Scan 6457-9

H. W. Lenstra, Jr.,

1 page

add to 3 sequence

§91 **MATHEMATISCH INSTITUUT**
der Universiteit van Amsterdam
Roetersstraat 15, telefoon (020) ~~21 10 44~~ 5222299

6457
–6459

Amsterdam-C., November 28, 1975.

Dear Dr. Sloane:

Here are some integer sequences for you derived from the theory of Euclidean rings.

Theory: if R is a Euclidean ring with only finitely many units, then R has a so-called "smallest algorithm" ϑ, and for each $n \geqslant 0$ the number $a_n = \#\{x \in R \mid \vartheta(x) \leqq n\}$ is finite. Reference: Samuel, J. Algebra 19 (1971), 282-301.

01/7/91

Examples: (a) $R = Z$, then $a_n = 2^{n+1} - 1$ $(n \geqq 0)$.

(b) $R = Z[i]$, $i^2 = -1$. I proved

$a_{n+5} - 4a_{n+4} + 3a_{n+3} + 6a_{n+2} - 10a_{n+1} + 4a_n = 0$ $(n \geqq 0)$, 6457

and one computes $(a_n)_{n=0}^{29}$ = (1, 5, 17, 49, 125, 297, 669, 1457, 3093, 6457, 13309, 27201, 55237, 111689, 225101, 452689, 908885, 1822809, 3652701, 7315553, 14645349, 29311081, 58650733, 117342321, 234741877, 469565561, 939245693, 1878655105, 3757539461, 7515406469).

(c) $R = Z[\rho]$, $\rho^2 = -\rho - 1$. Then

$a_{n+6} - 5a_{n+5} + 5a_{n+4} + 5a_{n+3} - 4a_{n+2} - 8a_{n+1} + 6a_n = 0$ $(n \geqq 0)$,

and $(a_n)_{n=0}^{19}$ = (1, 7, 31, 115, 391, 1267, 3979, 12271, 37423, 113371, 342091, 1029799, 3095671, 9298147, 27914179, 83777503, 251394415, 754292827, 2263072411, 6789560412). 6458

(d) $R = Z[\sqrt{-2}]$. No relation as before seems to exist, and I computed $(a_n)_{n=0}^{17}$ = (1, 3, 9, 17, 31, 53, 85, 133, 197, 293, 417, 593, 849, 1193, 1661, 2291, 3139, 4299). (The values given by Samuel are erroneous). 6459

(e) Also for $R = Z[(1+\sqrt{-d})/2]$, $d = 7$ or 11, such sequences exist, but nobody seems yet to have calculated a substantial portion of them.

With kindest regards,

Sincerely yours,

HWLen

(H.W. Lenstra, Jr.)

file A6457.9