PPP Puzzles Pastimes Problems

Solutions

to Puzzles, Pastimes and Problems shown on page 20.

1. Nebuchadnezzar's Herb Garden

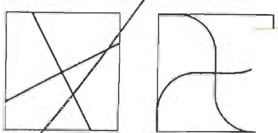


Figure 4

The first plan shows th
through the centre of the
equal plots, if they are
The other plans show sor
the paths meet at the c
other.

2. When is Division the sa

(i) $a = \frac{4}{3}$, $b = \frac{2}{3}$. (ii) $p =$
(iv) $x = n^2/(n+1)$, $y = n/($
sum are both n/m. (vi) $\frac{4}{15}$,
$\frac{7}{11}$. (x) $\frac{49^2}{5100}$, $\frac{49}{100}$.

3. More Fun with Fraction

(a) (i) 5. (ii) 8. (iii) 11. (iv)
(b) (i) 3. (ii) 10. (iii) 21. (iv

4. Quadrilaterals

(i) S, R, I. (ii) S, R, P, D.
(v) K, A. (vi) K, A.
(vii) Yes: see Fig. 5.

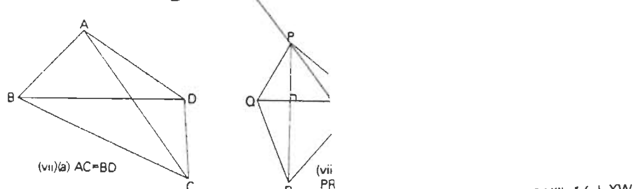


Figure 5

5. Mathematical Music

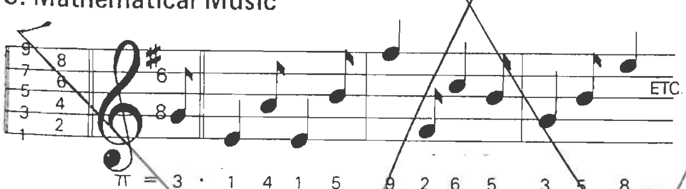


Figure 6

The notes on the five lines of the stave represent the odd numbers 1, 3, 5, 7, 9, counting upwards, and the spaces between the lines represent the even numbers 2, 4, 6, 8, whilst the space below the stave represents 0. So the tune represents $\pi = 3.14159265358\ldots$

6. True or False?

Statements about numbers can usually be proved by algebra, using n to represent any number, $2n$ any even number, and $2n+1$ any odd number.

(i) Take $2n-1$ and $2n+1$ as any two consecutive numbers. Then $(2n-1)+(2n+1) = 4n$, which is always a multiple of 4.
(ii) $n+(n+1)+(n+2) = 3n+3 = 3(n+1)$, a multiple of 3.
(iii) False, since $1+2+3+4 = 10$, which is not a multiple of 4.
(iv) $n+(n+1)+(n+2)+(n+3)+(n+4) = 5(n+2)$.
(v) True: $n^2+(n+1)^2+(n+2)^2 = 3(n+1)^2+2$.
(vi) True: $(2n-1)^2+(2n+1)^2+(2n+3)^2 = 12(n^2+n+1)-1$.
(vii) True: $(n+n+1)^2-n^2-(n+1)^2 = 2n(n+1) = 4T_n$, where T_n is the n^{th} Triangular number $= \frac{1}{2}n(n+1)$.

7. "Doublets"

(i) MOUSE-HOUSE-HORSE. (ii) CAT-COT-COG-DOG. (iii) LOVE-LAVE-LATE-HATE, or LOVE-HOVE-HAVE-HATE. (iv) LEAD-LOAD-GOAD-
BOOT-SOOT-SHOT-SHOE. (vi) TWO-
EN. (vii) APE-APT-OPT-OAT-MAT-
find any shorter chains, please send them
l to me.

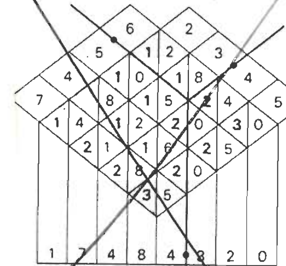
Scan
~~for two pages~~
One page
Add to four segs

124
125
127
6261

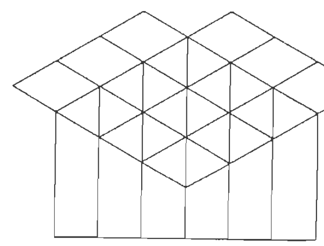
l point location
symmetrical display

Harris, Computer Department, BP

1975 issue, Frank Tapson described how
ed where to put decimal points on the gelosia
ay wish to test the effectiveness of the
ay below, for I am sure that by using it we
eborahs.

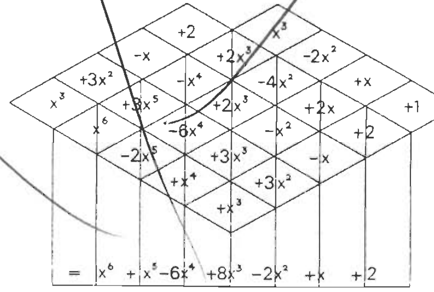


can easily be drawn on isometric paper.
grids of different sizes can be duplicated by



When the accuracy of the answer is required to a fixed number of decimal places, the symmetrical display allows the user to omit those products which do not contribute to the answer. Further, it is possible to simplify the multiplication of algebraic brackets e.g.

$$(x^3+3x^2-x+2)\ (x^3-2x^2+x+1).$$



Variations on a Geometric Progression

f91 124 125 127 f91 → 6261

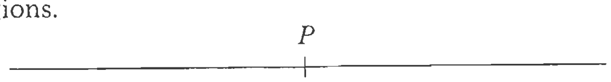
by M. L. Cornelius, Department of Education,
University of Durham

Mathematics in School
4, No. 3, 1975, May
p32

The following problems and ideas from different sources are related. They may provide useful investigations for pupils at almost any stage in their school careers.

1. Division of a line by points

(See Nuffield Mathematics Project, *Problems—Red Set*, No. 3.) One point on a line divides the line into two regions.



Into how many regions is a line divided by n points? Investigation quickly gives:

Table I

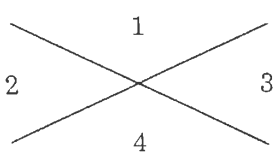
No. of Points:	0	1	2	3	4	5	....	n
No. of Regions:	1	2	3	4	5	6	....	$n+1$

If $R_n^{(1)}$ represents the number of regions obtained with n points, we have:

$$R_n^{(1)} = 1+R_{n-1}^{(1)} = 1+n = 1+{}^nC_1 \qquad (1)$$

2. Division of a plane by lines

(See Nuffield Mathematics Project, *Problems—Red Set*, No. 3)
One line in a plane divides the plane into two regions. Two lines produce a maximum of four regions.



What is the maximum number of regions formed by n lines?
Investigation by drawing reveals the pattern:

Table II

No. of Lines:	0	1	2	3	4	5	....	n
Maximum No. of regions:	1	2	4	7	11	16	....	$R_n^{(2)}$

Here it is apparent that $R_n^{(2)} = R_{n-1}^{(2)}+n$ and a little algebra soon gives:

$$R_n^{(2)} = R_{n-1}^{(2)}+n = 1+\tfrac{1}{2}n(n+1) = 1+{}^nC_1+{}^nC_2 \qquad (2)$$

3. Division of space by planes

(See G. Polya, *Induction and Analogy in Mathematics)*
What is the maximum number of regions into which three-dimensional space is divided by 1,2,3,4,5, . . . planes? It is easy to answer this question for 1,2 and 3 planes. Visualization of three dimensions has to be good to reach an answer for 4 and particularly 5 planes. The following table is obtained:

Table III

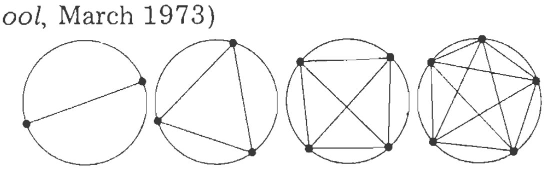
No. of Planes:	0	1	2	3	4	5	....	n
Maximum No. of Regions:	1	2	4	8	15	26	....	$R_n^{(3)}$

Algebraic investigation produces:

$$R_n^{(3)} = \tfrac{1}{6}n(n^2-1)+(n+1) = 1+{}^nC_1+{}^nC_2+{}^nC_3 \qquad (3)$$

4. Regions in a circle

(See A. G. Howson, A Problem Story, *Mathematics in School*, March 1973)



What is the maximum number of regions produced when we join 1,2,3, . . . points on a circle?

Table IV

No. of Points:	1	2	3	4	5	6	....	$n+1$
Maximum No. of Regions:	1	2	4	8	16	31	....	$R_{n+1}^{(4)}$

It can be shown that:

$$R_{n+1}^{(4)} = \tfrac{1}{24}(24+14n+11n^2-2n^3+n^4)$$
$$= 1+{}^nC_1+{}^nC_2+{}^nC_3+{}^nC_4 \qquad (4)$$

5. The problems compared

The sequences obtained in tables I, II, III and IV are:

1	2	3	4	5	6	7	. . .
1	2	4	7	11	16	22	. . .
1	2	4	8	15	26	42	. . .
1	2	4	8	16	31	57	. . .

and the general terms are:

$1+{}^nC_1$ 124

$1+{}^nC_1+{}^nC_2$

$1+{}^nC_1+{}^nC_2+{}^nC_3$ ← 125

$1+{}^nC_1+{}^nC_2+{}^nC_3+{}^nC_4$ — 127

Some of the patterns here seem worthy of further investigation. We have a basic geometric progression, 1 2 4 8 16 . . . and it "breaks down" one term later in each sequence as we move down the list.

The next sequence in the series viz: 6261

1 2 4 8 16 32 63 120 . . .

poses the challenge of finding a physical description similar to those for the previous sequences. Following the pattern established, could it represent some division of a finite three-dimensional space e.g. a sphere? Investigations along these lines are liable to turn an attractive new red Edam cheese into an unappetizing mass of crumbs. ■