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Abstract

In this paper, we propose to identify the dependence structure existing between the returns of equity and commodity futures and its evolution through the past 20 years. The key point is that we do not do not impose the dependence structure but let the data select it. To do so, we model the dependence between commodity (metal, agriculture and energy) and stock markets using a flexible approach that allows us to investigate whether the co-movement is : (i) symmetric and occurring most of the time, (ii) symmetric and occurring mostly during extreme events and (iii) asymmetric and occurring mostly during extreme events. We also allow for this dependence to be time-varying from January 1990 to February 2012. Our analysis uncovers three major stylized facts. First, we find that the dependence between commodity and stock markets is time varying, symmetric and occurs most of the time (as opposed to mostly in extreme events). Second, not allowing for time-varying parameters in the dependence distribution generates a bias toward evidence of tail dependence. Similarly, considering only tail dependence may lead to wrong evidence of asymmetry. Third, a growing comovement between industrial metals and equity markets is identified as early as in 2003, a comovement that spreads to all commodity classes and becomes unambiguously stronger with the global financial crisis after Fall 2008.

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JEL Classification: C20, G10, F30, Q

1 Introduction

After almost 30 years of low and moderately fluctuating prices, commodity prices have soared in 2003. The break observed during the global crisis was only temporary and the prices have strongly rebounded since the end of 2009. In total, non-oil commodity prices have grown three-fold since 2000.

The dramatic price spikes urged the European Commission and the United States Senate to investigate their potential causes. The increase in non-energy commodity prices is consistent with the strong increase in world demand, especially from China and India, and the deterioration of the supply conditions, due to declining productivity growth rates, low stock levels and adverse weather conditions. However, the increasing number of financial investors operating on the commodity market may have exacerbated these price changes. Clearly, the declining returns of assets in the aftermath of the dotcom bubble and the increasing commodity prices led investors to diversify their portfolio, encouraging the creation of commodity indices. As a result, the number of futures and options contracts outstanding on commodity exchanges has increased fivefold between 2003 and 2012. Similarly, physical hedgers that represented almost 80% of positions in commodity future markets in 1998, account for less than 30% in 2012, according to the Commodity Futures Trading Commission (CFTC). Have these evolutions changed the behavior of the commodity returns? In particular, what can we say about the cross-market linkages between traditional assets and commodities since the 2000s? The answers matter because the adverse effects of volatility in the physical commodity market on the balance of payment, public finances and households is a primary source of concern.

Unfortunately there is no consensus in the growing body of empirical papers investigating the cross-market linkages of conventional asset and commodity markets. On the one hand, Gorton and Rouwenhorst (2005) provide influential evidence in favor of the diversification benefits of commodity futures over the period 1959- 2004. They find that commodity future contracts have the same average returns as equities along with a negative correlation with bonds and equities and present less volatile returns. Chong and Miffre (2010), Hong and Yogo (2009) reach similar conclusions over a more recent period (see also Büyükşahin, Haigh and Robe (2010), Kat and Oomen (2006), Erb and Harvey (2006)). On the other hand, Büyükşahin and Robe (2011), Daskalaki and Skiadopoulos (2011), Silvennoinen and Thorp (2010), Tang and Xiong, (2010), among others, find evidence of integration among traditional and commodity markets.

One possible explanation for this lack of consensus is the different dependence maesures considered. We can identify two major approaches in the commodity litterature. First, several studies, such as Gorton and Rouwenhorst (2005), continue to rely on standard correlation which, implicitely, assumes that individual returns are normally distributed. Yet, Erb et al. (1994) and Longin and Solnik (2001), among others, show that asset classes are not normally distributed and conclude that the correlation coefficient may be a misleading measure of the dependence structure between two returns. Second, Engle (2002) introduces a Dynamic Conditional Correlation (DCC) GARCH model, a methodology applied recently to commodity and equity comovement (Chong and Miffre (2010), Büyükşahin, Haigh and Robe (2010)). The intuitive interpretation of correlations makes this time-varying parametric approach widely used when studying financial series.[1] However, the DCC dependence structure varies with the choice of marginal distributions for the individual returns, which are unknown. As a result, they are imposed along with the dependence structure, which makes this dependence measure too limited.

As an alternative, the copula approach provides a measure for financial markets comovements that presents several adantages and adresses some of the points raised above. First, it disentangles the particular characteristics of each return series from the dependence stucture that links them. This properties allows for a wide range of models capturing different types of dependence between variables, such as tail and asymmetric dependence. Second, it does not require elliptically distributed returns. Third, the dependence captured by a copula is invariant with respect to increasing and continuous transformations of the marginal distributions, that is the copula does not change with returns or logarithm of returns. To our knowledge, no academic work uses copula to model the comovement between commodities and traditional assets.[2]

In this paper, we propose to identify the dependence structure existing between the returns of equity and commodity futures for the past 20 years. The key point is that we adopt a totally agnostic approach, letting the data select the dependence structure. Clearly, we rely on Patton (2006)'s extension of Sklar (1959) theorem to the conditional case and his parametric model on the evolution of the copula. That is, we consider three types of dependence structure ((i) symmetric and occurring most of the time, (ii) symmetric or asymmetric and occurring mostly during extreme events and (iii) occurring mostly during

[1] Engle and Kroner (1995) BEKK is also widely used, yet it shares the same downside than the DCC: the structure of the dependence depends on the margins.

[2] Works using copula to model the dependence structure across financial markets include Ning (2011), Chollete et al (2011) and Demoulin-Lebeault, Kharoubi, (2012), among others.

extreme and negative events, that is asymmetric) and allow the stength of the relationship to be constant and time varying within each structure. Finally, we retain the two most likely types of dependence out of these six considered.

As half of the exposure to commodity price movements is based on investment in commodity index, we first investigate the dependence between the total returns of the two most popular commodity indices and their sub-indices (the Goldman Sachs Commodity Index (SP-GSCI) and Dow-Jones UBS Commodity Index (DJ-UBS)) and major equity indices (SP500, FTSE100, CAC40, DAX30).[3] Second, we account for the heterogeneity among commodities by focusing on a sample of individual commodity futures covering industrial metal, agricultural and energy markets. We study the period from January 1990 to February 2012, daily data.

Our analysis uncovers three stylized facts. First, we find that the dependence between commodity and stock markets is time varying, symmetric and occurs most of the time (as opposed to mostly in extreme events). This result holds for the indices as well as for the 21 commodities under investigation. This result leads to the second stylized fact: not allowing for time-varying parameters in the dependence distribution generates a bias toward evidence of tail-dependence. Similarly, considering only tail-dependence may falsely lead to evidence of asymmetric relation between the returns. Finally, the last stylized fact highlights the impact of the 2008 crisis: the time-varying parameter shows that the comovement between commodity and asset markets becomes stronger starting in September 2008 with the bankruptcy of Lehman brothers and the strengthening of the financial crisis.

The paper is organized as follows. Section 1 presents the main results in the recent empirical literature on co-movements between commodities and traditional asset markets. Section 2 focuses on presenting the concept of copulas and the different models considered, while Section 3 describes the data and discusses the results. Section 4 concludes.

2 A brief literature review on comovement

Most questions raised in the empirical literature dealing with commodities and traditional assets focus on the diversification benefits of commodities: are these asset markets related to each other? What is the sign of the relationship? How does the relationship evolve with time? Is the relationship symmetric? Is there tail-dependence? Answers vary substantially.

Gorton and Rouwenhorst (2006) are among the first to produce some stylized facts to

[3] Commodity indices are weighted average of selected commodity prices, calculated based on future prices.

characterize commodity futures after the 2000s. They construct an index of commodity futures covering the period between July 1959 and December 2004. During their sample period, the standard deviation of commodity future returns is lower than that of stocks and bond returns. The distribution of their commodity index returns is positively skewed contrary to equity returns. And they find a negative correlation between stocks and commodity returns, as well as bonds and commodity returns, with the longer the time horizon the higher the negative correlation. In sum according to their findings, commodities provide positive diversification benefits. However their index is equally-weighted and rebalanced, hence bears no resemblance to any existing index, a fact that most probably influences its performance as pointed by Smith (2006).

Chong and Miffre (2010) estimate the time-varying conditional correlation between commodity futures, equity returns (S&P) and fixed-income securities (T-bills) using a GARCH-DCC model. They find that correlations fall over time and tend to fall in turbulent periods. They attribute this asymmetric pattern to investors' flight-to-quality strategy, a welcome news for long institutional investors (see also Kat and Oomen (2006)). However more recent studies tend to contradict these findings. For example, Silvennoinen and Thorp (2010) report time-varying correlations between commodity futures and stock markets that increase in volatile markets. They show that a higher proportion of non-commercial traders raises the correlations with stock and oil markets. Tang and Xiang (2011) also report an increase in the correlations between the returns of different commodity futures, starting in the 2000's. In particular, they show that this trend is significantly more pronounced for commodities in the two popular SP-GSCI and DJ-UBS commodity indices, a result attributed to the growing importance of index trading. In turn, using non-public data from the CFTC, Büyükşahin and Robe (2011) find that the comovement between equities and commodities has not increased until 2008, providing subtantial diversification opportunities. However, they show a positive correlation between the returns after fall 2008. Along this line, Daskalaki and Skiadopoulos (2011) show that including commodity index in investor's portfolio yield significant diversification benefits during the 2005-2008 commodity boom period, a benefit that dramatically vanishes after 2008 (see also Bicchetti and Maystre (2012)).

Overall, there is a lack of consensus in studies covering the arrival of financial investors on commodity markets: while many emphasize the asymmetric aspect of the comovement between commodities and equities, many do not agree on the timing of the strengthening of that relation. One possible explanation is the different dependence mesures considered and the fact that strong hypotheses are imposed regarding the joint distribution of the series. In the next section, we present an alternative that circumvent these limitations.

3 Methodology

3.1 Measures of dependence

In empirical financial works, some studies take a different approach in the dependence structure measurement using the copula framework. Indeed, the dependence structure in DCC framework, used by most works mentioned above, relies on the marginal distributions of the returns. In contrast, the dependence structure estimated via copula is, in that sense, more robust as this approach separates the dependence structure from the choice of margins.

Sklar(1959) shows that any multivariate distribution function can be decomposed into marginal distributions that describe the individual behavior of each series and the copula that fully captures the dependence between them. Furthermore, a copula can link any given set of marginal distributions in order to construct a joint distribution, providing a lot of flexibility in the specification of the marginal distributions and the dependence structure between them. This theory, however, was developed for applications where the data could be assumed to be independent and identically distributed, Patton (2006) extends it to the conditionnal case.

While the copula methodology is widely known, its application to financial assets is quite recent (Embrechts et al. (2002), Malevergne and Sornette (2003), Patton (2006) or Kole et al. (2007)). Most of these studies, however, treat the relationship described by the copula as constant over time, ignoring the work of Erb et al. (1994), Longin and Solnik (1995) or Engle (2002), among many others, that show otherwise.

In this paper, we focus on the dependence between two variables at a time, hence the copula becomes a bivariate joint distribution function. The choice of a specific distribution allow us to impose the structure of the dependence between the two variables, while the corresponding density function measures the strength of this dependence. Within each structure, we consider two cases: the dependence is constant and varies througth time.

Among the types of dependence available when dealing with copulas, we are particularly interested in the ability of capturing the potential joint occurrence of extreme events. Clearly, we want to measure the probability for the variables to observe jointly extremely high or low values. This so-called tail dependence can be described as follows: let $X_{1,t+1}, X_{2,t+1}$ be two random variables, and $F_{it}(X_{i,t+1})$,$i = 1, 2$, their conditional marginal distributions forming the conditional joint distribution $F_t(X_{1,t+1}, X_{2,t+1})$. Both marginal and joint distributions are conditioned on the information set Γ_t available at time t. In his extension of Sklar's theorem to continuous conditional distributions, Patton (2006) shows that there is a unique conditional copula function $C_t(.,.)$ such that $F_t(X_{1,t+1}, X_{2,t+1}) = C_t(F_{1t}(X_{1,t+1}), F_{2t}(X_{2,t+1}))$. Then, the right ($\tau^U$) and left ($\tau^L$)) tail dependence follow:

$$\tau^L = \lim_{\varepsilon \to 0} \Pr[F_{1t}(X_{1,t+1}) \le \varepsilon \mid F_{2t}(X_{2,t+1}) \le \varepsilon] = \lim_{\varepsilon \to 0} \frac{C_t(\varepsilon, \varepsilon)}{\varepsilon}$$

$$\tau^U = \lim_{\varepsilon \to 0} \Pr[F_{1t}(X_{1,t+1}) > \varepsilon \mid F_{2t}(X_{2,t+1}) > \varepsilon] = \lim_{\varepsilon \to 0} \frac{1 - 2\varepsilon + C_t(\varepsilon, \varepsilon)}{1 - \varepsilon}$$

provided that the limit exists, $\tau^L$and $\tau^U \in [0.1]$.

Several works, such as Longin and Solnik(2001), find evidence extreme and asymmetric dynamics in several asset markets, hence it seems realistic to investigate the presence of these effects in the commodity market.

For the analysis of the dependence between commodity and equity returns, we narrow our choice for the copula function $C_t(.,.)$ down to three types of structures: (i) symmetric and occuring most of the time, (ii) a/symmetric and occuring mostly during extreme events and (iii) asymmetric and occuring mostly during extreme and negative events. The corresponding models and their dependence parameters that measure the strength of this dependence, are briefly presented below, their distribution functions can be found in Table 1:

1. The *Gaussian copula* allows for equal degrees of positive and negative dependence but does not allow for tail dependence ($\tau^L = \tau^U = 0$). A nice feature of this copula is that the dependence parameter is the Pearson's correlation coefficient: the $-1 < \rho < 1$,.

2. The *symmetrized Joe-Clayton copula* allows for asymmetric dependence in the tails, yet symmetric tail-dependence ($\tau^L = \tau^U$) is a special case. $\tau^U = 0$ ($\tau^L = 0$) implies left (right) tail-dependence. The dependence parameters are $\kappa = [\log_2(2 - \tau^U)]^{-1}$ and $\gamma = [-\log_2(\tau^L)]^{-1}$. It does not allow for negative dependence.

3. The *rotated Gumbel copula* is better suited for strongly correlated variable at low values: with the dependence parameter $\delta \in [1, \infty)$ that does not allow for negative dependence and takes a value of 1 for the case of independence. The lower-tail dependence then becomes $\tau^L = (2 - 2^{1/\delta})$

Finally, we use Patton (2006)'s parametric model to describe the evolution of the dependence parameters as a function of an autoregressive term to capture any persistence in the dependence term, and a forcing variable to capture any variation in dependence.

For the Gaussian copula, the evolution equation for ρ_t is:

$$\rho_t \;= \Lambda_1 \left[\varpi_\rho + \beta_\rho(\rho_{t-1}) + \alpha \frac{1}{N} \sum_{j=1}^{N} \Phi^{-1}(F_{1t}(X_{1,t+1-j}))\Phi^{-1}(F_{2t}(X_{2,t+1-j})) \right]$$

where $\Lambda_1(z) = \frac{1-e^{-z}}{1+e^{-z}}$ is a transformation function designed to hold the correlation parameter ρ_t in the interval (-1,1), and N the number of lags considered. The forcing variable is the average of the product of the last N observations of the transformed variables..

For the non-Gaussian copulas, the evolution equation for the dependence parameters is:

$$\theta_t \;= \Lambda_2 \left[\varpi + \beta\theta_{t-1} + \alpha \frac{1}{N} \sum_{j=1}^{N} |F_{1t}(X_{1,t+1-j}) - F_{2t}(X_{2,t+1-j})| \right]$$

where $\Lambda_2(z)$ is an appropriate transformation function to ensure that the parameter always remains in its domain: $1+e^{-z}$ for the rotated Gumbal copula ($\theta_t = \delta_t$) and $\frac{1}{1+e^{-z}}$ for the symmetrized Joe-Clayton copula.($\theta_t = \tau_t^i$, with $i = U, L$). The forcing variable is the mean absolute difference between the transformed variables over the previous N periods.

3.2 Estimation

As discussed previously, the copula representation allows for great flexibility in the specification of the individual variables and Patton (2006) suggests to estimate parametric copulas using the two-stage maximum likelihood approach. First, we estimate and select the model that provides the best fit for the individual variables, then we estimate the dependence structure of the copula.

Daily asset returns have a tendency to show fat-tails, conditional heteroscedasticity and autoregressive characteristics. As a result, they are commonly described via AR(k)-t-GARCH(p,q) models. Thus, the model for the (log) returns X_{it} is described as follows:

$$\begin{aligned} X_{it} &= \mu_i + \sum_{j=1}^{k} \phi_{ij} X_{it-j} + \varepsilon_{it} \\ \varepsilon_{it} &= \sqrt{h_{it}} z_{it} \\ & \quad h_{it} = \omega_i + \sum_{l=1}^{p} \beta_l h_{t-l} + \sum_{m=1}^{q} \alpha_m \varepsilon_{i,t-m}^2 \end{aligned}$$

where i is the index of the analyzed series and z_{it} are standard t-distributed with

υ_i degrees of freedom. The CDF of the estimated standardized residuals are then used to replace F_{it} in the conditional copula model in order to estimate the dependence parameter.

For each pair of returns, the six different copulas (three constant and three time-varying) are estimated on the transformed residuals along with the corresponding log-likelihood and information criteria. The best model is the one maximizing the log-likelhood and minimizing the information criteria.

4 Data and Results

We collect daily spot and futures prices on 21 commodities from the markets of agriculture, industrial metal, and energy from January 1990 to January 2012. To provide the closest measure of the future curve, we extract monthly from daily prices, on all actively traded contracts with maturity dates up to one year. Then, we take the average across all the (log) returns in each period and collateralize by adding the 3-month Treasury Bill rate (adjusted to monthly).[4]

First, we investigate the dependence between the total returns of the two most popular commodity indices and their sub-indices on agriculture, industrial metal, and energy, the Goldman Sachs Commodity Index and the Dow-Jones UBS Commodity Index and four major equity indices, CAC40, DAX30, FTSE100 and SP500. Second, we examine the cross-linkage between equity indices and individual commodity futures covering industrial metal, agricultural and energy markets. This disaggregation allows us to investigate the presence of heterogeneity across different markets as well as to measure the integration of these markets.[5]

Table 2 reports some summary statistics of the data and highlights several expected features. For all the (log) returns, both skewness and kurtosis excesses confirm that the returns are not normally distributed, hence confirming the necessity of using an alternative to the linear correlation in our analysis.

For each pair of commodity/commodity index and equity-indices, we estimate six copula models previously discussed. Table 3 summarizes the results by reporting the two best models as well as the best model when forcing the parameter of dependence to remain constant.

A first glance at the results allows us to highlight some keys points. First, the comovement between commodity and equity markets is best described by a time-varying copula as 24 out of the 25 pairs considered choose time-varying models as the two best choices. Second, whether we consider the returns of commodity indices or individual commodity futures, all

[4] Daily and weekly returns were also considered without having an impact on the results.

[5] All the data have been collected from Bloomberg.

the pairs select the time-varying normal copula as the best fitting dependence structure. Third, imposing the wrong restrictions may lead to spurious results. Indeed, not allowing the level of dependence to vary during the period considered may lead to wrong evidence of tail dependence between equity and commodity returns while, in reality, it is the strength of that dependence that is changing: a comparison of the columns "constant" and the other two show that 22 out of the 25 pairs would lead to a tail dependence. Similarly, investigating solely tail-dependence may lead to false evidence of asymetric relation between the returns: in absence of normal (time-varying) copula, all the pairs would select to an asymmetric tail-dependence.

The dependence structure selected being a time-varying normal copula, Figures 1 to 8 plot the parameter ρ_t for each copula estimated, describing how the dependence strength is behaving through time. As discussed previously, ρ_t is the Pearson correlation coefficient that varies between (-1,1).

Figure 1a and b show that results are quite robust to the choice of equity indices (CAC40, DAX30, FTSE100 or SP500) and of commodity index (SP-GSCI or DJ-USBCI). Hence, we only report results using SP500 and SP-GSCI in the remaining of the analysis.

Figures 2 to 8 show that, while the dependence strcuture between the equity index and commodities (indices and market future returns) is best decribed by a time-varying normal copula, the behavior of ρ_t tends to be more heterogenous than in the previous analysis. Hence, the reading of these graphs is facilitated by Tables 4 and 5 which report the average ρ_t and its corresponding standard deviation for 4 different periods. Following Büyükşahin et al. (2010), we divide our sample in several sub-periods: (i) from 1990 up to May 1997, a relatively calm period that can be a benchmark period prior to the commodity investment boom; (ii) June 1997-May 2003, a period accounting for the late 1990s dotcom bubble, recessions in the US and Europe, the Asian crisis and the Russian and Argentinian sovereign defaults; (iii) June 2003-May 2008, a period with an increasing participation of financial traders in commodity futures markets but prior to the financial crisis and (iv) post October-2008.

Figures 3.a, b, c and d report the evolution of the dependence between equity index and the overall commodity index, the agriculture, the industrial material and the energy sub-indexes. Their overall pattern is quite similar as they show a drastic increase in ρ_t around 2008. Considering the commodity index, the relation is almost inexitant in the 90s ($\bar{\rho}_t \approx 0$) but drastically increases starting in 2008 ($\bar{\rho}_t \approx 0.41$). The results are quite similar for the energy sub-index ($\bar{\rho}_t^{ener} \approx 0.38$). In contrast, the industrial metal sub-index reports a dependence on average already positive prior to 2008 ($\bar{\rho}^{ind}_{97-08} \approx 0.16, \bar{\rho}^{ind}_{08-12} \approx 0.38$). The agriculture sub-index also reports an early but much weaker strengthening of the comovement before 2008 ($\bar{\rho}^{ag}_{90-97} \approx 0.03, \bar{\rho}^{ag}_{97-08} \approx 0.07, \bar{\rho}^{ag}_{08-12} \approx 0.24$).

Figures 4 to 8 plot the dependence parameter for the relation between equity index and individual commodities. Figures 4 to 5 focus on the agricultural markets sorting the individual commodities around 3 sub-groups: food and fibers (that is coffee, cotton, orange juice and sugar), meat and livestock (lean hogs, live cattle and pork belly) and grains and oilseeds (corn, soybeans, soybean oil and wheat). While the comovement between the different returns are best decribed by a time-varying normal copula, the behavior of each ρ_t is quite heterogenous.

Food and fiber commodities as well as grains and oilseeds report clear evidence of a strengthening of the dependence with the equity index after 2008. In contrast, the meat and livestock display a steadier dependence and, in particular, the dependence of pork belly with equity remains close to zero over the whole period. [6]

Figure 7 displays the dependence parameter for the relation between SP500 and base metals. They show that the comovement tends to strengthen in the late 90s ($\bar{\rho}^{copper}_{97-08} \approx 0.12$). This tendency is clearly enhanced during the last period ($\bar{\rho}^{copper}_{08-12} \approx 0.35$).

Finally, most of the energy markets, that is brent, crude oil and heating oil, observe a clear increase in dependence with the equity index mid-2008. More specifically from no relation it increases to a relatively strong and positive one after the beginning of the global financial crisis ($\bar{\rho}^{brent}_{post08} \approx 0.4$). In turn, the dependence parameter remains close to zero between equity and natural gas ($\bar{\rho}^{gas}_{upto08} \approx 0.00, \bar{\rho}^{gas}_{post08} \approx 0.05$).

Finally, unlike the comovement strength (ρ_t), its volatility remains quite stable across the four periods considered. We can notice, however, that the standard deviations are quite heterogeneous across the commodities. The overall relation between the returns of equity and commodity index or the energy sub-index are the most volatile ($0.14 < std(\rho^{commo}_t) < 0.18$ and $0.16 < std(\rho^{energy}_t) < 0.18$). Similarly, the relation with the energy market future returns are the most volatile, with the exception of natural gas ($0.14 < std(\rho^{brent}_t) < 0.17$).

To sum up, our analysis shows that the comovements between commodities (future or indices returns) and (US and european) stock index returns are best described by a relation that varies with time while being symmetric around the central tendency. That is, the

[6] It is worth mentionning that pork belly is off the GSCI-index, a fact that may confirm the findings of Tang and Xiong (2010) that the off-index commodities have only mild increases in correlation. Yet, this result is not confirmed by soybean oil and tin , that are also off the index.

relation between these market returns exist most of the time and is equally positive and negative; but its intensity differs depending on the period observed. In the 90s, equity and commodity market returns tend to show on average almost no relation. Hence, in the early 2000s, the index managers could promote commodity futures as a new asset class for institutional investors and praise its risk-diversification benefits. In the 2000s, most of the commodities report an extremelly weak relation with assets up to 2008 with the exception of industrial metals that show a strengthening of the comovement with equity markets in 2003. These results contrast with Silvennoinen and Thorp (2010) and Tang and Xiang (2011) who found that the co-movement increases before the crisis. Finally, we uncover a stronger comovement for all commodities following the 2008 crisis, similarly to Büyüksahin and Robe (2012). The two categories mostly integrated to equity markets are energy and industrial metals (bent and copper have the greatest integration).

Contrary to previous studies we find no asymmetry nor tail-dependence i.e. the returns of most commodities and major equity indices tend to move together most of the time with an equal degree of positive and negative dependence. However, we also show that not allowing for time-varying parameter or considering only tail-dependence may force evidence of tail-dependence or asymmetric dependence, respectively.

5 Concluding remarks

In this paper, we investigate the cross-market linkages of equity and commodity markets using copula approach. Unlike the DCC approach, our method considers a wide range of dependence structures to describe the comovement between these markets, and let the data choose the best fitting one. More specifically, we allow for three types of comovements: (i) occuring most of the time, in a symmetric manner (ii) occuring mostly during extreme events, in a symmetric or asymmetric manner and (iii) occuring mostly during negative and extreme events, that is in an asymmetric manner.

Our analysis highlights three major stylized facts. First, the dependence between commodity and stock markets is time varying, symmetric and occurs most of the time. Second, imposing the relationship to be constant in time may lead to false evidence of tail-dependence, while imposing tail-dependence may lead to spurious evidence of asymetrie. Third, a stength-ened comovement between industrial metals and equity markets is identified as early as in 2003, and spreads to all commodity classes with the global financial crisis after Fall 2008. In sum, the integration of some commodities with equity indices has started mildly in the

2000's, a trend that the global crisis has definitely strenghtened.

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Table 1: Copula distributions

Gaussian copula
$C_N(F_{1t}(X_{1,t+1}), F_{2t}(X_{2,t+1}); \rho) = \Phi_\rho(\Phi^{-1}(F_{1t}(X_{1,t+1}), \Phi^{-1}(F_{2t}(X_{2,t+1})))$
with Φ^{-1} the inverse cdf of a standard normal distribution, and $-1 < \rho < 1$,
the dependence parameter is the Pearson's correlation coefficient
symmetrized Joe-Clayton copula
$C_{SJC}(F_{1t}(X_{1,t+1}), F_{2t}(X_{2,t+1}) \mid \tau^L, \tau^U) = 0.5(C_{JC}(F_{1t}(X_{1,t+1}), F_{2t}(X_{2,t+1}) \mid \tau^L, \tau^U)$
$+C_{JC}(1 - F_{1t}(X_{1,t+1}), 1 - F_{2t}(X_{2,t+1}) \mid \tau^L, \tau^U) + F_{1t}(X_{1,t+1}) + F_{2t}(X_{2,t+1}) - 1)$
$C_{JC}(F_{1t}(X_{1,t+1}), F_{2t}(X_{2,t+1}) \mid \tau^L, \tau^U) = 1 - (1 - \left\{ \begin{array}{c} [1 - (1 - F_{1t}(X_{1,t+1})^\kappa]^{-\gamma} \\ +[1 - (1 - F_{2t}(X_{2,t+1}))^\kappa]^{-\gamma} - 1 \end{array} \right\}^{-1})$
with the dependence parameters, $\kappa = [\log_2(2 - \tau^U)]^{-1}$ and $\gamma = [-\log_2(\tau^L)]^{-1}$,
where τ^U and τ^L measure the tail dependence.
rotated Gumbel copula
$C_{RG}(F_{1t}(X_{1,t+1}), F_{2t}(X_{2,t+1}); \delta) = F_{1t}(X_{1,t+1}) + F_{2t}(X_{2,t+1}) - 1 + \exp \left\{ \begin{array}{c} -((\log(F_{1t}(X_{1,t+1}))^\delta \\ +(-\log(F_{2t}(X_{2,t+1})))^\delta)^{1/\delta} \end{array} \right\}$
the dependence parameter $\delta \in [1, \infty)$ that does not allow for negative dependence
and takes a value of 1 for the case of independence.
The lower-tail dependence then becomes $\tau^L = (2 - 2^{1/\delta})$

Table 2: Model Selection for Copulas

Indices total returns	Skewness	Kurtosis	Commodities	Skewness	Kurtosis
DJ commodity index	-0.63	5.93	Agriculture		
_agriculture	-0.22	5.44	Corn	-0.46	4.73
_industrial metals	-0.32	5.27	Wheat	-0.38	4.07
_energy	-0.76	7.31	Soybeans	-0.54	4.55
SP GSCI index	-0.59	6.17	energy		
_agriculture	-0.20	5.67	Crudel oil	-0.42	6.65
_industrial metals	-0.31	5.42	Brent	-0.61	5.86
_energy	-0.51	6.77			
			Industrial metals		
Stock return indices			Copper	-0.72	6.02
CAC	-0.71	7.85	Aluminium	-0.44	4.42
DAX	-0.70	7.71			
S&P	-0.88	8.68			
FTSE	-1.50	18.40			

Table 3: Dependence between commodities and equity index, copula selected

	Copula selected				Copula selected		
	Constant	Time varying			Constant	Time varying	
		1st choice	2nd choice			1st choice	2nd choice
Commodity index	SJC	TV normal	TV rot. Gumbel	Industrial metals			
_agriculture	SJC	TV normal	TV rot. Gumbel	Aluminium	SJC	TV normal	TV SJC
_industrial metals	SJC	TV normal	TV SJC	Copper	SJC	TV normal	TV SJC
_energybloombegr	SJC	TV normal	TV rot. Gumbel	Lead	SJC	TV normal	TV SJC
				Nickel	SJC	TV normal	TV SJC
Commodities				Tin	SJC	TV normal	TV rot. Gumbel
Agriculture				Zinc	SJC	TV normal	TV SJC
Coffee	normal	TV normal	TV SJC				
Corn	SJC	TV normal	TV rot. Gumbel				
Cotton	SJC	TV normal	TV rot. Gumbel				
Lean hogs	SJC	TV normal	TV rot. Gumbel				
Live cattle	SJC	TV normal	TV rot. Gumbel				
Orange juice	SJC	TV normal	-	Energy			
Pork belly	normal	TV normal	TV rot. Gumbel	Brent	SJC	TV normal	TV rot. Gumbel
Soybeans	SJC	TV normal	TV rot. Gumbel	Crude oil	SJC	TV normal	TV rot. Gumbel
Soybean oil	SJC	TV normal	TV rot. Gumbel	Heating oil	SJC	TV normal	TV rot. Gumbel
Sugar	SJC	TV normal	TV rot. Gumbel	Natural gas	normal	TV normal	TV rot. Gumbel
Wheat	SJC	TV normal	TV rot. Gumbel				

TV stands for time varying,1st/2nd choice based on min(AIC, BIC, -LL)

Table 4: Average time-varying parameter

	up to May 97	June 97-May 03	June 03-Aug 08	Sept 08-end
Commodity index	−0.04 (0.18)	0.02 (0.18)	−0.00 (0.18)	0.41 (0.14)
_agriculture	0.03 (0.10)	0.07 (0.12)	0.07 (0.10)	0.24 (0.09)
_industrial metals	0.09 (0.10)	0.16 (0.12)	0.15 (0.10)	0.35 (0.09)
_energy	−0.04 (0.18)	0.01 (0.18)	−0.02 (0.16)	0.38 (0.17)
Commodities				
Agriculture				
Coffee	0.09 (0.07)	0.09 (0.09)	0.10 (0.08)	0.20 (0.08)
Corn	0.03 (0.08)	0.04 (0.09)	0.06 (0.09)	0.16 (0.09)
Cotton	0.04 (0.07)	0.04 (0.07)	0.05 (0.07)	0.13 (0.08)
Lean hogs	0.03 (0.04)	0.02 (0.05)	0.02 (0.04)	0.06 (0.04)
Live cattle	0.08 (0.08)	0.05 (0.08)	0.05 (0.08)	0.18 (0.10)
Orange juice	0.03 (0.06)	0.04 (0.05)	0.04 (0.06)	0.08 (0.04)
Pork belly	0.03 (0.04)	0.02 (0.04)	0.04 (0.04)	0.03 (0.04)
Soybeans	0.02 (0.11)	0.04 (0.11)	0.08 (0.10)	0.22 (0.07)
Soybean oil	0.00 (0.11)	0.04 (0.11)	0.04 (0.12)	0.25 (0.09)
Sugar	−0.01 (0.10)	0.02 (0.10)	−0.01 (0.10)	0.15 (0.10)
Wheat	0.03 (0.06)	0.04 (0.09)	0.04 (0.07)	0.16 (0.07)

standard deviation in parentheses

Table 5: Average time-varying parameter

	up to May 97	June 97-May 03	June 03-Aug 08	Sept 08-end
Industrial metals				
Aluminium	0.06 (0.09)	0.13 (0.12)	0.11 (0.14)	0.31 (0.09)
Copper	0.05 (0.12)	0.15 (0.13)	0.13 (0.16)	0.35 (0.07)
Lead	0.07 (0.08)	0.09 (0.09)	0.11 (0.10)	0.28 (0.10)
Nickel	0.05 (0.06)	0.10 (0.08)	0.11 (0.1)	0.22 (0.05)
Tin	0.03 (0.09)	0.05 (0.11)	0.08 (0.10)	0.24 (0.09)
Zinc	0.06 (0.10)	0.13 (0.09)	0.11 (0.10)	0.27 (0.07)
Energy				
Brent	-0.05 (0.17)	0.00 (0.19)	-0.02 (0.17)	0.40 (0.14)
Crude oil	-0.05 (0.18)	0.01 (0.19)	-0.02 (0.17)	0.40 (0.17)
Heating oil	-0.05 (0.18)	0.02 (0.18)	-0.03 (0.16)	0.37 (0.15)
Natural gas	-0.00 (0.09)	-0.00 (0.09)	0.00 (0.08)	0.05 (0.10)

standard deviation in parentheses

Figure 1: Contour Plots

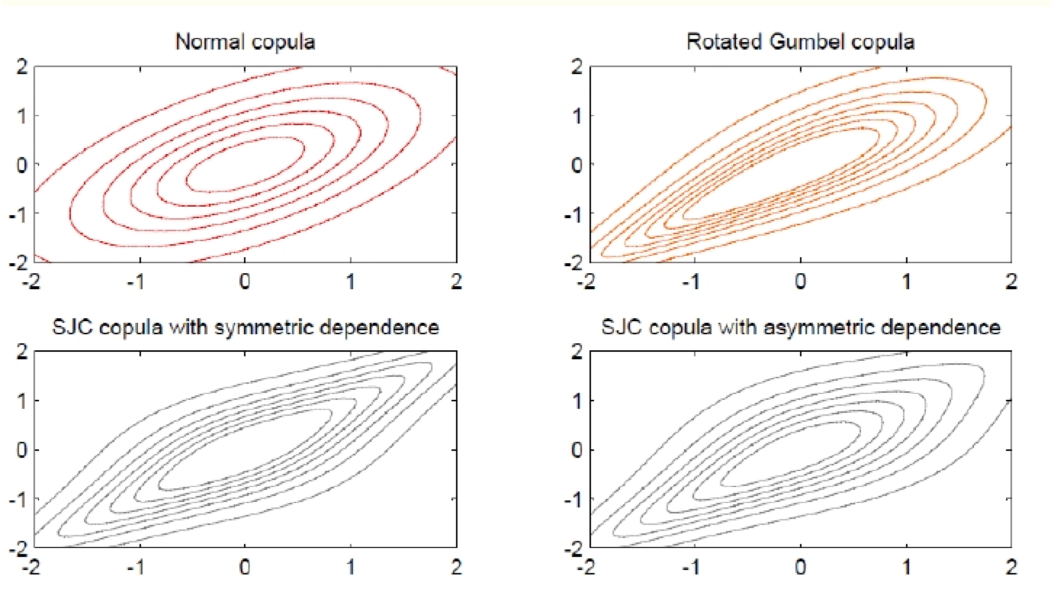
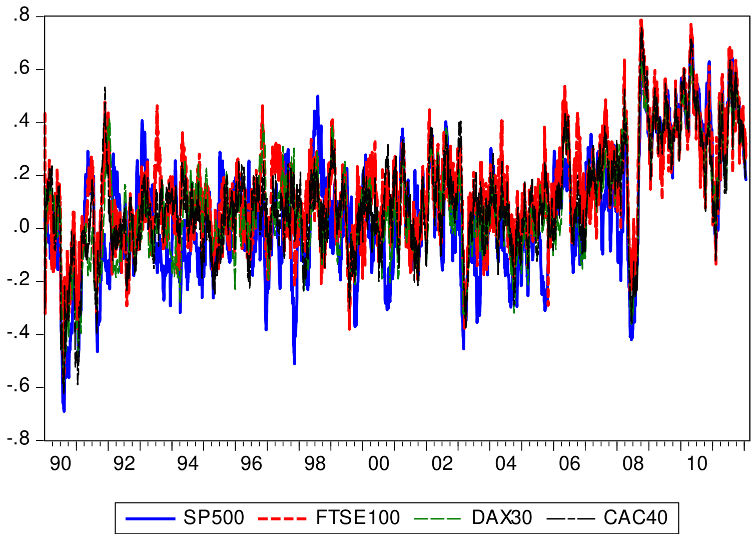


Figure 2: Model Selection for Copulas

a. SP GSCI index, Time varying normal copula.



b. SP 500, Time varying normal copula.

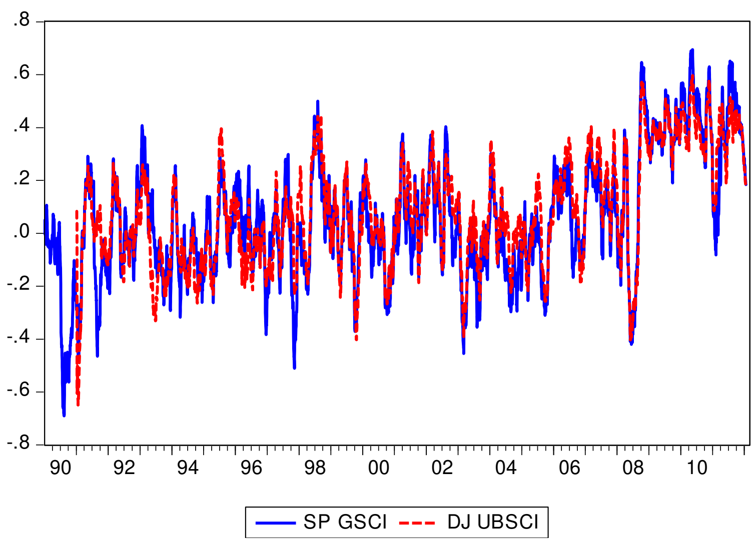


Figure 3: Model Selection for Copulas

Commodity indexes, time varying normal copula.

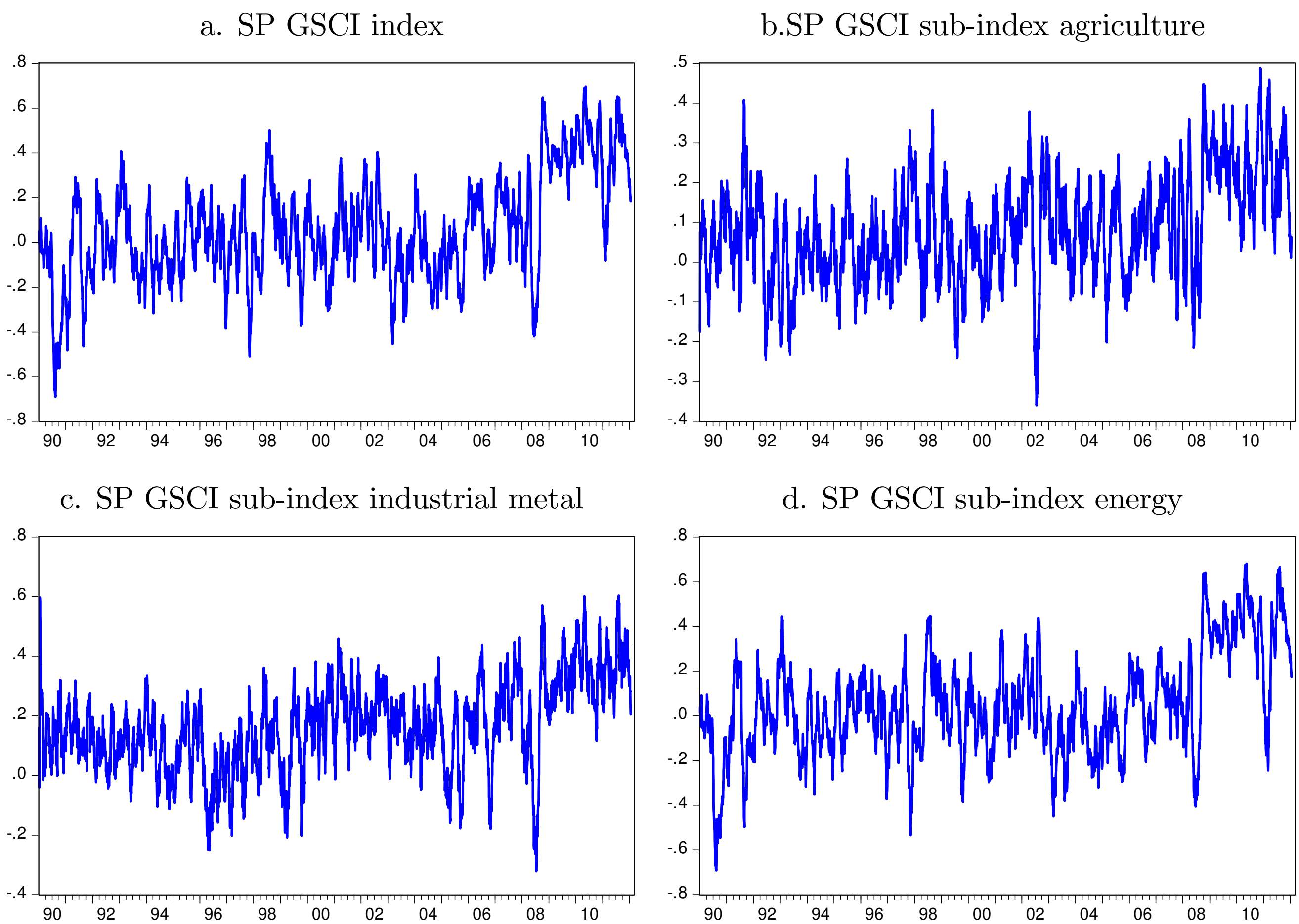
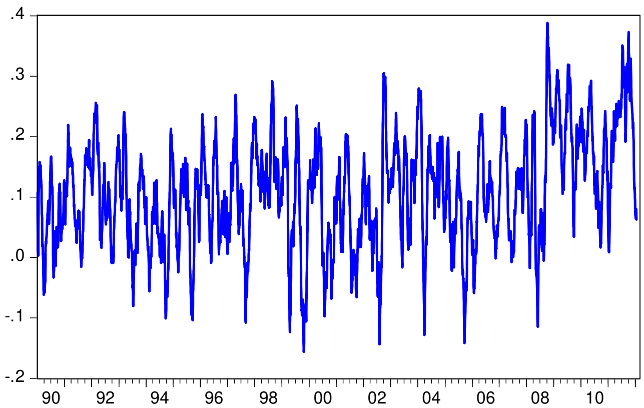


Figure 4: Model Selection for Copulas

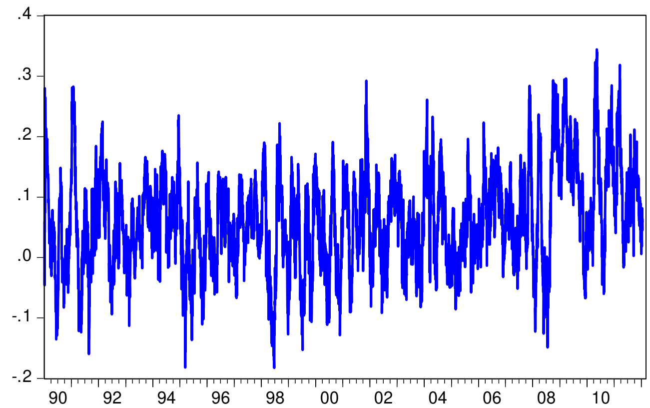
Agricultural markets, time varying normal copula

Food and Fiber - S&P 500

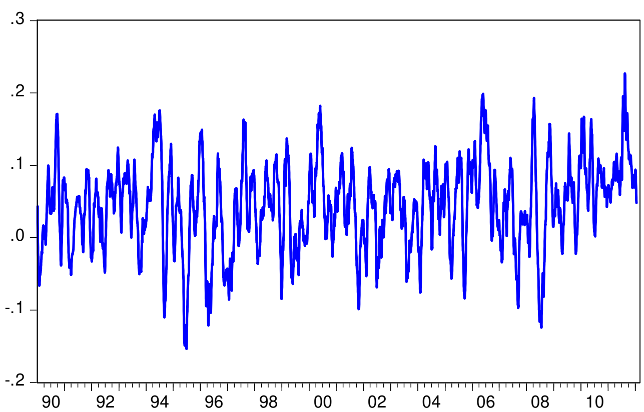
a. Coffee



b. cotton



c. Orange jus



d. Sugar

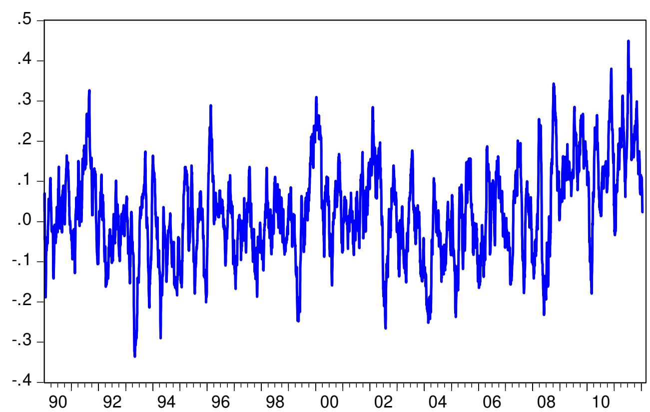
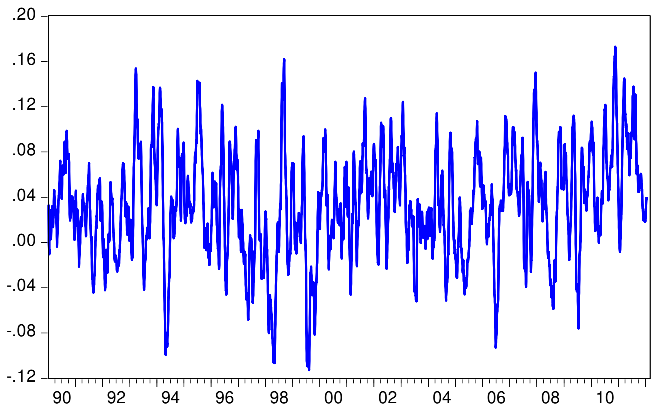


Figure 5: Model Selection for Copulas

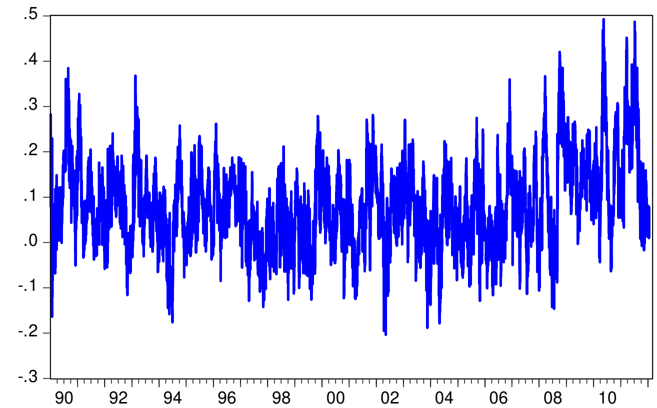
Agricultural markets, time varying normal copula (cont.)

Meat and livestock-S&P 500

a. Lean hogs



b. Live cattle



c. Pork belly

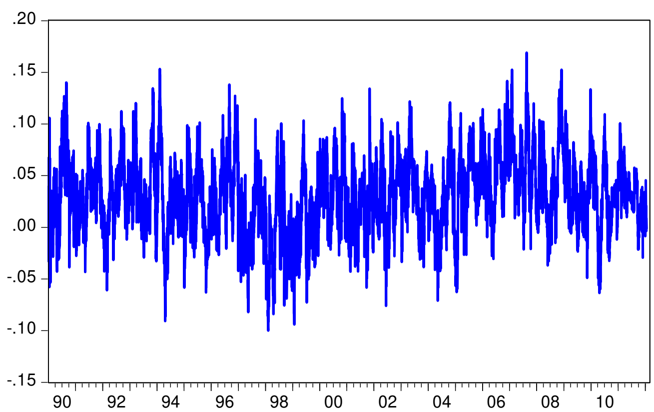
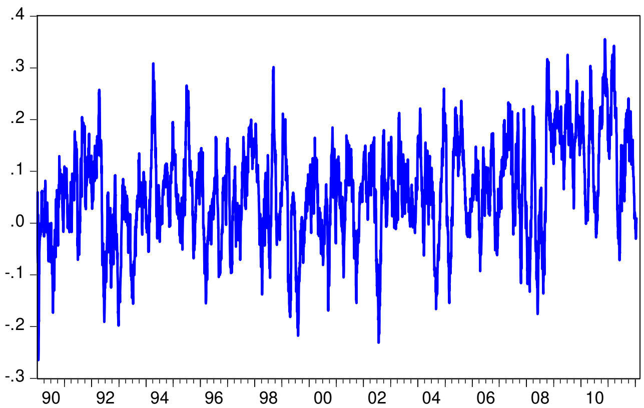


Figure 6: Model Selection for Copulas

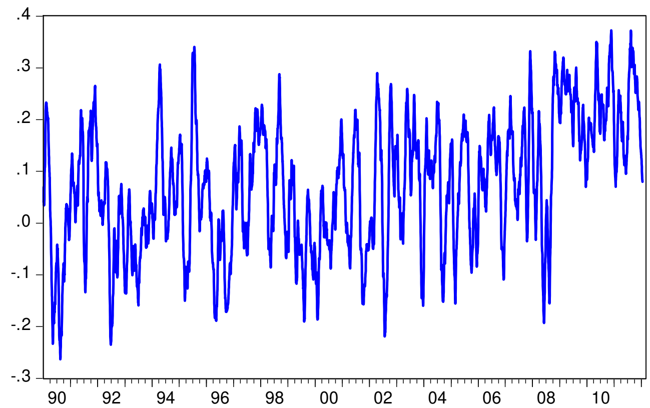
Agricultural markets, time varying normal copula (cont.)

Grains and oilseeds - S&P 500

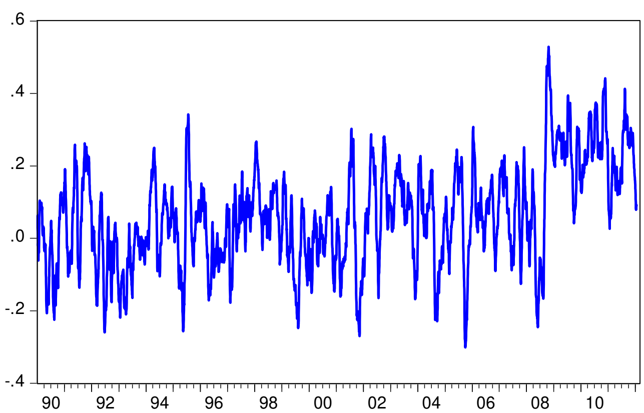
a. Corn



b. Soybeans



c. Soybean oil



d. Wheat

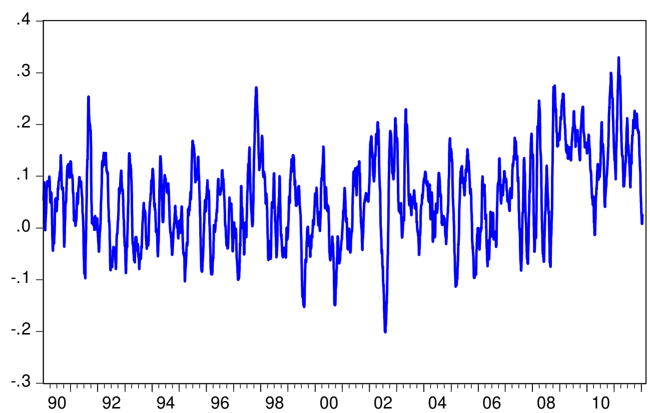
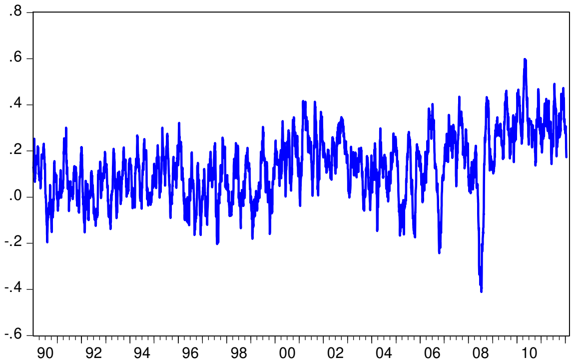


Figure 7: Model Selection for Copulas

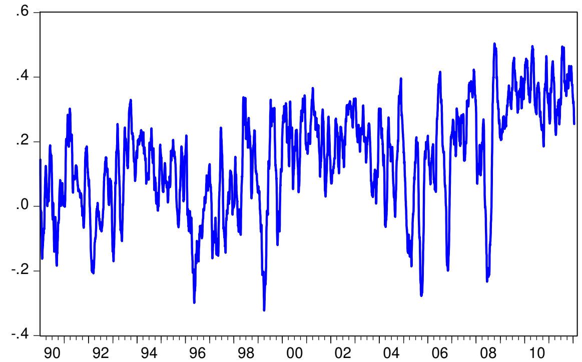
Metal markets, time varying normal copula (cont.)

Base metals - S&P 500

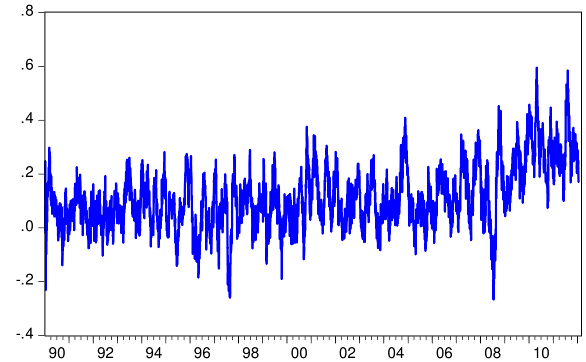
a. Aluminium



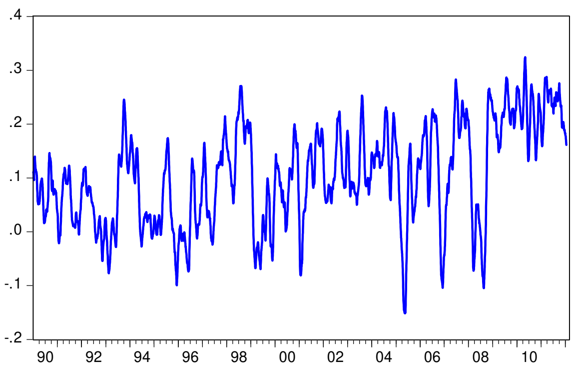
b.Copper



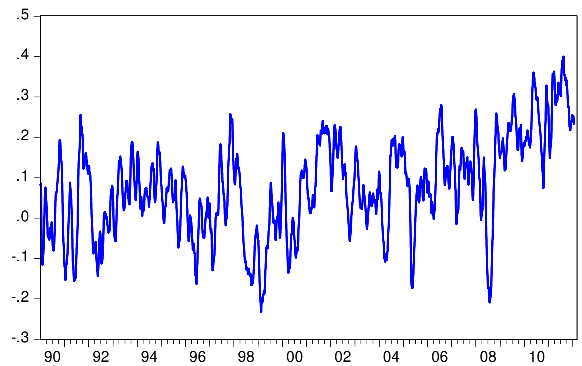
c. Lead



d. Nickel



e. Tin



f. Zinc

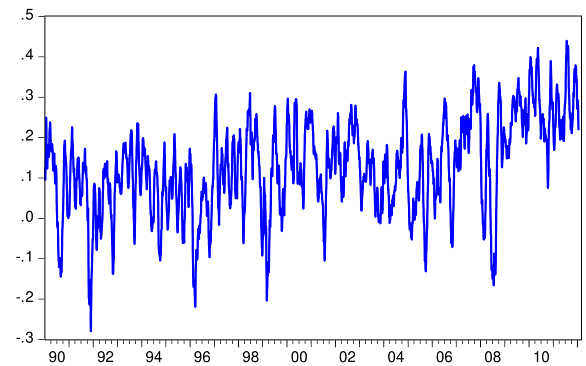


Figure 8: Model Selection for Copulas, Individual Commodities

Energy Markets