1514
—1518
1858
1519

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Sequences 1514-1518.

Sequence 1519

Sequence 1858

Dear John,

There is a small mountain of sequences to be included in the catalog which require some work on my part, (other than just copying them out), and I'm starting to work through it now.

I've just included some sequences you suggested in a letter of July 16, 1968; namely the sequences

$y_n(1)$, $y_n'(1)$, $y_n''(1)$, $y_n(2)$, $y_n(3)$

where $y_n(x)$ is a Bessel polynomial. I describe them as "From Bessel Polynomials" and refer to Comb. Identities p. 77 and to a personal communication from you.

Also f_{2n}, the bisection of the Fibonacci series, referring to Comb. Identities p. 86.

Also $A_n^*(1)$: (1), 1, 2, 7, 38, 291, ..., "Forests of labeled trees", referring to Journal Comb Theory 5 96 68 (your paper) and to a personal communication from you (on October 16, 1970, for which many thanks

As to their Types, $y_n(1)$, $y_n'(1)$, ... are Type 1, (implying that there is an explicit formula for the n^{th} term) since $y_n(1) = (2n-1)\, y_{n-1}(1) + y_{n-2}(1)$ is explicitly solvable, and so are the others. f_{2n} is also Type 1 of course. So is $A_n^*(1)$ $= Y_n(1, \cdots, n^{n-2})$.

Sequence 262

Also $G_n(1)$: (1,) 1, 3, 13, 73, ... (number 262), with a new name "Forests of Greatest Height", and the type corrected to Type 1.

From the xeroxed table of planted trees enclosed in your letter of October 16, 1970:

Sequence 1678

The column sums, (1, 0, 1,) 1, 2, 3, 6, 10, 19, ... are what Harary calls "homeomorphically irreducible planted trees" (Acta Mathematica 101 (1959) p 150) - David Cantor at UCLA computed many terms of this sequence for me on his polynomial manipulating system on the IBM 360/85 (or 95 I think - it is a much bigger computer than Bell's). (Of course his numbers agree with yours.) I've changed the name to "series-reduced".

Sequence 1669

I've used the same name for the row sums (1,) 1, 2, 5, 12, 33, ..., referring to Cayley and to a p.c. from you, and for

1 & 1860

$Q_{n,n-3}$ and $Q_{n,n-4}$

referring to a p.c. from you. "~~Homeomorphically irreducible~~" changed to series-reduced ~~is slightly more musical than "with no points of degree two"?~~

I classified the row and column sums as Type 2, since the recurrence doesn't look solvable, and $Q_{n,n-3}$ and $Q_{n,n-4}$ as type 1 because of their nice differences.

Thank you also for your letter of October 19, 1970, with many more sequences and comments on the draft of chapter 2 - excellent (the comments, not the draft). All of your suggestions will be used.

Sequence 1861

$B_n(2)$: 1, 2, 6, 22, ... I've called Values of Bell Polynomials, Type 1 (since Stirling numbers are), Reference p.c. from J.R.

Sequence 1862

$L^*_n(1)$: (1,) 1, 2, 7, 26, 111, 562, 3151, ... I've called
Forests of Least Height, Type 1, Ref JCT 5 97 68 and p.c.
from J.R. again.

Sequence 669

<u>From letter of September 29, 1970.</u>

Thank you for twenty terms of B_n, series reduced trees, which
you accomplished without any mechanical aids. ~~The~~ I would
have thought that some manufacturer of calculating machines -
such as Monroe - would have given you one in gratitude,
and so that they could mention you in their advertisements
- Riordan uses a Monroe 1102 : shouldn't you? Laver has all
his tennis rackets given to him, for just this reason.

Sequence 735

<u>From letter of August 31, 1970.</u>

The numbers W_n of our joint paper appear in the form
$\frac{W_n}{n}$: 1, 8, 78, 944, ... , the form they originally took in my
thesis, and a sequence I worried over for a long time.
It is not the most natural, so now W_n : 1, 4, 26, ... and
W_n : 2, 24, 312, ... are also included as "Trees by total
height". I haven't been able to decide on the best way to
enter a sequence with lots of common factors — W_n
still has a common factor of 2! — here are the present
rules (a) use whatever form the original author decided
to tabulate (b) take out common factors of 2, 3, 4, (and
rarely 5, 6) if it seems appropriate and include the
reduced sequence as well as the original (highly subjective
- lately I haven't bothered to do this much) (c) when in
doubt use a transformation only if it moves a sequence
towards the beginning of the table, i.e. decreases the
second term. Important sequences of even ~~odd~~ numbers
(noncyclic simple groups, differences between primes) also
& tangent numbers)

Sequence 1863-4

appear divided by 2.

C_n and C_{nn} went in.

Four New Sequences

Let t_{pd} be the number of unlabeled trees with p points and diameter d
" T_{pd} " labeled " "
" r_{ph} " unlabeled " height h
" R_{ph} " labeled " "

then the new sequences are

$$\bar{t}_p = \sum_d d\, t_{pd} \quad : (0,)\ 1, 2, 5, 9, 21, 44, 103, \ldots$$

$$\bar{T}_p = \sum_d d\, T_{pd} \quad : (0,)\ 1, 6, 44, 430, 5322, \ldots$$

$$\bar{r}_p = \sum_h h\, r_{ph} \quad : (0,)\ 1, 3, 8, 22, 58, \ldots$$

$$\bar{R}_p = \sum_h h\, R_{ph} \quad : (0,)\ 2, 15, 148, 1785, \ldots$$

(they are measures of the average diameter and height of trees, and are suggested by Meir & Moon's problem at the bottom of p.99, JCT 8 (Jan 1970))

Comtet's Book

It is a brilliant idea to use italic figures for the page numbers of volume 2!

On page 67 (not 67) he has copied Guy's erroneous value for the seventeenth Catalan number (Guy 1967a):

q_{17} should be 35357670 (not 37638810)

This is already too long. More soon, with a map. We are very much looking forward to seeing you both on the 29th.

Regards

Neil