Scan ~~A3477~~

A3471

(delicate) + many

John Riordan

~~Q NJA~~ 2 letters + 2 pages

1974

5 pages

add to ~~many~~ sequences

8 of many

(8) f91 N98

A3471 ~~283~~ 2083
A316 3470
A459 3481
A1788 3482
Oct. 24, 1974 A1818
A3472 A248
A245094 A1792
A259859 A1333
A129
A59300
A1858
A138464
A1862

Neil Sloane, Esq.
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Dear Neil:

Thanks for sending the Lucas pages so promptly. In trying to make sense of them, I ran into the following, for the Handbook

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1. The number of permutations of n with no "hits" on the two main diagonals ($A_n(0)$ of I.C.A, prob 9 of chap. 7, p. 187) is not one of Lucas' and does not appear in the Handbook. It starts

3168
459
3471

n	3	4	5	6	7	8	9	10	11
$= A_n(0)$	0	4	16	80	672	4752	48768	440192	5380160
		1	4	20	168	1188			

$A_7(0)$ is given incorrectly in ICA as 528

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2. Some time ago, working over Cayley on partitions, I ran into the sum $\binom{l+1}{n} = \sum a_{nk}\binom{l}{n-k}$, which entails $a_n(x) = 2a_{n-1}(x) + x\,a_{n-2}(x)$, $a_0(x) = 1$, $a_1(x) = 1+2x$. $a_n(1)$ is sequence ~~1064~~ A1333; $a_{n0} = 2^n$ and $a_{n1} = (n+1)2^{n-2}$ is sequence 1100. Should it be noticed that 1100 is $(n+1)2^{n-2}$? A1792

done already

N1729
1788

3. The same polynomial $a_n(x)$, with $a_0(x) = 1$, $a_1(x) = 2$, has $a_n(1)$ as sequence 552, $a_{n1} = (n-1)2^{n-2}$ as 1398, $a_{n2} = \binom{n-2}{2}2^{n-4}$ as 1729 A129, $\binom{n-3}{3}2^{n-6} = a_{n3}$ as 1916, $a_{n4} = \binom{n-4}{4}2^{n-8}$ does not appear; it goes 1, 10, 60, 280, 1120, 4032, 13440 = A3472

For these boundary conditions $a_n(x) = \sum_0 \binom{n-k}{k} 2^{n-2k} x^k$

There are more sequences in my budget but I will defer to next week

John J. Riordan

N297 = 2083

3470 –
3481 –
3482 –

f91

Neil Sloane, Esq.
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Nov 14, 1974

~~208~~

Dear Neil:

Here are some more sequences (promised in my letter of 11/24/74).

1. Random Tournaments sequence 297, I ran into the paper by Capell & Narayana which you reference and found they had neglected the simple recurrences implicit in their definitions. If T_n is the number, their definitions may be compressed to

N297

$T_1 = 1$, $T_n = \sum_{1}^{m} T_{n-k}$, $m = [n/2]$ which implies $T_{2n} = 2T_{2n-1}$ $\forall n \geq 2$, $T_{2n+1} = 2T_{2n} - T_n$. Making the table

f

n	1	2	3	4	5	6	7	8	9	10	
T_{2n}	1	2	6	22	84	330	1308	5210	20796	83100	(A245094)
$2T_{2n}$	2	4	12	44	168	660	2616	10420	41592	166200	
T_n	1, 1	2	2	3	6	11	22	42	84		A2083
T_{2n+1}	1	3	11	42	165	654	2605	10398	41550	166116	

f91

Thus there are 10 additional numbers for 297

2. ICA 187ff Prob 10 Chap 7. Write $d_n(x) = \sum d_{nk} x^k$ $[d_0 = 1, d_1 = x, d_2 = 1 + x + x^2]$; then $d_{n0} = D_n = nD_{n-1} + (-1)^n$

$$d_{nk} = (n-k) d_{n-1,k} + d_{n-1,k-1}, \quad k = 1, 2$$

$$k d_{nk} = d_{n,k-1} + d_{n-1,k-1}$$

f91

A3470

imply

f

$$d_n(x) = n d_{n-1}(x) - x d_{n-2}(x) + x^n + (-1)^n, \quad d_n'(x) = d_n(x) + d_{n-1}(x) - x^n$$

and $d_n(1) = d_n = n d_{n-1} - d_{n-2} + 1 + (-1)^n$, $d_n'(1) = d_n + d_{n-1} - 1$

3470 =

N1109.5

n	0	1	2	3	4	5	6	7	8	9	10	
d_n	1	1	3	8	31	147	853	5824	45741	405845	4012711	A3470 — new
$d_n'(1)$	0	1	3	10	38	177	999					A259859

~~Corrected~~ which ~~is~~ are not in the handbook.

f91

N297 2083

DUMP2 April 25, 1975 15:18:57

Page 1

SMS

(1 , 1 , 1 , 2 , 3 , 6 , 11 , 22 , 42 , 84 , 165 , 330 , 654 , 1308 ,
2605 , 5210 , 10398 , 20796 , 41550 , 83100 , 166116 , 332232 , 664299 ,
1328598 , 2656866 , 5313732 , 10626810 , 21253620 , 42505932 ,
85011864 , 170021123 , .null. , .null. , .null. , .null.)

f91

Cut^d 3470

(1 , 1 , 3 , 8 , 31 , 147 , 853 , 5824 , 45741 , 405845 , 4012711 ,
43733976 , 520795003 , 6726601063 , 9_3651619881 , 139_8047697152 ,
2227_5111534553 , 37727_8848390249 , 676874_4159489931 ,
12822886_0181918440 , 255780845_9478878871 , 5358574878_8874537851 ,
.null. , .null. , .null. , .null. , .null. , .null. , .null. , .null. ,
.null.)

SSC

f91

from Riordan's letter but not used

(0 , 1 , 3 , 10 , 38 , 177 , 999 , 6676 , 51564 , 451585 , 4418555 ,
47746686 , 564528978 , 7247396065 , 10_0378220943 , 149_1699317032 ,
2367_3159231704 , 39955_3959924801 , 714602_3007880179 ,
13499760_4341408370 , 268603731_9660797310 , 5614355724_8353416721 ,
.null. , .null. , .null. , .null. , .null. , .null. , .null. , .null. ,
.null.)

DO   Lib.

3. A paper by SM Tanny and M Zuker, On a unimodal sequence of binomial coefficients, Discrete Math 9(1974), 79-89 contains the table

3481 = N837.5
3482 = N1652.7

k	1	2	3	4	5	6	7	8	9	
n_k	2	20	143	986	6764	46367	317810	2178308	14930351	Ent'd
r_k	0	5	39	272	1869	12815	87840			Ent'd

Get ref & article (5) Tanny sent me a reprint and I found

$$5\, r_{n+2} = 9 f_{4n+3} + 2 f_{4n} - 4 \qquad f_0 = f_1 = 1,\ f_{n+2} = f_n + f_{n-1}$$

which goes with the Tanny Zuker result that $n_k = f_{4k-1} - 1$
The recurrences for the sequences are $n_{k+2} = 7n_{k+1} - n_k + 5$, $k = 1, 2, \ldots$
$r_{k+2} = 7r_{k+1} - r_k + 4$, $k = 1, 2, \ldots$

(8) omitted
4. I enclose two tables which might have been included in my Forests of Labeled Trees. The first is on Forests of Free Trees. The second on trees of least height. As you see, my energy ran out on the first

N1997
A1818
f91

5. Your sequence A1818 1997, for which the reference is Central Factorial Numbers (Comb. Identities, 217) is actually $[1\cdot 3 \cdots (2n-1)]^2$ $= (-4)^n\, t(2n+1, 1)$ which appears in CI, p 235. Curiously, it is also the number of permutations of 2n with even cycles denoted by e_{2n} in ICA, p. 86 ff, from with you took the table for odd cycles (Handbook #1137). Of course $o_{2n} = e_{2n}$, $o_{2n+1} = (2n+1)\, e_{2n} = \frac{1}{2n+1}[1\cdot 3 \cdots (2n+1)^2]$.

7. Once you mentioned not having some of my reprints, but didn't say which ones. If you let me know, perhaps I can find something

Yours
John

3-2 (12-55)

Rooted and free trees of least height

$L_n(x) = Y_n(x, 2x, \ldots, nx) = \sum \binom{n}{k} k^{n-k} x^k$

$L_n^*(x) =$

	$L_n(1)$	1	1	3	10	41	196	1057	6322	41393	293604	2237911	— have N1148 = A248
	$k \backslash n$	0	1	2	3	4	5	6	7	8	9	10	
	0	1											
	1		1	2	3	4	5	6	7	8	9	10	
#1729	2			1	6	24	80	240	672	1792	4608	11520	— have N1729 = A1788
Not in H	3				1	12	90	540	2835	13608	61232	262440	
" H	4					1	20	240	2240	17920	129024	860160	
"	5						1	30	525	7000	78750	787500	Δ is now A59300
	6							1	42	1008	18144	272160	
	7								1	56	1764	41160	
										1	72	2880	
											1	90	
												1	
	$L_n^*(1)$	1	1	2	7	26	~~116~~ 111	562	3151	16252	114905	925226	— ~~new~~ have A1862 when corrected
$L_n^*(x)$	0	1											
	1		1	1	3	4	5	6	7	8	9	10	
	2			1	3	15	56	180	567	1624	4356	11160	
	3				1	6	45	255	1470	7728	38930	185580	
	4					1	10	105	875	7105	53424	386400	
	5						1	15	210	2380	14175	254205	
	6							1	21	378	5544	75285	
	7								1	28	436	11550	
	8									1	36	990	
	9										1	45	
	10											1	

column doesn't add to 116

Forests of Free Trees

$A_n^x(x) = B(x)\,(n + B(x))^{n-1} = Y_n(xT_1, \ldots, xT_n) \quad T_n = n^{n-2} \qquad A_0^x = 1$

$A_{n+1}^x(x) = x \sum_k \binom{n}{k} T_{k+1} A_{n-k}^x(x)$

A1858

A138464

$A_{n1}^x = n^{n-2}$

$A_{n2}^x = \frac{1}{2}(n+6)(n-1)\,n^{n-4}$

$A_n^x(1)$	1	2	7	38	291	2932	36961	561998	10026505	205608536
$k \backslash n$	1	2	3	4	5	6	7	8	9	10
1	1	1	3	16	125	1296	16807	224384	3687609	66025360
2		1	3	15	110	1080	13377			
3			1	6	45	435	5250			
4				1	10	105	1295			
5					1	15	210	3220		
6						1	21	378	7056	
7							1	28	430	14070
8								1	36	990
									1	45
										1