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Eberhart
– Sloane
Letters
(4 letters)

Several Corrections to book! 7650

25 January 1977

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Dear Mr. Sloane,

After eyeing your book (H. of I.S.) for many months on the shelf of Foyle's Bookstore in London and watching the price creep up (as on everything else) I finally broke down my sales-resistance and bought a copy. Take heart — I have not regretted it!

I do have a few queries, however, regarding some statements in the third chapter. Probably many people have already written to you about two mis-prints, but in case they haven't: p. 18, the Catalan Sequence, is of course 577 not 557; and on p. 20, the "pancake" sequence is 419 not 491.

Regarding the latter, I can not for the life of me see what the given sequence (419) has to do with pancake slicing! But I can see that the given sequence is definitely not in agreement with $(n+2)(n+3)/6$, but rather with either $(n+2)(n^2+n+6)/6$ if one begins $n = 0,1,2,\ldots$, or else with $(n+1)(n^2-n+6)/6$ if one begins $n = 1,2,3,\ldots$ (I refer to the formula given on p. 20, line 3.)

I cannot see what is meant by the cryptic definition of seq. 1181 (p. 20, 1st line), either, but for that I assume I must go to the literature indicated (alas — I have no library at my disposal).

I once tackled a problem that sounds related to the description of seq. 1181, namely the following: Let n straight lines be arranged in the plane so that no more than 2 cross at any point (i.e. so as to form a simple 2-arrangement in the projective plane). How many polygonal regions do they bound? This is given easily by seq. 391, as described on the bottom of p. 19; it is interesting that the same quantitative total is reached no matter how the lines are arranged. Counting the qualitative total is harder: in how many distinct ways can the n lines be arranged (counting two arrangements as the same iff they have the same no. of p'gons of each kind and the p'gons of one can be further matched 1-1 with the p'gons of the other so that neighboring p'gons, also, are of the same kind)? (— there exist arrangements having same nos. of all kinds of components but differently arranged, neighboringly.) For from 1 to 5 lines there is essentially only 1 arrangement, the one you show on p. 20, letting the circles grow to become the respective lines at infinity. For 6 lines there are suddenly 4 different possibilities (1 you've shown, 2 you haven't with axial symmetry, and 1 more corresp. to the icosidodecahedron net), for 7 lines 11 possibilities, and when I last heard (2 yrs. ago, Branko Grünbaum) the count for 8 lines was still uncertain. Now 1,4,11,? could be any seq. from 1377 to 1390, if it's in your list at all (probably 2 more entries would decide that).

What fascinated me was an empirical result I have checked using about 100 lines, drawn by myself and about 100 students over the years as a geometry teacher: Draw "lots" of lines, willy-nilly on your paper, framed by 4 in a rectangle around the border to provide a neat finish but otherwise avoiding parallels and aiming to have the drawing look as though a handful of chopsticks had been just dropped randomly (no more than 2 crossing at any corner). Count the no. of 3-, 4-, 5-, etc. sided p'gons and work out the percentages of their distributions. I thought of doing this originally as a lead-in, pedagogically, toward studying the regular and semi-regular mosaics in the plane, of which there are 10, which may be grouped as 4 in which three meet per corner, 3 in which four meet, 2 in which five meet, and 1 in which six meet, neatly 4,3,2,1.* To my great surprise, if one lumps the rare occasional 7- or larger-sided p'gons together with the 6's, then one finds approx. 40% six- or more-sided, 30% five-sided, 20% four-sided, and 10% three-sided, again 4,3,2,1! The variations are ±1% or less, based on the experience cited, but I have not yet devised a rigorous way of even defining, much less carrying out, what I mean by "random" 2-arrangements of n lines, and taking the limit as n goes to infinity. But I have done very many drawings with very many lines, and the results are always the same. I mentioned the problem once orally to Paul Erdös, who expressed mild interest, but also was unable to suggest a way to tackle it theoretically. I would not expect any direct correlation, as the former 4,3,2,1 sequence is qualitative in nature (different ways to pack p'gons, so-many at a corner) whereas the latter 4,3,2,1 sequence is quantitative (what per cent of the meshes cought in the line-weaving are so-many sided?). Nevertheless, 4,3,2,1 is such an archetypal sequence that its "cosmic" emergence out of such "chaos" (order out of disorder) is tantalizing.

But that's enough time spent on that subject!

The suggested error-corrections are offered for the (hopeful) event that your book enjoys a 2nd revised printing. In the meanwhile I would appreciate being placed on your up-date list to receive any mailings you may put out.

The material on pancake (proj. plane) cutting I leave up to you to enjoy yourself or pass on to whomever you think might enjoy it, or chuck in the circular file.

Please give my greetings to Bruce Bogert, if you see and know him. We used to play bassoon together many years ago when I was still in high school in Summit. I've been through Oberlin Conservatory (in music history) and the Univ. of Washington (in mathematics) now.

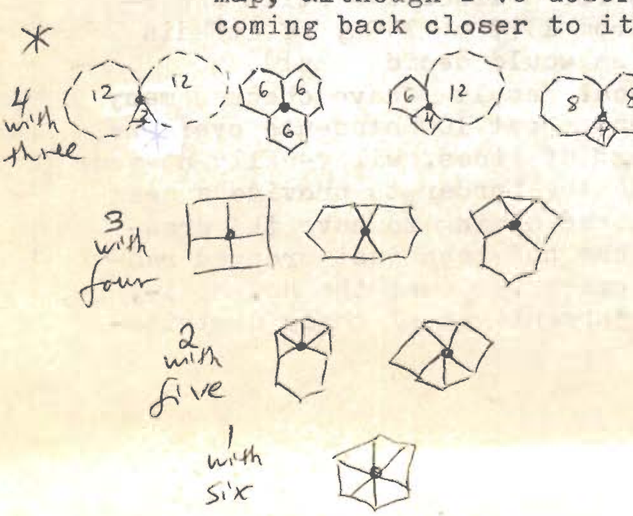
And in the very unlikely event that you could suggest any job openings for a combinatorialist (with FORTRAN experience) like myself in the Labs, I'd appreciate hearing about it. I wrote a year ago (to Personnel) and sent a curriculum vitae. Four months later I received an answer requesting a curriculum vitae, and that I please fill out the enclosed questionnaire — there was no enclosed questionnaire.

Both my Dad (Edgar = "Ebbie") and uncle (Frank Hardy) as well as step-uncle (Clyde Keith) and many other friends have been Lab men, so I've always felt it to be sort of the origin of my mental map, although I've described a rather wide arc away from it before coming back closer to it again.

With best regards,

Stephen Eberhart

Stephen Eberhart



7650

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February 8, 1977

Dr. Stephen Eberhart
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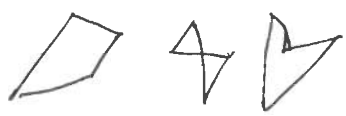
Dear Dr. Eberhart:

Thank you very much for your letter of January 25. Very few corrections have been found in the main table of sequences, where I checked and rechecked each sequence carefully. But obviously th e text at the beginning was not checked so carefully, and I must plead guilty to the three errors you caught. Obviously (n+2)(n+3)/6 on page 20 is nonsense.

The definition of sequence 1181 on page 20 I think is correct: For example in the fourth pancake of figure 14 there are 3 quadrilaterals, namely:



(allowing crossings and non-convex figures). Sequence 419 also means what it says: the maximum number of pieces you can obtain by slicing a cake (a dense chocolate cake, say) with n slices of a very sharp knife. The third cut is made perpendicular to the first two, giving 8 pieces, for example. Obviously this wouldn't work with a pancake, which has zero thickness.

I enjoyed hearing about your experiments with random lines, and about your sequence which begins 1, 4, 11. If Branko Grünbaum doesn't know of further terms then probably no-one does.

The job situation here is pretty tough, but still you should have been better treated by Personnel than that. You could always try them again, and mention my name (though I'm not sure it would help!).

With very best wishes.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
Reprints

16 Feb. 77 7650

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Mr. N. J. A. Sloane
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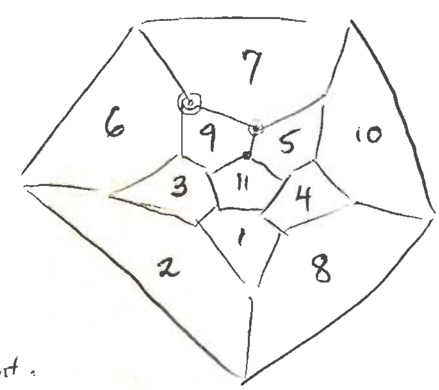
Dear Sir,

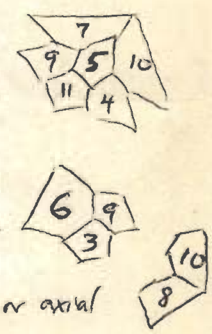
The Jan. 77 Am. Math. Monthly (p. 73) gives a "telegraphic review" of your 1975 Short Course on Error-Correcting Codes, which sent me scuttling into Foyles Bookstore in London (since my present library of books by Johnson, Anderson, Cameron & van Lint do not "tell me all about Golay Code I really want to know") — but, searching the lists of Springer Verlag publications, we could find no such book. Can you send me particulars? Better yet, can you arrange to have the book sent? (Pref. air mail, as I may be moving in June.)

I have done quite a bit of work on my own on v,k,λ designs, but am only just beginning to submit things to journals (and I suffer currently from non-access to libraries). Here's one example: Imagine wanting to read codes sent using 11, 5, 2 design given by translations of p.d.s. gen. by powers of 3 mod 11 — they could, I find, be visually displayed in a readily "Gestalt"-graspable manner as 11 faces of a dodecahedron, exploded Schlegel-style, as follows:



Note sums at outer corner points ◎ are all ≡ 11, at middle ones ◎ they're twice powers of 3, and at inner ones • they're powers of 3 again, duplicating the facial arrangement.

E.g. Block 5 is of shape and Block 6 is of shape with clear radial or axial symmetry.



Block no:	11	10	9	8	7	6	5	4	3	2	1
Display shape											
Elements: $3^5 \equiv 3^0 = 1$	2	3	4	5	6	7	8	9	10	11	
$3^1 = 3$	4	5	6	7	8	9	10	11	1	2	
$3^2 = 9$	10	11	1	2	3	4	5	6	7	8	
$3^3 \equiv 5$	6	7	8	9	10	11	1	2	3	4	
$3^4 \equiv 4$	5	6	7	8	9	10	11	1	2	3	

Blocks contain el't of same no., [illegible] do not but are centered on it.

I have tried to do something similar with the 19, 9, 4 design and the icosahedron, but sofar without success!

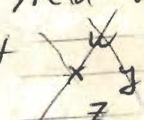
With regard to your _Handbook of Int. Seq._, since I have no access to libraries currently I cannot tell what is in your reference SIAMR 12 277 70 in which most of the following seq's occur (except Pell Nos., Tribonacci, & #1621), but I find a remarkable family of sequences contained in the following triangle:

Horizontal rows sum to Pell Nos, (552) $t_n = 2t_{n-1} + t_{n-2}$

30° Diagonals sum to Tribonacci (406) $t_n = t_{n-1} + t_{n-2} + t_{n-3}$

```
                    1  — odds
                  1 | 1  — 1567
                1 | 3   1  — 1844
              1   5 | 5   1  — 1974
            1   7 | 13   7   1  — 2045
          1   9   25 | 25   9   1  — 2102
        1   11   41 | 63   41   11   1  — 2139
      1   13   61   129 | 129   61   13   1
    1   15   85   231 | 321   231   85   15   1
                     |       |
                   1621    1184
```

cf. in Pascal Δ Horizontal Rows yield $t_n = 2t_{n-1}$ (powers of 2) and 30° Diagonals yield $t_n = t_{n-1} + t_{n-2}$ (Fibonacci)

The generalization, however, fails as I see it to yield any well-known (i.e. ref'd in your book) series: Let 

be grid points of Δ; then calling $z = y + x + \lambda w$ with
$\begin{matrix} 0 & 1 & 0 \\ 0 \;\; 1 & & 1 \;\; 0 \end{matrix}$
initiation and $\lambda = 0$ gives the Pascal Δ, $\lambda = 1$ gives the Δ above, and other λ values give interesting Δs but with no familiar series. Initiating instead
$\begin{matrix} 0 & 1 & 0 \\ 0 \;\; 1\tfrac{1}{2} & & 1\tfrac{1}{2} \;\; 0 \end{matrix}$
and ruling $z = 2(y+x) - 3w$ gives a <u>plane-filling</u> grid whose horizontal rows sum to all (+ and −) integral powers of 3 and whose 30° diagonals sum to a series approaching $1 : \frac{1+\sqrt{13}}{2}$ (tantalizingly 99.99⁺% close to $\log_e 10$ — a red herring, of course), but no other well-known series. Powers of 4 are given by horiz. row sums of Δ initiated

```
  0 1 0
 0 2 2 0
  3 10 3
4 28 28 4
```

with rule $z = 2(y+x+1)$ <u>except</u> end-nos. (not plane-filling) while powers of both 3 and 4 are found in

```
          1
        1   3
      1   6   9
    1   9   27   27
  1   12   54   108   81
1   15   90   270   405   243
```

whose ± 30° diag's sum to terms approaching respective ratios of $1 : \frac{2 \mp 1 + \sqrt{13}}{2}$.

(again, <u>not</u> plane-filling)

I wrote you once before about 3-4 weeks ago to note some misprints in intro. to H. of I. S. when I first bought the book (which I find strongly addictive!) and enquired at that time, from a human point of view (as opposed to Personnel Dep't who last year asked for a curriculum vitae after I'd sent one and told me to fill out a questionnaire without sending one), whether you could envisage work at the Labs for a M. Sc. in combinatorics ('75, under Branko Grünbaum, U. of W. — thesis on discovery of 32-dim. alg. embedding 31 copies of Cayley alg. circulantly). I am still very much searching for a way back to the U.S. this summer or fall.

Any response to any of the above would be appreciated!

Yours, Stephen Eberhart

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P.S.

Lab connections:

132 2996

Bruce Bogart — former fellow bassoonist in N.J. orch's
Edgar (Ebbie) Eberhart — father
Frank Hardy — uncle
Clyde Keith — step-uncle
Herbert Lewis — friend of family
Paul Oncley — former music teacher

(my undergrad work was at Oberlin Conservatory but I've been a math's teacher.)

7650

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March 3, 1977

Mr. Stephen Ebergart
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Dear Mr. Eberhart:

Thank you very much for your letter of 16 and 18 Feb; I trust by now you have received my reply to your earlier letter.

This was intended to be a quick reply, as I am off on a trip tomorrow. The book "A short course on error-correcting codes" is published by Springer-Verlag, and it is in their latest catalog. I don't have any copies left myself. Their main office in Berlin is at

Heidelberger Platz 3
D-1000 Berlin 33
Germany

and probably your best bet is to write directly to them.

As a matter of fact F. Jessie MacWilliams and I have just finished writing what we like to think will be the book on coding theory. It is simply called "The Theory of Error-Correcting Codes" and it will be published this Spring by North-Holland Publishing Co., Amsterdam. We are furiously checking proofs right now. But the "short course" is admittedly shorter.

Your description of the 11, 5, 2 design is cute. It must be tied up with properties of the symmetry group of the dodecahedron, that is, the alternating group A_5. Have you looked at Klein's The Icosahedron? (It won't help much.)

Ah yes: I see that my first letter has now reached you.

Thank you for the two very nice new sequences. They will go into the second edition.

I find your 32-dimensional structure extremely interesting. There is one fact in particular I should like to know. Cayley numbers have magnitudes (or norms), and there are 240 of unit norm. Question: how many things in your structure have the smallest norm? I could not follow your description too well. If you have written anything down, maybe you could send me a copy.

Muses's 24-dim. structure also interests me a great deal. May I take you up on your offer to send me a copy? Especially in view of the errors.

I have been playing with complex lattices recently, and a short paper is being typed. I will send.

That's all for now.

With best regards,

MH-1216-NJAS-mv

N. J. A. Sloane