IAS 12(1940)

2053
8836
7421
2819

Liouville

ON A TABLE OF VALUES OF L (n)

By Hansraj Gupta

(*Department of Mathematics, Government College, Hoshiarpur*)

Received September 11, 1940

(Communicated by Dr. B. L. Bhatia, D.SC., F.N.I.)

1. In what follows, all small letters denote integers $\geqslant 0$, p denoting a prime $\geqslant 2$.

8836

We define $\lambda(n)$ by the relations:

$$\lambda(n) = -\lambda(n/p) \text{ where } p \mid n; \text{ and } \lambda(1) = 1. \tag{1}$$

Thus $$\lambda(475) = -\lambda(95) = \lambda(19) = -\lambda(1) = -1.$$

2819

The function L (n) is now defined by the relation:

$$\mathrm{L}(n) = \sum_{m=1}^{n} \lambda(m). \tag{2}$$

At the suggestion of Dr. Chowla, I have recently calculated a table[1] which gives the values of L (n) for values of n up to 20,000. Every composite $n \leqslant 20{,}000$ is first of all broken into two factors one of these being the least prime that divides n. $\lambda(n)$ is then given immediately by (1). Thus when $\lambda(n)$ and L (n) have been calculated for every value of n up to 474, we have

$$\lambda(475) = \lambda(5\cdot 95) = -1, \text{ because } \lambda(95) = 1.$$

Moreover $$\begin{aligned}\mathrm{L}(475) &= \lambda(475) + \mathrm{L}(474), \\ &= -1 - 22 = -23.\end{aligned}$$

I find that[2]

$$\mathrm{L}(n) \leqslant 0 \text{ for } 2 \leqslant n \leqslant 20{,}000. \tag{3}$$

Most probably L (n) is negative or zero for all $n \geqslant 2$.

2. Let s be the greatest value of $n \leqslant 20{,}000$, for which

$$\mathrm{L}(n) = -t.$$

[1] The work has been carefully checked with the help of some of my students—Satnam Singh in particular.

[2] Polya verified this for values of n up to 1000.

See *Jahresberichte der deutsche Mathematiker Vereinigung*, 1919.

Please enter

2053

Then the following table gives the values of s for values of t upto 10:

t ..	0	1	2	3	4	5	6	7	8	9	10
s ..	586	591	592	1411	1422	1425	1432	3281	3296	5645	19680

3. Let h be the least value of n for which $L(n) = -k$. Then the following table gives the values of h for values of k upto 150:—

2053

Read across

k	0	1	2	3	4	5	6	7	8	9
0	1, 2	3	8	13	20	31	32	53	76	79
1	80	117	176	181	182	193	200	283	284	285
2	286	293	440	443	468	661	678	683	684	1075
3	1076	1087	1088	1091	1092	1093	1106	1109	1128	1129
4	1130	1131	1132	1637	1638	1753	1756	1759	1760	2699
5	2700	2703	2712	2713	2714	2715	2720	2731	2732	2739
6	2742	2769	2770	2801	2802	2803	2804	4157	4256	4261
7	4364	4373	4526	4527	4528	6317	6318	6381	6390	6391
8	6392	6397	6398	6399	6480	6481	6482	6575	6582	6859
9	6864	6877	6878	6969	6972	6975	6976	6977	6978	6987
10	6988	6997	7026	7027	9686	9689	9690	9695	9696	9697
11	9698	9699	9700	9719	9720	9721	9724	9725	9726	9789
12	9792	9795	9816	9817	9822	9823	9836	9837	9840	15669
13	15670	15671	15672	15675	15676	15679	15680	15745	15750	15753
14	15756	15791	15792	15795	15798	15799	15804	15805	15806	15807
15	15810	> 20000								

Within the limits of my table, $\{L(n)\}^2/n$ is the greatest for $n = 9840$. For this value of n, $\{L(n)\}^2/n$ is just $< \frac{5}{3}$. It thus appears that

$$|L(n)| = O(\sqrt{n}). \tag{4}$$

4. The following table will show the main variations in the value of $L(n)$ as n increases from 2 to 20,000.

n	L (n)	n	L (n)
2	0	3281	− 7
468	− 24	4528	− 74
586	0	5645	− 9
684	− 28	7027	− 103
880	− 6	8512	− 14
1132	− 42	9840	− 128
1411	− 3	12798	− 32
1760	− 48	15810	− 150
2264	− 8	19680	− 10
2804	− 66		