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Uncrossed Knight's Tours

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A variant of the classical problem of the Knight's tour in a square (or more generally, rectangular) board arises if we consider only those paths for the Knight which do not cross themselves. That is, if the path of the Knight is defined as the line connecting the center of one square visited by the Knight to the center of the next square the Knight visits, we forbid any future moves of the Knight which would cross that path, or visit a square previously visited, except for the re-entrant move to be considered soon.

In a 3 x 3 board, a Knight can make only two moves, visiting three squares, before running out of room: any further move must cross the path of the first move or double back on the second, which is not permitted.

Of course, in making an investigation of problems of this type, one can select one's own rules. For example, one may wish to consider paths of the Knight for which the maximum number of squares are visited, or alternatively paths in which the Knight makes the maximum number of moves. These two sets of paths are equivalent (why?) except for the case in which the Knight is permitted to return, on the last move of the tour, to his original square. Solely for the purpose of including a couple of pleasingly symmetrical tours, we will consider the latter problem: maximizing the number of moves the Knight can make without crossing or otherwise intersecting his path (again, except for the last move, which may be *re-entrant*, that is, return the Knight to his starting point).

TABLE 1

Maximal Uncrossed Knight's Tours

m \ n	3	4	5	6	7	8	9
3	2						
4	4	5					
5	5	7	10				
6	6*	9	11	~~16~~ 17			
7	8	11	16	21	24*		
8	9	13	19	24	29	35	?
9	10	15	20	27	32	40	~~41~~ 47 — not necess'y max!

* Re-entrant solutions

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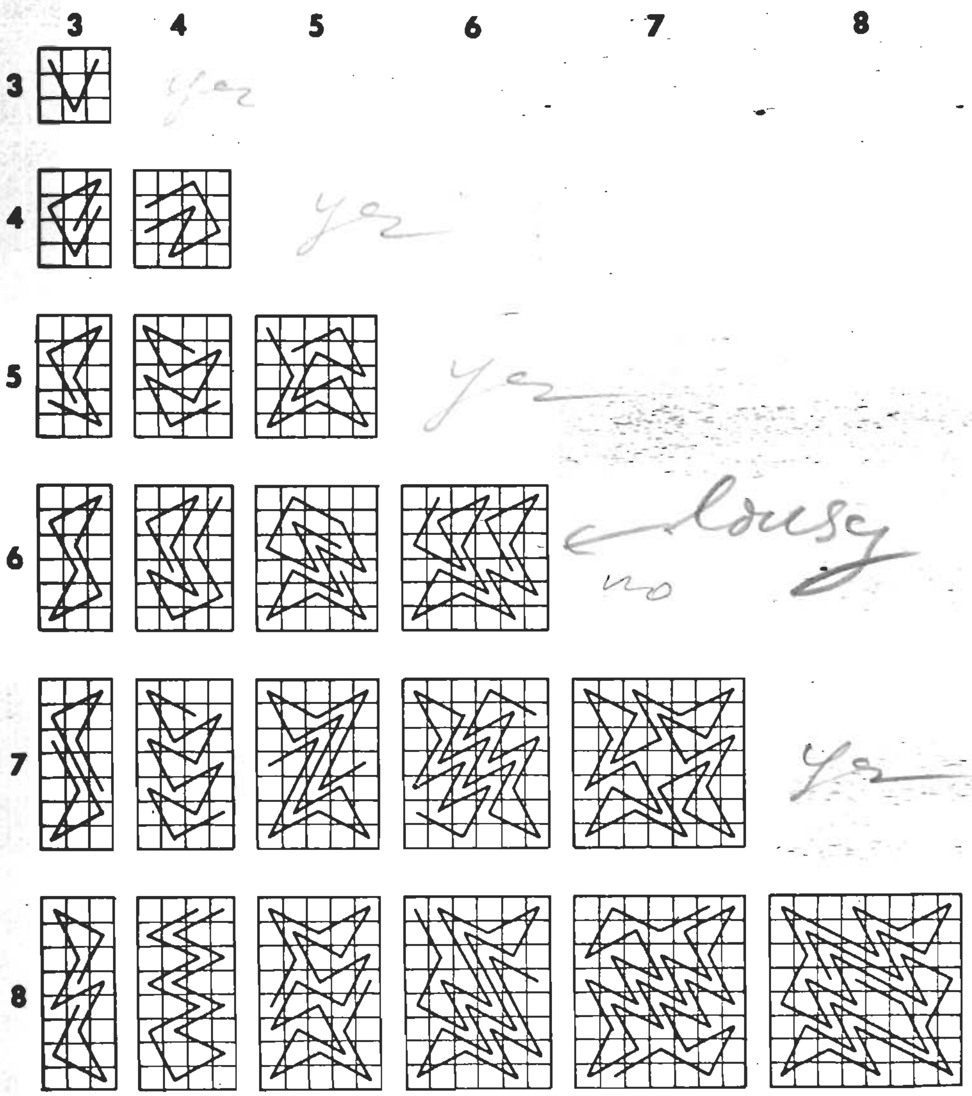
It happens that while there are many re-entrant paths in rectangles of various dimensions, only rarely does such a path also prove to be maximal: the paths in the 3 x 6 and 7 x 7 boards are the pleasant exceptions.

Table 1 summarizes my own investigation of the rectangles up to 9 x 9. At present too little is known about this problem area to offer a proof that the larger ones are indeed optimal, but the reader will surely have difficulty in improving on them. Figure 1 gives some of the solutions for boards up to 8 x 8 and the reader is invited to try his hand on these and higher order boards. I would be pleased to hear of improvements on these results.



yes

yes

yes

lousy

no

yes

yes

FIGURE 1

What values should one expect this maximal tour length to take as a function of the dimensions of the board, on the basis of the empirical data in Table 1? Is there some simple function of the number of squares on each side which will accurately predict the expected tour length? If there were, one could use this as a guide in attempting to find maximal tours for larger boards. Two such functions, which appear about equally useful for the data at hand, are:

$$mn/2 \text{ and } (m-2)(n-2).$$

If the reader is interested in evaluating these functions for the values associated with Table 1, he will observe some interesting relations between these two functions and the tabulated data. Also, if we call the first function $(mn/2)$, F_1 and the second function $(m-2)(n-2)$, F_2, one may observe that $aF_1 + bF_2$, for any a and b for which $a + b = 1$, also is a good approximation to the table. Further, one may observe that for very large m and n, F_2 gets very much larger than F_1, so that at any rate one of the functions is definitely better than the other. I leave to the reader the decision as to which, if either, should be used.