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Dear Neil:

Lest I forget, here is a new remark on the Handbook. Sequence 708 (Related to the Gamma Function, Reference SE2 78: J. Ser Les Calculs Formels des Series de Factorials, Gauthier-Villars, Paris, 1933) is also $S_n(1)$, with $S_n(x) = \sum_0^n k! \binom{n}{k}^2 x^k$, the rook polynomial for squares. ICA p. 170 & 184. = RI, 170+184

$$S_n(x) = [1+(2n-1)x]\, S_{n-1}(x) - (n-1)^2 x^2 S_{n-2}(x), \quad n = 1, 2, \ldots$$

$S_0(x) = 1$, $S_1(x) = 1+x$, $S_2(x) = 1+4x+2x^2$.

Hence

$$S_n(1) = 2n\, S_{n-1}(1) - (n-1)^2 S_{n-2}$$

Extend!

New values

n	0	1	2	3	4	5	6	7	8	9
$S_n(1)$	1	2	7	34	209	1546	13327	130922	1441729	17572114

10
234662231

→ Values for $n = 8, 9, 10$ are not in Seq. 708.

Where are you?

John (Riordan)

P.S. I am sending separately a reprint of a paper by Kreweras, 1970, which may have escaped your net. Note especially the tables on pp. 38 & 39.

Gen Stirling II Numbers

$S(n,k;\ell) = S(n-1,k-1;\ell) + k^{\ell}\, S(n-1,k,\ell)$ Comtet, Takacs

$S_n(x;\ell) = \sum S(n,k,\ell)\, x^k = x S_{n-1}(x;\ell) + (xD)^{\ell} S_{n-1}(x;\ell)$

Not in H

$S_n(x;2)$	$S_n(1;2)$	1	2	7	37	264	2433	27913	272504		
	k \ n	1	2	3	4	5	6	7	8	9	10
	1	1	1	1	1	1	1	1	1	1	1
	2		1	5	21	85	341	1365	5461	21845	87381
	3			1	14	147	1408	13013	118482	1071799	9667936
	4				1	30	627	11440	81653	1424930	23870679
	5					1	55	2002	61490	1618903	41897505
	6						1	91	5276	2[illegible]50320	10630423
	7							1	140	12136	843984
	8								1	204	25192
	9									1	285
	10										1

Not in H

$S_n(x;3)$	k	1	2	11	111	1732	41153				
	1	1	1	1	1	1	1	1	1	1	
	2		1	9	73	585	4681	37449	299593	2396745	19173961
	3			1	36	1045	28800	782281	21159036		
	4				1	100	7445	505280	33120201		
	5					1	225	35570	4951530		
	6						1	441	130826		
	7							1	784		
	8								1	1296	
	9									1	2025
	10										1

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Bell Stirling Transforms

$S(c,r,s) = \sum_i S(c,i,s-t)\, S(i,r,t) \qquad S(c,r,1) = S(c,r)$ - Stirling II

H 1178 $S(n,k;2)$	$B_n(2)$ / $k \backslash n$	1	3	12	66	338	2471	19302	167894	1606137	16733779
		1	2	3	4	5	6	7	8	9	10
$S(n,n;2) = 1$	1	1	2	5	15	52	203	877	4140	21147	115975
$S(n,n-1;2) = 2\binom{n}{2}$	2		1	6	32	175	1012	6230	40819	283944	2098424
$S(n,n-2;2) = 5\binom{n}{3} + 12\binom{n}{4}$	3			1	12	110	945	8092	70756	638423	5971350
$S(n,1;2) = B_n$	4				1	20	280	3465	40992	479976	5660615
	5					1	30	595	10010	156072	2350950
	6						1	42	1120	24570	487704
	7							1	56	1932	63350
	8								1	72	3120
	9									1	90
	10										1

H 1455 $S(n,k;3)$	$B_n(3)$	1	4	22	154	1304	12915	146115	1855570	26097835	402215465
$S(n,1;3) = B_n(2)$	1	1	3	12	66	358	2471	19302	167894	1606137	16733779
$S(n,n;3) = 1$	2		1	9	75	660	6288	65051	728556	8792910	113805204
$S(n,n-1;3) = 3\binom{n}{2}$	3			1	18	255	3465	47838	685580	10285488	162016200
	4				1	30	675	12495	235193	4444188	85653900
	5					1	45	1365	35706	877653	21107625
	6						1	63	2562	86940	2703141
	7							1	84	4410	188370
	8								1	108	7110
	9									1	135
	10										1

JR
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