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Polynomials  
for 'same game'

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In N.J.A. Sloane's On-Line Encyclopedia of Integer Sequences the sequence A035615 deals with the winning n-digit binary strings in 'same game'. Erich Friedman describes this game as: Strings that can be reduced to null string by repeatedly removing an entire run of two or more consecutive digits. Example: 11011001 is a winning string since  $110(11)001 \rightarrow 11(000)1 \rightarrow (111) \rightarrow \text{null}$ .

The first values of this sequence are : 0, 2, 2, 6, 12, 26, 58, 126, 278, 602, 1300, 2774, 5878, 12350, 25778, 53470, 110332, 226610, 463602, 945214, 1921550, 3896642, 7885092, 15927086, 32121582, 64697726.

**Algorithm:**

```

win(string s, int links, int rechts)
    if rechts-links==0 return false
    if all digits in s are equal return true
    for all entire runs of two or more consecutive digits r do
        if win(s-r,links',rechts')==true return true
    return false

main()
    solutions=0;
    for all binary strings s with length n do
        if win(s, l, r) = true solutions++

```

In A035617 the 'same game' is treated for ternary strings. More general let  $a(n,b)$  denote the number of winning n-digit b-ary strings in 'same game'. The above algorithm can obviously be used for the general problem. Because of the fact, that a winning n-digit b-ary string can only have  $\lfloor \frac{n}{2} \rfloor$  different digits there exist for  $a(n,b)$  a polynomial with maximal degree  $\lfloor \frac{n}{2} \rfloor$ .

The first few polynomials are:

$$\begin{aligned}
 a(1, b) &= 0 \\
 a(2, b) &= b \\
 a(3, b) &= b \\
 a(4, b) &= 2b^2 - b \\
 a(5, b) &= 5b^2 - 4b \\
 a(6, b) &= 5b^3 - 3b^2 - b \\
 a(7, b) &= 21b^3 - 35b^2 + 15b \\
 a(8, b) &= 14b^4 - 36b^2 + 23b \\
 a(9, b) &= 84b^4 - 204b^3 + 162b^2 - 41b \\
 a(10, b) &= 42b^5 + 60b^4 - 405b^3 + 465b^2 - 161b \\
 a(11, b) &= 330b^5 - 990b^4 + 990b^3 - 341b^2 + 12b
 \end{aligned}$$

**Literatur:**

N.J.A. Sloane (2001), The On-Line Encyclopedia of Integer Sequences, published electronically at <http://www.research.att.com/njas/sequences/>.