N1215 = 3275
3276

[MI2]

f91

THE EQUATION $\phi(n) = \phi(n + 1)$

by Kathryn Miller, DePauw University

The following table gives a list of solutions to $\phi(n) = \phi(n + 1)$ for $1 \leq n < 500{,}000$. The solutions for $n < 100{,}000$ have been found previously [1]. No new solutions to $\phi(n - 1) = \phi(n) = \phi(n + 1)$ were found (the equations are true for $n = 5187$), nor was any new n found such that the odd one of n and $n + 1$ is not divisible by 15 (the only such n known is $n = 5187$).

value of $\phi(n) = \phi(n+1)$

enter

new terms! →

N1215

N741.5
= 3275

n	$n + 1$	$\phi(n) = \phi(n + 1)$
1	2	1
3	$4 = 2^2$	2
$15 = 3 \cdot 5$	$16 = 2^4$	8
$104 = 2^3 \cdot 13$	$105 = 3 \cdot 5 \cdot 7$	48
$164 = 2^2 \cdot 41$	$165 = 3 \cdot 5 \cdot 11$	80
$194 = 2 \cdot 97$	$195 = 3 \cdot 5 \cdot 13$	96
$255 = 3 \cdot 5 \cdot 17$	$256 = 2^8$	128
$495 = 3^2 \cdot 5 \cdot 11$	$496 = 2^4 \cdot 31$	240
$584 = 2^3 \cdot 73$	$585 = 3^2 \cdot 5 \cdot 13$	288
$975 = 3 \cdot 5^2 \cdot 13$	$976 = 2^4 \cdot 61$	480
$2204 = 2^2 \cdot 19 \cdot 29$	$2205 = 3^2 \cdot 5 \cdot 7^2$	1008
$2625 = 3 \cdot 5^3 \cdot 7$	$2626 = 2 \cdot 13 \cdot 101$	1200
$2834 = 2 \cdot 13 \cdot 109$	$2835 = 3^4 \cdot 5 \cdot 7$	1296
$3255 = 3 \cdot 5 \cdot 7 \cdot 31$	$3256 = 2^3 \cdot 11 \cdot 37$	1440
$3705 = 3 \cdot 5 \cdot 13 \cdot 19$	$3706 = 2 \cdot 17 \cdot 109$	1728

TABLE 1. Solutions of $\phi(n) = \phi(n + 1)$ for $1 \leq n \leq 500{,}000$.

TABLE 1 continued.

n	$n + 1$	$\phi(n) = \phi(n+1)$
$5186 = 2 \cdot 2593$	$5187 = 3 \cdot 7 \cdot 13 \cdot 19$	2592
$5187 = 3 \cdot 7 \cdot 13 \cdot 19$	$5188 = 2^2 \cdot 1297$	2592
$10604 = 2^2 \cdot 11 \cdot 241$	$10605 = 3 \cdot 5 \cdot 7 \cdot 101$	4800
$11715 = 3 \cdot 5 \cdot 11 \cdot 71$	$11716 = 2^2 \cdot 29 \cdot 101$	5600
$13365 = 3^5 \cdot 5 \cdot 11$	$13366 = 2 \cdot 41 \cdot 163$	6480
$18315 = 3^2 \cdot 5 \cdot 11 \cdot 37$	$18316 = 2^2 \cdot 19 \cdot 241$	8640
$22935 = 3 \cdot 5 \cdot 11 \cdot 139$	$22936 = 2^3 \cdot 47 \cdot 61$	11040
$25545 = 3 \cdot 5 \cdot 13 \cdot 131$	$25546 = 2 \cdot 53 \cdot 241$	12480
$32864 = 2^5 \cdot 13 \cdot 79$	$32865 = 3 \cdot 5 \cdot 7 \cdot 313$	14976
$38804 = 2^2 \cdot 89 \cdot 109$	$38805 = 3 \cdot 5 \cdot 13 \cdot 199$	19008
$39524 = 2^2 \cdot 41 \cdot 241$	$39525 = 3 \cdot 5^2 \cdot 17 \cdot 31$	19200
$46215 = 3^2 \cdot 5 \cdot 13 \cdot 79$	$46216 = 2^3 \cdot 53 \cdot 109$	22464
$48704 = 2^6 \cdot 761$	$48705 = 3 \cdot 5 \cdot 17 \cdot 191$	24320
$49215 = 3 \cdot 5 \cdot 17 \cdot 193$	$49216 = 2^6 \cdot 769$	24576
$49335 = 3 \cdot 5 \cdot 11 \cdot 13 \cdot 23$	$49336 = 2^3 \cdot 7 \cdot 881$	21120
$56864 = 2^5 \cdot 1777$	$56865 = 3 \cdot 5 \cdot 17 \cdot 223$	28416
$57584 = 2^4 \cdot 59 \cdot 61$	$57585 = 3 \cdot 5 \cdot 11 \cdot 349$	27840
$57645 = 3^3 \cdot 5 \cdot 7 \cdot 61$	$57646 = 2 \cdot 19 \cdot 37 \cdot 41$	25920
$64004 = 2^2 \cdot 16001$	$64005 = 3 \cdot 5 \cdot 17 \cdot 251$	32000
$65535 = 3 \cdot 5 \cdot 17 \cdot 257$	$65536 = 2^{16}$	32768
$73124 = 2^2 \cdot 101 \cdot 181$	$73125 = 3^2 \cdot 5^4 \cdot 13$	36000
$105524 = 2^2 \cdot 23 \cdot 31 \cdot 37$	$105525 = 3^2 \cdot 5^2 \cdot 7 \cdot 67$	47520
$107864 = 2^3 \cdot 97 \cdot 139$	$107865 = 3^3 \cdot 5 \cdot 17 \cdot 47$	52992
$123824 = 2^4 \cdot 71 \cdot 109$	$123825 = 3 \cdot 5^2 \cdot 13 \cdot 127$	60480
$131144 = 2^3 \cdot 13^2 \cdot 97$	$131145 = 3 \cdot 5 \cdot 7 \cdot 1249$	59904
$164175 = 3 \cdot 5^2 \cdot 11 \cdot 199$	$164176 = 2^4 \cdot 31 \cdot 331$	79200
$184635 = 3^2 \cdot 5 \cdot 11 \cdot 373$	$184636 = 2^2 \cdot 31 \cdot 1489$	89280
$198315 = 3^3 \cdot 5 \cdot 13 \cdot 113$	$198316 = 2^2 \cdot 43 \cdot 1153$	96768
$214334 = 2 \cdot 31 \cdot 3457$	$214335 = 3^2 \cdot 5 \cdot 11 \cdot 433$	103680
$215775 = 3^2 \cdot 5^2 \cdot 7 \cdot 137$	$215776 = 2^5 \cdot 11 \cdot 613$	97920
$256274 = 2 \cdot 97 \cdot 1321$	$256275 = 3^2 \cdot 5^2 \cdot 17 \cdot 67$	126720

TABLE 1 continued.

n	n + 1	$\phi(n) = \phi(n+1)$
$286995 = 3\cdot5\cdot19^2\cdot53$	$286996 = 2^2\cdot157\cdot457$	142272
$307395 = 3^5\cdot5\cdot11\cdot23$	$307396 = 2^2\cdot31\cdot37\cdot67$	142560
$319275 = 3^3\cdot5^2\cdot11\cdot43$	$319276 = 2^2\cdot19\cdot4201$	151200
$347324 = 2^2\cdot31\cdot2801$	$347325 = 3\cdot5^2\cdot11\cdot421$	168000
$388245 = 3\cdot5\cdot11\cdot13\cdot181$	$388246 = 2\cdot17\cdot19\cdot601$	172800
$397485 = 3^2\cdot5\cdot11^2\cdot73$	$397486 = 2\cdot23\cdot8641$	190080
$407924 = 2^2\cdot11\cdot73\cdot127$	$407925 = 3^2\cdot5^2\cdot7^2\cdot37$	181440
$415275 = 3\cdot5^2\cdot7^2\cdot113$	$415276 = 2^2\cdot17\cdot31\cdot197$	188160
$454124 = 2^2\cdot11\cdot10321$	$454125 = 3\cdot5^3\cdot7\cdot173$	206400
$491535 = 3^3\cdot5\cdot11\cdot331$	$491536 = 2^4\cdot31\cdot991$	237600

It is also of interest to note the distribution of solutions to $\phi(n) = \phi(n+1)$ [1], as in Table 2. It seems that the number of solutions varies roughly as the logarithm of n, but more evidence is necessary before any definite relationship can be established.

n	# of solutions to $\phi(n) = \phi(n+1)$ in the interval	Cumulative # of solutions
0 - 1	1	1
2 - 10	1	2
11 - 100	1	3
101 - 1000	7	10
1001 - 10000	7	17
10001 - 100000	19	36

TABLE 2. Number of solutions to $\phi(n) = \phi(n+1)$, $n \leq 100{,}000$.

Reference

1. M. Lal and P. Gillard, On the equation $\phi(n) = \phi(n + k)$, Mathematics of Computation, vol. 26, 1972, pp. 579 - 583.

Addendum

In Table 1, each of the pairs (n, n + 1) has one member divisible by 15, except for (1, 2), (3, 4), (104, 105), (5186, 5187), and (5187, 5188). It might be tempting to conjecture that all larger pairs would have one member divisible by fifteen, but the next two solutions of $\phi(n) = \phi(n + 1)$ do not have this property:

n	n + 1	$\phi(n) = \phi(n + 1)$
$525986 = 2 \cdot 181 \cdot 1453$	$525987 = 3^3 \cdot 7 \cdot 11^2 \cdot 23$	261360
$546272 = 2^5 \cdot 43 \cdot 397$	$546273 = 3^2 \cdot 7 \cdot 13 \cdot 23 \cdot 29$	266112