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Various authors ...

4 pages

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"Various authors ..."

Letters to the Editor

"Snowball" Primes

Dear Mr. Madachy:

Several of the question marks in Table 2 of Leslie E. Card's "Patterns in Primes —Addenda" (*JRM*, Vol. 1, No. 4, October 1968, pp. 250-252) can now be removed.

Those primes below which are underlined were verified as prime by J. B. Muskat, Department of Computer Science, University of Pittsburgh. The primes which are not underlined are those originally given by Prof. Card.

1-19-197-1979-19793-197933-1979339-19793393-197933933-1979339333 (or 1979339339)

409-4099-40993-409933-4099339-40993391-409933919-4099339193-40993391939-409933919393- ?

829-8293-82939-829399-8293993-82939939-829399397-8293993973

829-8293-82939-829399-8293993-82939939-829399399-8293993991

829-8293-82939-829399-8293993-82939939-829399399-8293993993-82939939933

829399399331 (or 829399399333)

The question marks after the 12th and 24th "snowball" rows on page 251 should be removed, as well as the question marks after the 2nd, 8th, 11th, 12th, and 13th "snowball" rows on page 252. Addition of the digits 1, 3, 7, or 9 to these entries produces composites.

Only two question marks remain: the one after 409933913 in the 7th row on page 252, and the one after 409933919393 as indicated earlier in this letter.

Edgar Karst
University of Arizona
Tucson, Arizona

* * * * *

Uncrossed Knight's Tours

Dear Mr. Madachy:

I have been able to improve on L. D. Yarbrough's uncrossed Knight's tours ("Uncrossed Knight's Tours," *JRM*, Vol. 1, No. 3, July 1968, pp. 140-142). The improvements are on the 7 × 8, 5 × 9, 6 × 9, 7 × 9, 8 × 9, and 9 × 9 boards.

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Mr. Yarbrough's table should be revised accordingly (and corrected, since an error appeared in his 5×6 entry).

Ronald E. Ruemmler
Roselle, New Jersey

* * * * *

Dear Mr. Madachy:

Mr. Yarbrough's uncrossed Knight's tours are a pleasant recreation but, not being too good at it by myself, I programed it for our computer.

Well over three billion cases were examined by the computer—without getting into any 7×9, 8×9, or 9×9 boards (too big for the computer!). Here is a summary of the findings:

The maximum 8×8 tour length is 35, as found by Yarbrough. The improvements on his 5×6, 6×6, 6×8, and 7×8 tours are the maximum length tours and they are unique (except for rotations and reflections). There are two symmetrical maximum tours for the 5×9 board of length 22, which betters Yarbrough's findings.

There are maximum *re-entrant* tours for the 5×5, 4×6, 4×8, 5×8, 6×8, 7×8, and the 5×9 boards. There are also more aesthetic, but still maximal, tours on the 4×5 and 4×7 boards, and more symmetrical maximal tours on the 5×7, 6×7, 7×7, and 4×9 boards.

Several interesting general properties of these Knight's tours were also discovered. The maximal 5×8 tour shown [in the Editor's Note which follows these letters] will yield tours of length $6n - 5$ on any $5 \times 2n$ board with $n \geqq 2$.

It is easy to show that the maximal re-entrant paths on the $3 \times n$ boards are essentially the same for $n = 4k + 2$, $4k + 3$, $4k + 4$, and $4k + 5$ where $k \geqq 1$. They will have length $4k + 2$ with the general form indicated in Figure 2 below. Likewise, the longest re-entrant path on the $4 \times n$ board has length $2n - 4$ for $n \geqq 4$, but there are non-unique solutions.

A $4n \times 4n$ board gives a re-entrant path of length $n^2 - 8n + 12$, where n is a multiple of 4. This is illustrated in Figure 3 (below) for the 16×16 board with a path length of 140.

Donald E. Knuth
Institute for Defense Analyses
Princeton, New Jersey

* * * * *

Dear Mr. Madachy:

The Knight's tours article proved very interesting and S. Kobayashi and I have managed to improve several of Mr. Yarbrough's results. These are shown below for the 6×6, 6×8, 5×9, 7×9, and 9×9 boards.

Michio Matsuda
Tokyo, Japan

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NOTE: Mr. Yarbrough's uncrossed Knight's tours seem to have "caught on." In addition to reader reaction (as indicated by the previous letters), Martin Gardner presents his "Mathematical Games" readers (*Scientific American,* Vol. 220, No. 4, April 1969, page 125) with the problem of finding the length-17 tour on the 6 × 6 board.

The table below is a revised listing of the maximal uncrossed Knight's tours, incorporating reader findings to date. Figure 1 shows the various tours mentioned in the three letters above; Figures 2 and 3 are discussed by Mr. Knuth above.

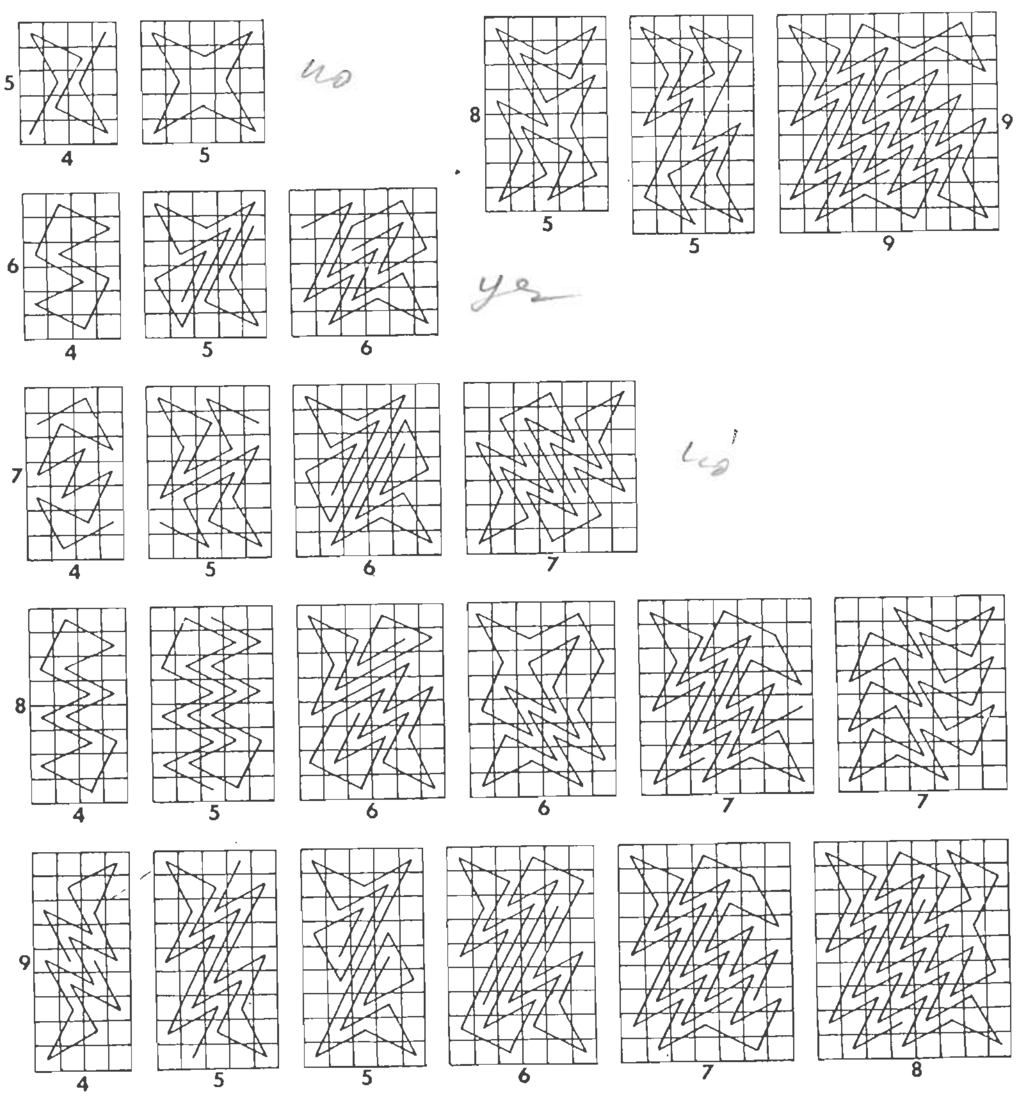

FIGURE 1

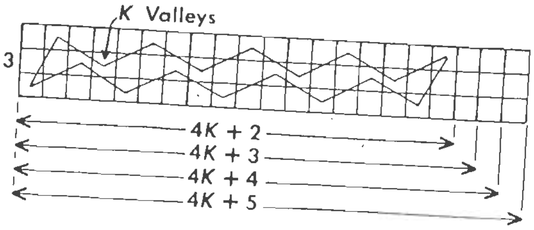

FIGURE 2

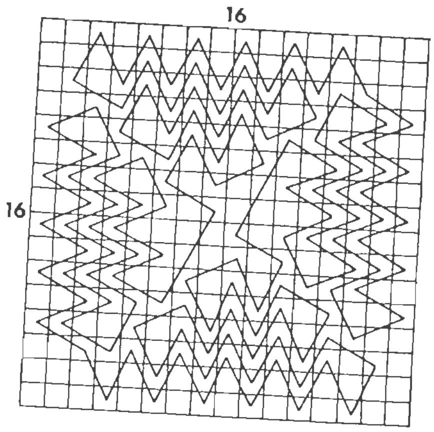

FIGURE 3

TABLE 1—Maximal Uncrossed Knight's Tours

m \ n	3	4	5	6	7	8	9
3	2						
4	4	5					
5	5	7	10 (8*)				
6	6*	9 (8*)	14	17			
7	8	11	16	21	24*		
8	9	13 (12*)	19 (18*)	25 (22*)	30 (26*)	35	
9	10	15	22 (20*)	29	35	42	47

* Maximal *Re-entrant* solutions

What this editor finds most interesting is the fact that although tours for the larger boards (7 × 9, 8 × 9, 9 × 9) were too much for a computer, Mr. Ruemmler, S. Kobayashi, and Michio Matsuda were able to find better tours without examining (I'm sure) several billion cases! Of course, we don't know if these tours are really maximal.