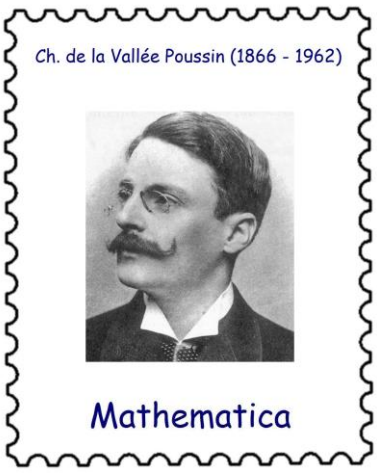
CHARLES DE LA VALLÉE POUSSIN (August 14, 1866 – March 2, 1962)

by HEINZ KLAUS STRICK, Germany



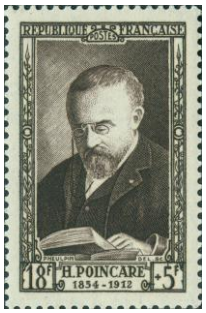
If you look for the name CHARLES-JEAN GUSTAVE NICOLAS BARON DE LA VALLÉE POUSSIN in a keyword index, you will probably find it under the initial letter L, because originally the family from France was called Lavallée. When one of the great-grandfathers of the mathematician-to-be honoured here married a descendant of the French baroque painter NICHOLAS POUSSIN (1594-1665), the latter – himself an artist – added the name POUSSIN. And finally, in 1930, the title of Baron was added (in honour and recognition of the mathematician's merits by the Belgian King).

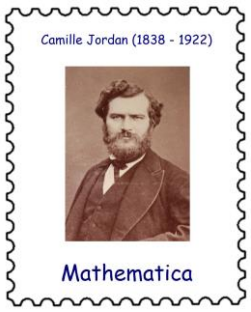
CHARLES grew up as the son of a professor of mineralogy and geology in Leuven (Louvain), Belgium. LOUIS-PHILIPPE GILBERT, a cousin of his father and professor of mathematics at the *Université Catholique de Louvain*, tried in vain to interest his nephew in mathematics but the boy insisted on becoming a priest and transferred to the Jesuit College *Saint-Stanislas* in Mons (in the Walloon province of Hainaut/Hainaut).

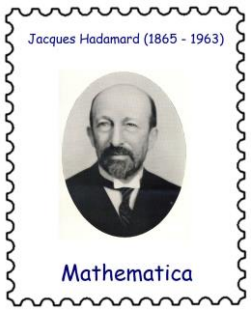
At the age of 17, CHARLES began to study philosophy, but was soon disappointed by this subject and changed to engineering studies, which he completed with a diploma. It was only during his engineering studies that his interest in mathematics grew. He attended the lectures of his uncle LOUIS-PHILIPPE GILBERT, was employed as his assistant in 1891 and also completed his doctorate under him. When his uncle died unexpectedly, he was appointed by the university administration to take over the lectures in differential and integral calculus.

In 1892 he was awarded a competition prize of the Belgian *Academy of Sciences* for his work on differential equations and in the same year he was appointed associate professor, and in 1897 a full professor.

In the meantime, he spent another year studying at the universities of Paris (with CAMILLE JORDAN, HENRI POINCARÉ and ÉMILE PICARD) and Berlin (with HERMANN SCHWARZ, FERDINAND FROBENIUS and LAZARUS FUCHS).







A publication in 1896 made CHARLES DE LA VALLÉE POUSSIN famous at a stroke. Around the same time as the French mathematician JACQUES SALOMON HADAMARD (1865-1963) and independently of the latter, he succeeded in proving the so-called *prime number theorem*, which states the following:

- The function $p(x)$, which gives the number of primes less than or equal to *x*, satisfies:

$$\lim_{x\to\infty}\left(\frac{\pi(x)}{\ln(x)}\right)=1.$$

The first investigations into this theorem had been carried out – independently of each other – by CARL-FRIEDRICH GAUSS (1755-1855) and ADRIEN-MARIE LEGENDRE (1752-1833).

In the first edition of his book *Théorie des nombres* from 1798, LEGENDRE had published the connection with the logarithm as a conjecture, then in the second edition he gave an approximation: $\pi(n) \approx \frac{n}{\ln(n) - 1.08366}$.

This led to a dispute over priorities with GAUSS, who had already studied prime numbers and their distribution as a 15-year-old, but had not published his empirical results further. GAUSS had found that the increase of $\frac{x}{\pi(x)}$ per power of ten is approximately ln(10) ≈ 2.3.

x	10	100	1.000	10.000	100.000	1.000.000	10.000.000
π(x)	4	25	168	1229	9592	78498	664579
x/π(x)	2.5	4.0	6.0	8.1	10.4	12.7	15.0
increase		+2.0	+2.1	+2.3	+2.3	+2.3	

In 1851, the Russian mathematician PAFNUTI LVOVICH CHEBYSHEV (1821-1894) was then able to show that the quotient of the two functions was: $0.92929 \le \frac{\pi(x)}{\ln(x)} \le 1.1056$,

which restricted the ratio to about ten per cent above and below.

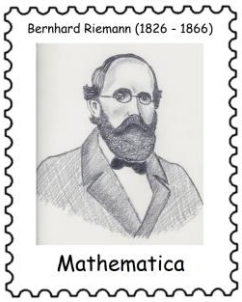
From investigations of $\frac{\pi(x)}{x / \ln(x)}$ as well as the integral function $\mathrm{Li}(x) = \int_2^x \frac{t}{\ln(t)}\, dt$, which had been introduced by PETER GUSTAV LEJEUNE DIRICHLET (1805-1859), he concluded:

- If the limit $\lim_{x\to\infty}\left(\frac{\pi(x)}{\ln(x)}\right)$ exists at all, it *must* be equal to 1.

In 1892, the English mathematician JAMES JOSEPH SYLVESTER (1814-1897) succeeded in further improving CHEBYSHEV's estimate: $0.95695 \le \frac{\pi(x)}{\ln(x)} \le 1.04423$.

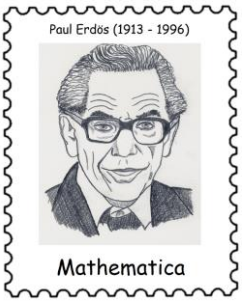
The proofs of DE LA VALLÉE POUSSIN and HADAMARD relied on a connection between the prime distribution and the *zeta function* introduced by EULER:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \frac{1}{1 - \frac{1}{p^s}}$$



Its complex continuation had been studied by BERNHARD RIEMANN (1826-1866) in 1859.

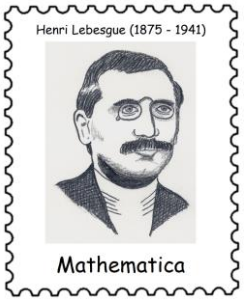
The two extremely complicated analytic proofs of DE LA VALLÉE POUSSIN and HADAMARD were indeed simplified later by ATLE SEBERG and PAUL ERDŐS; but even their approach can hardly be called elementary.



Even if it was the proof of the *prime number theorem* that made VALLÉE POUSSIN famous, it was above all a textbook that brought him great recognition. In 1882, the French mathematician CAMILLE JORDAN had published the elaboration of his lectures on calculus.

According to G H HARDY, this was the textbook that "opened the eyes" of him and his fellow students to what calculus actually was. In 1903, VALLÉE POUSSIN published in continuation of this Jordanian work the elaboration of his lectures on differential calculus for functions with one and with several variables, as well as the introduction to integral calculus as Volume I of the *Cours d'Analyse Infinitésimale*.

In 1906, the second volume on multiple integrals, differential equations and differential geometry followed. It was set in two font sizes: in normal font the explanations for beginners and engineering students, in smaller font the in-depth texts for "specialists".



In the opinion of critics, the two volumes surpassed all comparable works in terms of comprehensibility and accuracy. In 1909 and 1912, the second edition of the books followed, in which VALLÉE POUSSIN included, among other things, HENRI LEBESQUE's current contributions to integral calculus as well as recently published findings on set theory. At the beginning of 1914, the third edition of the first volume appeared and a translation into German was prepared.

In August 1914, the manuscript of the second volume was also available. However, it could no longer be printed, as the German army expelled the population of Leuven after the invasion of neutral Belgium in a "punitive action" on 29 August and set fire to the city. Immeasurable art treasures were lost, including the 300,000 volumes of the university library, which was over 500 years old and the manuscript of the second volume was also burnt.

VALLÉE POUSSIN fled abroad and took on lectures at Harvard and Cambridge (Massachusetts). From 1916 he taught at the *Collège de France* and the *Sorbonne* in Paris. After the war he returned to Belgium and he was elected the first president of the newly founded *International Union of Mathematicians*.

In the years that followed (until 1959), further editions of his two volumes on analysis appeared, also in English and Russian, although he omitted the sections for "specialists". He shifted his research activities to investigations on the approximation of polynomials by FOURIER series (sums of trigonometric functions).

During the 1920s VALLÉE POUSSIN lectured worldwide and he became an honorary member of various scientific academies (Paris, Strasbourg, Haarlem, Utrecht, Rome, Naples, Madrid, Oslo, Boston, Toronto) and received four honorary doctorates. He was twice named *Mathematician of the Decade* by the Belgian *Academy of Sciences*.

It was not until 1943 (i.e. at the age of 79) that he ended his lecturing activities in Leuven.

At the end of 1961, DE LA VALLÉE POUSSIN had such an unfortunate fall that he never recovered from the consequences of the shoulder fracture he suffered and he died at the age of 95.

Incidentally, one of VALLÉE POUSSIN's doctoral students was the astrophysicist and theologian GEORGES LEMAÎTRE, the founder of the Big Bang theory.

First published 2018 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/charles-de-la-vallee-poussin-1866-1962/1662422

Translated 2021 by John O'Connor, University of St Andrews

Here an important hint for philatelists who also like individual (not officially issued) stamps. Enquiries at europablocks@web.de with the note: "Mathstamps".