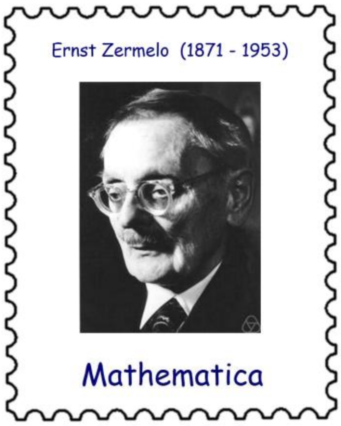
ERNST ZERMELO (July 27, 1871 – May 25, 1953)

by HEINZ KLAUS STRICK, Germany



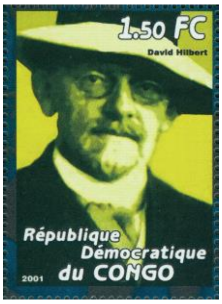
The photograph used in the *Mathematica stamp* on the right, taken from the archives of the *Mathematical Research Institute* in Oberwolfach (MFO), shows the 81-year-old ERNST FRIEDRICH FERDINAND ZERMELO shortly before his death in 1953.

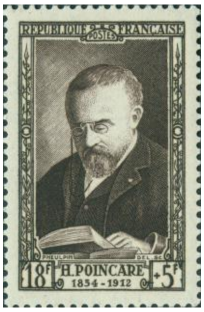
ERNST ZERMELO was born the son of a grammar school professor in Berlin; his home environment provided him with the best possible support during his school years. After passing the *Abitur* examination at the Luisenstädtisches Gymnasium, he began a broad-based course of study with a focus on mathematics, physics and philosophy. As was customary at the time, he also studied for a few semesters at other universities (Halle, Freiburg).

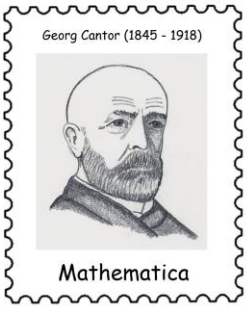
In 1894 ZERMELO was awarded his doctorate in Berlin with a thesis on the topic *Investigations on the Calculus of Variations* (his supervisors were LAZARUS IMMANUEL FUCHS and HERMANN AMANDUS SCHWARZ). In this work he extended the methods developed by KARL WEIERSTRASS for determining the extrema of integrals.

After his doctorate, he obtained a permanent assistant position to the Chair of Theoretical Physics with MAX PLANCK. In the course of this work, he was mainly concerned with questions of hydrodynamics, a branch of fluid mechanics. In 1897 he moved to Göttingen, where he was able to complete his research in 1899 in preparation for his *habilitation* thesis (*Hydrodynamic Investigations on Vortex Motions in a Spherical Surface*).

In the following semesters, the 28-year-old *Privatdozent* – in keeping with his doctoral and *habilitation* topics – took on lectures on statistical mechanics and the calculus of variations in Göttingen. In the winter semester of 1900-1901, ZERMELO switched to a topic that was completely different from his previous focus: from now on, his lectures were mainly on the set theory, founded by GEORG CANTOR.







(drawing © Andreas Strick)

In the 1870s, CANTOR had worried many mathematicians with his discoveries (countability of the set of rational and algebraic numbers, non-countability of transcendental numbers).

The opinions of contemporary colleagues differed widely. For some, CANTOR was a *corrupter of youth* (KRONECKER), for others a *gifted researcher* (DAVID HILBERT 1925: *No one will be able to drive us out of the paradise that CANTOR has created for us*.).

When CANTOR himself later encountered paradoxes connected with the definition of a set, HENRI POINCARÉ, next to HILBERT probably the most important mathematician of his time, saw himself confirmed in his view of set theory: Most of the ideas of CANTOR's set theory should be banished from mathematics once and for all.

In 1878, CANTOR had formulated the so-called *continuum hypothesis*:

➢ *There is no set whose cardinal lies between the cardinal of the natural numbers and the cardinal of the real numbers.*

(There are indeed infinite sets whose cardinals are greater than that of the real numbers, but – according to the hypothesis – there are no sets that cannot be counted and whose cardinal is *smaller* than that of the real numbers).

Until the famous mathematical congress in Paris in 1900, there was no progress in clarifying whether the continuum hypothesis held. In his list of the most important problems to be solved, HILBERT placed this at number 1. However, he assumed that further investigations were unnecessary if another of CANTOR's conjectures was proved first:

➢ *For every set there is an order relation with respect to which this set can be "well-ordered".*

A set M is called *well-ordered* if there is an order relation $\triangleleft$ such that the following hold:

- For every two elements $a, b \in M$ either $a \triangleleft b$ or $b \triangleleft a$ or $a = b$ holds. (Trichotomy)
- For every three elements $a, b, c \in M$ it follows from $a \triangleleft b$ and $b \triangleleft c$ that $a \triangleleft c$. (Transitivity)
- Every non-empty subset of M has a smallest element.

Examples: The set of natural numbers is obviously well-ordered with the usual ordering relation "<" because one can specify a smallest element for each subset.
The set of integers in its usual order is not well-ordered; however, the elements of this set can be arranged in such a way that a well-ordering arises, e.g. by the arrangement 0, +1, -1, +2, -2, ... Obviously, every countable set is well-ordered, because by counting the elements, a well-ordering of the set is established.

In 1901, ZERMELO published his first contribution to set theory, in which he dealt with the question of how *transfinite cardinal numbers* can be added.

CANTOR had introduced the concept of the *cardinal number* within the framework of his set theory:

– For finite sets, this was equal to the number of elements of the set in question; for infinite sets, he used the transfinite cardinal numbers $\aleph_0$ for countable sets and $\aleph_1$ for the set of real numbers (assuming the validity of the continuum hypothesis).

Then, in 1904, ZERMELO's treatise *The proof that any set can be well-ordered* appears.

Essential to his proof is an *axiom*, i.e. a principle that is actually immediately obvious and does not require proof, the so-called …

Axiom of choice: For each set of mutually disjoint non-empty sets there exists a function by which each of these sets is assigned exactly one element from this set, i.e. exactly one element is selected from this set.

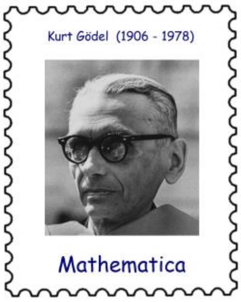
In 1908, ZERMELO published his first axiom system for set theory, which contained, among other things, an *axiom of separation* in order to circumvent the *ZERMELO-RUSSELL paradox*:

➢ *The set of all sets that do not contain themselves*.

However, he did not succeed in showing that this system was internally consistent (free of contradictions). Therefore, DAVID HILBERT urged the verification of the consistency of axiom systems within the different fields of mathematics (HILBERT's programme).

In 1922 ADOLF ABRAHAM FRAENKEL discovered a gap in ZERMELO's axiom system and proposed additions. In 1930, ZERMELO published a final version.

In the literature, the ZERMELO-FRAENKEL axiom system of 1930 is abbreviated to ZF or ZFC – depending on whether the axiom of choice is included or not. It was later proved that the axiom of choice cannot be derived from the other axioms of set theory.



The proof that the ZFC system is free of contradictions did not succeed; rather, KURT GÖDEL showed that such a proof is not possible within the framework of the ZFC system (*Second Incompleteness Theorem*).

With the proof of the well-ordering theorem in 1904, ZERMELO became famous in one fell swoop. The University of Göttingen awarded him the title of professor, but did not have a full professorship for him.

When he fell ill with tuberculosis in 1906, he had to considerably curtail his lecturing and research activities and stays at health spas in Switzerland and Italy had only a temporary effect. After various unsuccessful applications, he finally became a full professor at the University of Zurich in 1910, but could hardly meet the obligations of his new post.

Despite various surgical interventions (including with FERDINAND SAUERBRUCH), there was no improvement, so that the university administration suggested that ZERMELO apply for early retirement. In order to supplement ZERMELO's low pension income, HILBERT arranged for ZERMELO to be awarded 5000 Reichsmarks from the income of the WOLFSKEHL Foundation.

In 1921 ZERMELO moved to the Black Forest. In the 1920s he published a series of writings, among others on the assessment of the playing strength of participants in chess tournaments and on the navigation of airships as a problem of the calculus of variations.

He began the translation of the *Odyssey* of HOMER and compiled the collected works of GEORG CANTOR, which were published in 1932.

In 1926 ZERMELO was appointed honorary professor by the University of Freiburg, and he again took on individual lectures. When he refused to begin his lectures with the Hitler salute in 1935 and also made (semi-)public derogatory comments about the *Führer* and the *3rd Reich*, he was threatened with disciplinary proceedings, which would have resulted in the withdrawal of his teaching licence and removal from the university. The 64-year-old evaded this punishment by writing a letter of renunciation to the dean of the faculty.

In 1946, ZERMELO was rehabilitated and, at the request of the university senate, was once again appointed a full honorary professor at the University of Freiburg. However, in the meantime he had become almost blind and he could no longer take up teaching.

ERNST ZERMELO died in 1953 – in his 82nd year – in Freiburg.

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https://www.spektrum.de/wissen/ernst-zermelo-meister-der-ordnung/1640104

Translated 2021 by John O’Connor, University of St Andrews

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