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Hendricks

9 N 7th

Correspondence



AG052

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

February 12, 1974

John Robert Hendricks
c/o Journal of Recreational Mathematics
Joseph S. Madachy, Editor
4761 Bigger Road
Kettering, Ohio 45440

Dear Dr. Hendricks:

In your interesting articles on magic cubes of order 3 in J.R.M., you mention that W.S. Andrews asserts there are 192 magic cubes of order 3. Is it known how many magic squares of order n there are, for small n ?

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

Wrote to ask abt magic \square 's of
order n

The Pan-3-Agonal Magic Cube

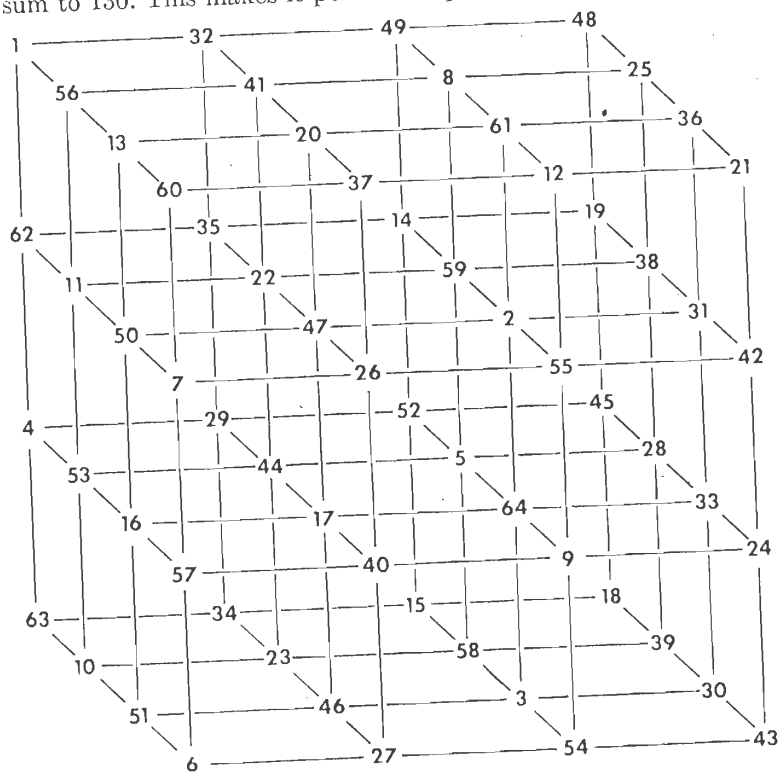
John Robert Hendricks
Regina, Saskatchewan

Although it was known that there were 192 magic cubes of order three, W. S. Andrews [1] left it up-in-the-air as to whether or not absolutely perfect cubes existed through the statement, "It is not easy—perhaps it is not possible—to make an absolutely perfect cube of (order) 3." This question has now been solved in the previous article in this issue of *JRM* [2]. In other words, to have the planar two-dimensional diagonals all adding to the magic sum in a magic cube, as well as the three-dimensional diagonals, is a bit much to expect.

Thus, when it comes to considering the fourth order magic cube, it is suggested that the conditions in the succeeding paragraphs are all that can be expected.

This cube contains the positive integers from 1 to 64. The sum of each row, each column, each pillar, and each three-dimensional diagonal of numbers is 130. This completes the requirements for a cube to be magic.

However, all broken 3-agonals which are parallel to the main three-dimensional diagonals sum to 130. This makes it possible to place the top layer of numbers on



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124 CHISHOLM ROAD
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19 February 1974

Mr. N. J. A. Sloane,
Bell Laboratories,
600 Mountain Avenue,
Murray Hill, New Jersey 07974,
United States of America.

Re: MH-1216--NJAS-mv, 12 February 1974

Dear Sir,

Kraitchik¹ has shown on pages 146-148, especially Figure 27 of page 147, that there is only one magic square of order 3. This may be taken in 8 aspects due to rotations and reflections.

Kraitchik has shown further, on pages 182-192, especially the table at the bottom of page 190, that there are 880 magic squares of order 4. These may be taken in 8 aspects due to rotations and reflections for a combined total of 7040. He also indicates a sub-class of $48 \times 8 = 384$ pandiagonal squares in the table at the top of page 190.

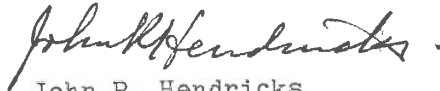
Andrews² has shown that there are 192 magic cubes of order 3; but, leaves it a bit of a mess on page 70, where, by using his rules, you would find some to be rotations and reflections of the others. Then, on page 352 Andrews leaves a remark, "It is not easy — perhaps it is not possible — to make an absolutely perfect cube of (order) 3," which implies doubt to whether or not there are any more.

Hendricks³ has shown conclusively, that there are only four magic cubes of order 3, which may be taken in 48 aspects due to rotations and reflections, yielding 192 in total — and, there are no other varieties.

To the best of my humble knowledge, this is all that is known about the numbers of possible magic squares and cubes; and, for that matter, hypercubes of any order and any dimension.

I hope that I have been of service.

Sincerely yours,



John R. Hendricks

1. Kraitchik, Mathematical Recreations, second revised edition, Dover Publications Incorporated, New York, ~~1973~~ 1953.
2. Andrews, Magic Squares and Cubes, second edition revised and enlarged, Dover Publications, Inc. New York.
3. Hendricks, The Third-Order Magic Cube Complete, JRM, Vol 5, No.1, 1972

replied Feb 25, 1974

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FEB 28 1974

Dr. John R. Hendricks
124 Chisholm Road
Regina Saskatchewan
CANADA S4S 5P1

Dear Dr. Hendricks:

Thank you for replying so promptly to my request for information about the numbers of magic squares and cubes. (I am collecting sequences of integers for a supplement to my book - see enclosure.) It is disappointing that no-one has calculated the number of 5×5 magic squares. After all, the number of Latin squares of order n is now known for $n \leq 9$ (J. Riordan, *Combinatorial Analysis*, p. 210; *Journal of Combinatorial Theory* 3 (1967) p. 98 for $n=8$; and S. E. Bammel & Jerome Rothstein, Ohio State University, unpublished, for $n=9$).

Thank you again for your helpful letter.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.

APPROVAL

H. O. Pollak

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FROM 124 CHISHOLM RD., REGINA, SASK. S4S 5P1, CANADA
TEL. (306) 586-2603

Re: MH-1216-NJAS-MU.28 Feb 1974 / 7 March 1974.

Dear Sir,

Thank you for your reprints etc. I see partially why you are interested. In answer to your further questions, certain combinations of latin squares, as components, do not yield magic squares; and, conversely certain magic squares, broken into components, do not yield ordinary latin squares, eg. bordered variety. This is what makes my problem so frightfully difficult. A computer might help but too expensive. Sincerely, JRB.

J. R. Hendricks