

Scan

A5254

Board of
directors
problem

1 page

JRM v9n3p240

Board of directors problem 7517

Since shareholder 1 no longer holds any shares, the problem is reduced to one involving $N - 1$ shareholders, where

$$\left. \begin{aligned} S(N-1, 1) &= S(N, 2) - S(N, 1) \\ S(N-1, 2) &= S(N, 3) - S(N, 1) \\ &\vdots \\ S(N-1, N-1) &= S(N, N) - S(N, 1) \end{aligned} \right\}$$

AS254
A37254

Since we know the solution for $N - 1$ shareholder, we need only find the minimum value of $S(N, 1)$ consistent with the stated conditions. This is obtained by demanding that the $S(N, n)$ determined using system A just barely satisfy the most stringent requirement of condition (2), i.e.

$$S(N, 1) + S(N, 2) + \dots + S(N, m) = 1 + S(N, N) + S(N, N-1) + \dots + S(N, m+1)$$

where

$$m = \frac{1}{2}N \text{ For } N \text{ even} \qquad m = \frac{1}{2}(N+1) \text{ For } N \text{ odd} \qquad (B)$$

Substituting for $S(N, n)$, $n = (2, \dots, N)$ as given by (A).

$$\begin{aligned} S(N, 1) &= 1 + S(N-1, N-1) + S(N-1, N-2) + \dots + S(N-1, m) \\ &\quad - S(N-1, 1) - S(N-1, 2) - \dots - S(N-1, m-1) \end{aligned} \qquad (C)$$

Starting with the trivial result $S(1) = S(1, 1) = 1$ and using (A), (B), and (C) along with the expression for $S(N)$ in terms of the $S(N, n)$, the results shown in the table were computed:

N	m	$S(N)$	$S(N, n) \ n = 1, N$
1	1	1	1
2	1	3	1, 2
3	2	9	2, 3, 4
4	2	21	3, 5, 6, 7
5	3	51	6, 9, 11, 12, 13
6	3	117	11, 17, 20, 22, 23, 24
7	4	271	22, 33, 39, 42, 44, 45, 46
8	4	607	42, 64, 75, 81, 84, 86, 87, 88
9	5	1363	84, 126, 148, 159, 165, 168, 170, 171, 172
10	5	3013	165, 249, 291, 313, 324, 330, 333, 335, 336, 337

A37254

The table can be filled out in a matter of minutes.

Also solved by Frank Rubin, Wappingers Falls, NY, and B. E. Wynne and T. J. Narayana. The latter two solvers have incorporated their proof in their article "Tournament Configuration and Weighted Voting," which will appear in a forthcoming issue of *Cahiers du Bureau Universitaire de Recherche Operationnelle*. All solvers outwitted the editor by providing hand solutions.