f91

Mallows

6123
6124
6126
6128

Scan
add to 4 seqs

CL Mallows
and
NJAS
Emails June 1991

A6123

fai #N254 = A1405 = $\binom{n}{\lceil \frac{n}{2} \rceil}$ m=2

#456 = A1006
= Motzkin nos

#468 = A1998
= Folding a line

$\boxed{\text{ask CLM}}$

#468.5 1 2 4 10 26 75 215
Generalized Ballot (m=5) See #294 for m=2, #456 for m=3, #468 for m=4.
(Start with (1,2,...,m), add votes retaining strict inequalities)
(Also = # determinants in expansion of D^n(Hessian))

Colin, Ref? Defn? More terms?
(Please return!)
Neil

~~Neil - I think same as 6251~~
— omit unless he recalls ref

CLM

6124
6128
6126

$\left[D^{(ij)}(f) \right]_{k \times k}$

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From mhuxo!gauss!clm Wed Jun 19 17:42:22 EDT 1991
Status: R

$\boxed{6123}$

\>>>
\>>>#468.5 1 2 4 10 26 75 215
\>>>Generalized Ballot (m=5) See #294 for m=2, #456 for m=3, #468 for m=4.
\>>>(Start with (1,2,...,m), add votes retaining strict inequalities)
\>>>(Also = # determinants in expansion of D^n(Hessian))
\>>>

6123(a)

Consider a k x k Hessian matrix with entries D^(i+j)f, 0 <= i,j <= k-1. Denote this (0,1,...,k-1) by reading across the top row. Now differentiate it n times, working always by columns; for example after 3 steps you will get 2*(1,3) + (0,4) (many terms vanish because they have two equal columns). For k = 2,3,4,5 the seqs #294,456,468,468.5 give the sum of the coeffs after n steps . I don't know a ref. for this. Do your refs mention this interpretation? The calculation corresponds exactly to enumerating "strict ballot sequences", where we start with k candidates having respectively 0,1,...,k-1 votes, and add a total of n more votes, requiring that at each stage the candidates' totals are strictly increasing. The strict ballot problem is discussed in D.E.Barton and C.L.Mallows (1963) "Some aspects of the random sequence", Ann. Math. Statist. 36, 236-260. However the only formulas given there relate to the prob. that strict ordering holds throughout when a fixed number of votes is added, namely (a1,a2,...,ak) for the respective candidates. So maybe "Personal communication" is best.

✓

\>>>
\>>>#594.5 1 2 5 17 79
\>>>Pair-necklaces
\>>>

6124

kill!

$\boxed{\text{dead}}$

I don't know what this is. I know of two things it might be, but neither fits exactly. One thing it might have been is this, suggested by something I did with Frisch and Bovey on identifying polymers from NMR spectra: consider rings of two colors of beads, and code them by reading off for each adjacent pair whether they are the same or different color. Thus eg the ring 111122 is coded as sssdsd. Cyclic perms and reversals are not distinguished. Then the number of different codes is 1 2 2 4 4 8 9 19 which doesn't match either #114 or #115. Or, it might have to do with an enumeration I did with Ken Wachter related to moments of eigenvalues of random Wishart matrices. That also had to do with rings of bi-colored beads, but the details are messy and I haven't (yet) been able to reconstruct them. Nothing he or I published is relevant.

6|28

>>>
>>>#1010.5 1 3 6 12 20 325 54 86 128 192
>>>Sum{n{product from 1 to n(x/(1-x^i))}}
>>>
Yes, except for a misprint (325 should be 35) this is exactly what I said:

Sum from n=1 to infinity {n*{product from j=1 to n of {x/(1-x^j)}}}

But why?

>>>
>>>#1414.5 1 4 14 48 164 560
>>>Time for coin-toss difference to escape from (-3,+3).
>>>
At last one that I understand. Ref is Feller Vol 1, sect XIV.4 in my edition, to do with random walk with absorbing barriers at z=0 and z=a. Feller derives the generating function of the prob. that absorption occurs at 0 after exactly n steps starting from z, each step being with prob p,q for -> and <- respectively). Specializing to a=8 and z=4 and p=q=1/2, and replacing the argument s by 2y so that we enumerate paths, we get y^4/(1-4y^2 + 2y^4) which does generate 1,4,14,48,164 as claimed. I don't know why I wanted this.

~~6|28~~

Kill

>>>
>>>#2345.5 1 132 64988160 455760028510617600
>>>Euler paths
>>>
No idea. I must have copied this from somewhere, the third term has a factor 4231 which is prime and I don't remember ever counting that many things. The only thing that comes to mind is that I remember being interested in a "grand tour" on a 2^n cube, so maybe this has to do with that. If we are touring edges, n must be even; but we can visit all the vertices in any case. For the vertex tour on the 2^3 cube, there is only one kind of path (the "baseball seam"), but it can be positioned on the cube in many ways. Not 33, though. And I don't think there can be 33 paths on the 2^4 cube. There was a publication on "snakes" (maybe a problem somewhere?) which was somewhat related but not the same.

Omit

gauss$

6123

Take the matrix with $(i,j)^{th}$ entry

$$\left(D^{(i+j)} f\right)$$

as a function of variables

Differentiate it, using rule

$$D A = \sum_k (\text{Diff } k^{th} \text{ col, fix rest})$$

But now many cols are identical, so many of these terms vanish.

Eg

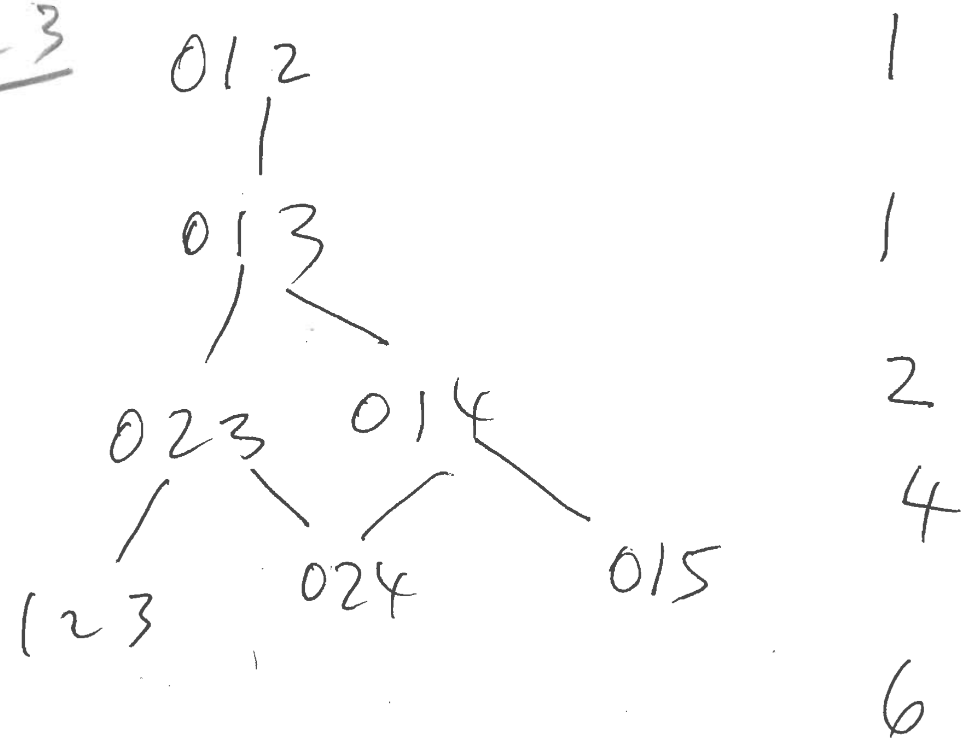
$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$$

Diff:

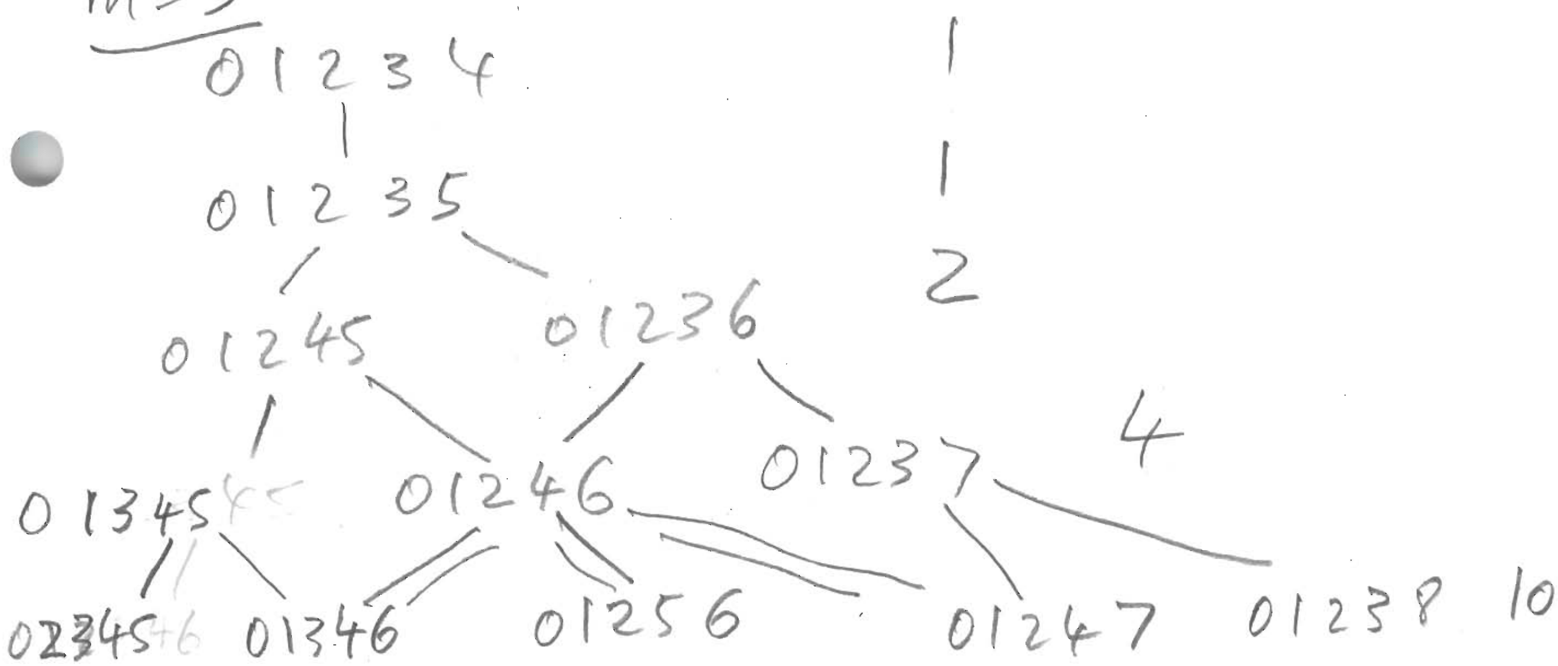
$$\cancel{\begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 3 & 3 & 4 \end{pmatrix}} + \cancel{\begin{pmatrix} 0 & 2 & 2 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix}} + \underline{\underline{\begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & 4 \\ 2 & 3 & 5 \end{pmatrix}}}$$

Only 1 survives

m=3


012
013
023 014
123 024 015
1
1
2
4
6

m=5


01234
01235
01245 01236
01345 01246 01237
02345 01346 01256 01247 01238
1
1
2
4
10

f91

A6124

#594.5 1 2 5 17 79
Pair-necklaces

Colin, Same questions!

Neil

Kill

fa1

A6128

#1414.5 1 4 14 48 164 560
Time for coin-toss difference to escape from (-3,+3).

Ref? Formula? More terms?

Rill

fall

#2345.5 1 132 64988160 455760028510617600
Euler paths

Colin, Defn? Ref?
(I suspect I may have this one in a different scaling)

Omit

New 6128 6128

#1010.5 1 3 6 12 20 325 54 86 128 192
Sum{n{product from 1 to n(x/(1-x^i))}}

~~Colin, I don't understand this one at all!~~

$$\sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{x}{1-x^i}$$

$$= 1 + x + 2x^2 + 3x^3 + 5x^4 + \cdots$$

~~(coefficients are partition function $p(n)$)~~

$$\boxed{\sum_{n=1}^{\infty} n \prod_{j=1}^{n} \frac{x}{1-x^j}}$$

ds