changed by a substitution so that the summand of the second highest power vanishes.

Tired of working as a lawyer, SYLVESTER applied for a professorship in mathematics at various universities in 1854 – initially without success. When a candidate, to whom he was initially judged inferior, suddenly died, he was able to finally take up a position at a college – if only at the *Royal Military Academy* in Woolwich.

In 1865 he was one of the founding members of the *London Mathematical Society*. DE MORGAN became the first president of the Society, SYLVESTER and CAYLEY being his two successors.



After SYLVESTER was forced to retire at the age of 55 in 1870 and – like all members of the military – he then spent most of his time at the London Athenaeum Club.

He immersed himself in the laws of verse and translated poems from French, German, Italian, Latin and Greek, but he was less successful with his own poems.

Only when CHEBYSHEV visited London and the two of them started talking about mechanical devices through which circular movements could be converted into linear movements, did he again engage in mathematical problems.

In 1877 he accepted a call to the newly founded *Johns Hopkins University* in Baltimore, where he finally found students interested in mathematical research. Within seven years he supervised nine doctoral theses. He founded the *American Journal of Mathematics* and again wrote his own articles.

However, he increasingly felt that he was no longer up to the great responsibility he took on when he accepted a professorship at an American university, in a country whose universities and research institutes had begun to outstrip European institutions.

So, at the age of 68, he decided to go to Oxford to take up the prestigious *Savilian Chair of Geometry*. In fact, he was obliged to spend the rest of his life lecturing mainly on classical Greek geometry, but he was more interested in other subjects. When his mental and physical condition deteriorated, a deputy was appointed to the chair. The last years he spent again in his club in London.

During his active life, SYLVESTER dealt with a wide range of topics, and in almost every contribution he invented new terms, some of which become established in the long run, such as the terms *matrix* and *invariant*.

SYLVESTER is still remembered today by numerous mathematical theorems bearing his name.

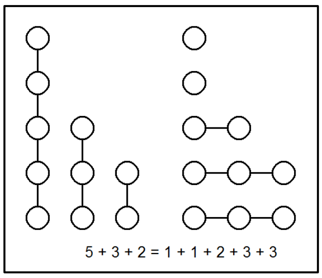
For example, a theorem of linear algebra is called *SYLVESTER's Law of Inertia*.

> This involves homogeneous quadratic forms, i.e. expressions in which the products of the variables are of the same degree (for example, for n = 2: $ax^2 + bxy + cy^2$ or for n = 3: $ax^2 + by^2 + cz^2 + dxy + exz + fyz$).
>
> By a suitable choice of the coordinate system every quadratic form can be represented in such a way that variables only occur as squares (thus in the two examples given: $(x')^2$ and $(y')^2$ respectively $(x')^2$, $(y')^2$ and $(z')^2$ and with coefficients only the numbers 0, +1, -1.
>
> SYLVESTER's theorem states that the number of coefficients 0, +1, -1 is independent (invariant) of the choice of the coordinate system.

In number theory, he coined the term *totient function*, which has since been used in English for EULER's φ-function, where φ(n) counts the positive integers up to a given integer n that are relatively prime to n.

In combinatorics, SYLVESTER introduced the idea of using graphs to illustrate *partitions* (representing numbers as sums).

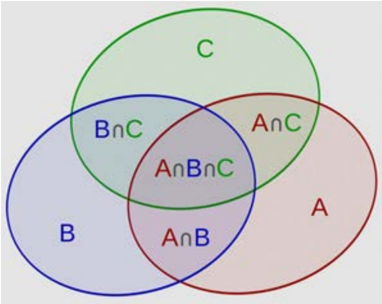
The use of the term *graph* in this context also comes from him.



A rule of stochastics was named after SYLVESTER: the so-called *inclusion-exclusion principle* for determining the probability P of one or more events.

For example for three events E_1, E_2, E_3:

$$P(E_1 \cup E_2 \cup E_3) = [P(E_1) + P(E_2) + P(E_3)] - [P(E_1 \cap E_2) + P(E_1 \cap E_3) + P(E_2 \cap E_3)] + [P(E_1 \cap E_2 \cap E_3)]$$



(source: Wikipedia)

SYLVESTER investigated *Egyptian fractions* and in particular the problem of representing a fraction by means of the smallest possible number of unit fractions (*SYLVESTER algorithm*).

Examples:

$\frac{5}{12}=\frac{1}{3}+\left(\frac{5}{12}-\frac{1}{3}\right)=\frac{1}{3}+\left(\frac{5}{12}-\frac{4}{12}\right)=\frac{1}{3}+\frac{1}{12}$

$\frac{4}{5}=\frac{1}{2}+\left(\frac{4}{5}-\frac{1}{2}\right)=\frac{1}{2}+\left(\frac{8}{10}-\frac{5}{10}\right)=\frac{1}{2}+\frac{3}{10}=\frac{1}{2}+\frac{1}{4}+\left(\frac{3}{10}-\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\left(\frac{6}{20}-\frac{5}{20}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{20}$

$\frac{3}{7}=\frac{1}{3}+\left(\frac{3}{7}-\frac{1}{3}\right)=\frac{1}{3}+\left(\frac{9}{21}-\frac{7}{21}\right)=\frac{1}{3}+\frac{2}{21}=\frac{1}{3}+\frac{1}{11}+\left(\frac{2}{21}-\frac{1}{11}\right)=\frac{1}{3}+\frac{1}{11}+\left(\frac{22}{231}-\frac{21}{231}\right)=\frac{1}{3}+\frac{1}{11}+\frac{1}{231}$

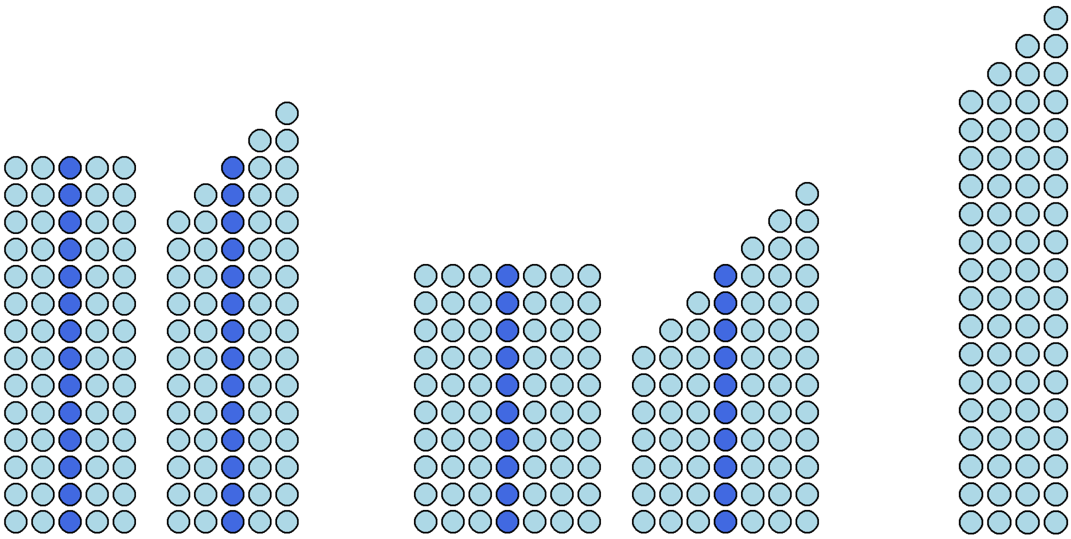
He proved that the zeros of the function f_n with $f_n(x)=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}$ satisfy:

if n is even, then f_n has no zero; if n is odd, then there is exactly one zero and this is negative.

He discovered the proposition:

- Any natural number $n > 2$ has exactly as many representations as the sum of successive natural numbers, as it has odd divisors (not including the number 1, but possibly including the number n itself).

Example: 70 has the odd divisors 5, 7 and 35; therefore there are three possibilities, namely:
12 + 13 + **14** + 15 + 16 = 7 + 8 + 9 + **10** + 11 + 12 + 13 = 16 + 17 + 18 + 19 = 70.



He sent several problems for publication to the journal *Educational Times*, including the following two. In fact, the proof of the second problem was not completed until 1933, by the Hungarian mathematician Tibor Gallai, a friend of Paul Erdős.

- *Suppose you have a large number of* 5*d and* 17*d stamps. What is the largest amount of postage that cannot be made up by a combination of the two values?*

(solution: 63)

- *A finite set S of points in the plane has the property that any straight line through two points also passes through a third point of the set S. Show that all points are then on a straight line.*

Because of his numerous contributions to various areas of mathematics, Sylvester was also widely honoured by foreign institutions; his merits, especially in combinatorics, the development of the theory of matrices and determinants and invariants are undisputed.

However, his work and his method of working were often severely criticized during his lifetime. In an obituary Max Noether extensively highlights Sylvester's achievements, but then takes him to court harshly:

... None of the works shows the desire to deepen and mature the subject matter in all directions: every mere supposition, often that which was conceived during printing, completely immature or false, was thrown out into the public eye with the greatest carelessness, always in complete ignorance of the literature, at the moment of its creation, without a trace of self-criticism ever having taken hold. ... Sylvester was not a harmonious or balanced mind, but an instinctively creative mind, without self-discipline.

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https://www.spektrum.de/wissen/james-joseph-sylvester-traegheitssatz/1303283

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