The aim ~~here~~ is to do the coding in such a way as to keep the entropy of the u sequences as low as possible, subject to this requirement of reproducibility with distortion d^* or less. Here the entropy to which we are referring ~~is the entropy to which we are referring~~ is the entropy per letter of the original source. Alternatively, we might think of the source as producing one letter per second and we are then interested in the u entropy per second.

We shall show that, for any d^* and any $\epsilon > 0$, coders and reproducers can be found that are such that $H(u) \leq R(d^*) + \epsilon$. As $\epsilon \to 0$ the block length involved in the code in general increases. This result, of course, is closely related to our interpretation of $R(d^*)$ as the equivalent rate of the source for distortion d^*. It will follow readily from the following theorem.

Theorem 2. Given an ergodic source, a distort... $_{ij}$, and rate
distortion function Γ ... the source),
given $d^* \geq d_{\min}$ an... its a set Λ
containing M words ... perties:

1) $\frac{1}{t} \log M \leq R(d$ ...

2) The average dis... arest (i.e.,
least distortion) word i...

This theorem impl... l later be
eliminated) the results ... erely uses
a device that maps any ... eproducer
is then merely an iden... er of the
coded sequence cannot ... nized at
$\frac{1}{t} \log M$ if all of the M ... M is by
the theorem to be less th...

This theorem will be ... consider
an ensemble of ways of ... average
distortion for this ensemb... w that
at least one code exists in ...

The ensemble of codes ... ll be a
set of transition probabilit... $R(d^*)$.
The set of letter probabiliti... luce a
measure $Q(Z)$ in the space ... gle Z
letter, say letter j, is $\sum_i P_i q$ ... word consisting of letters

$j_1, j_2, \ldots, j_t$ is $Q(Z) = \prod_{k=1}^{t} \left(\sum_i P_i q_i(j_k) \right)$.

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From la.tis.com!fermat!r Fri Jun 21 18:23:36 PDT 1991
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From: fermat!r@la.tis.com (Richard Schroeppel)
Message-Id: <9106220123.AA08650@rhmr.com>
To: njas@research.att.com
Subject: Magic Squares of Order 5
Status: R
```

The number of magic squares of order 5 is 4 * 68826306.

This count assumes that rotations and reflections are not counted separately. This is the counting method which finds 1 magic square of order 3, and 880 of order 4. The order 4 and 5 squares have additional group operations possible, beyond rotations and reflections. When these are taken into account, the numbers drop to 220 and 68826306.

If we count all configurations which are magic, with no reductions for isomorphisms, then the figures are multiplied by 8 for all squares bigger than 1^2. Even smaller numbers are obtained if the isomorphism "Subtract each cell from N^2+1" is applied, but it's not exactly a division by 2 since some squares will be isomorphic to reflections, etc. I don't know how these counts come out.

I wrote a program to count them in 1973, and Mike Beeler did a short paper describing the program. Martin Gardner did a column a couple of years later which mentioned the count. (I think the column was mostly about magic cubes.)

I have some old mag tapes with sufficient information to reconstruct the squares, but it's probably easier to write a new program.

A few years ago, I received a manuscript from a Japanese person, confirming my count. Unfortunately, my personal files are in a sorry state, and I can't easily retrieve it.

Rich Schroeppel
rcs@la.tis.com

****
this last sequence could be extended the formula as something to do with
catalan numbers.

Si[illegible].
next mail..........

From la.tis.com!fermat!r Sat Jun 22 00:33:23 PDT 1991
Received: by gauss; Sat Jun 22 03:36:09 EDT 1991
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Message-Id: <9106220733.AA12197@rhmr.com>
To: njas@research.att.com
Subject: Magic Squares of Order 5 -- further data
Status: R

I dug up the Japanese manuscript that I referred to in my previous note. The author is Kisaburo Okajima, and his letter to me is postmarked September 1988. I think he did his count somewhat earlier, perhaps a year or two. He finds 275305224, which is 4 * 68826306, as I claimed in my last note. He only divides out by rotations and reflections, and ignores the other factor of 4 from fancier derangements of the square.

His counting method is very different from mine, so I think that our results support each other.

I don't read Japanese, but his history section mentions a 1976 date with my number. This might be the Martin Gardner article.

On another topic, do you have semigroups-of-order-N? I counted the order 7s a while back, and got a number that differed slightly from the results of a German group. We both confirmed the order 6 number from the 1960s article by Plemmons.

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