

Twin primes and their applications

Page maintained by Jean-Claude Evard. Last update: June 17, 2003.

AMS classification numbers [Link](#) : 11A41 [Link](#) , 11N05 [Link](#) , 11Y35 [Link](#) ,
11Y60 [Link](#) .

Mathematical Review copyright: Copies of reviews from Mathematical Review cannot be posted on Web pages, but they can be seen through links to MathSciNet. These links work only in the networks of institutions or on the computers of users who are current subscribers to MathSciNet.

Main directions of research about twin primes

1. The twin prime conjecture and extensions of Chen's theorem [Link](#) .
2. Search for lower and upper bounds for the number of twin primes in intervals $[1, n]$.
3. Search for the largest known twin primes.
4. Computation of Brun's constant.
5. Computation of the twin prime constant.
6. Generalizations to Gaussian primes.
7. Applications of twin primes.

Twin primes on large Web sites

1. List of 114 Web sites about twin primes, with summaries [Link](#) , provided by Geometry, The Online Learning Center.
2. Web pages related to twin primes maintained by Eric Weisstein [Link](#) on the Web site MathWorld [Link](#) at Wolfram Research [Link](#) .
 - Introduction to twin primes [Link](#) .
 - The twin prime conjecture [Link](#) .
 - The twin prime constant [Link](#) .
 - The Brun's constant [Link](#) .
3. Web pages related to twin primes on the Web site of Chris Caldwell [Link](#) . at the University of Tennessee at St. Martin:
 - Introduction to twin primes [Link](#) .
 - The twin prime conjecture [Link](#) .
 - First generalization of the twin prime conjecture:
 - The prime k -uple conjecture [Link](#) .
 - Second generalization of the twin prime conjecture:
 - The Dickson's conjecture [Link](#) .
 - The largest twenty known twin primes [Link](#) .
 - The twin prime constant [Link](#) .
 - The Brun's constant [Link](#) .

4. Web page on twin primes, [Link](#) on the Web site of Number Theory [Link](#), maintained by Keith Matthews [Link](#) at the University of Queensland in Australia.
 5. Web page on Brun's constant [Link](#) maintained by Steven Finch [Link](#) at MathSoft [Link](#).
 6. Lists of twin primes [Link](#) provided by mathematical.com [Link](#).
-

Web pages about twin primes

(In alphabetic order of the name of the first author).

1. Rensselaer Scientists Test Famous Hypothesis, by Bruce Adams, Review, weekly newspaper published by Rensselaer Polytechnic Institute, issue of February 13, 1998 [Link](#)
2. Web pages on twin primes [Link](#) constructed by Patrick Fry [Link](#) and [Link](#), Jeffrey Nesheiwat [Link](#), and Boleslaw Szymanski [Link](#), when there were graduate student in the Department of Computer Science of Rensselaer Polytechnic Institute.
3. Twin Primes and Brun's Constant: A Distributed Approach, by Patrick Fry, Jeffrey Nesheiwat, and Boleslaw Szymanski, Seventh IEEE International Symposium on High Performance Distributed Computing, Chicago, IL, July 1998 [Link](#)
4. A Characterization of Twin Prime Pairs [Link](#), by Jeffrey Gold [Link](#) and Don H. Tucker, Proceedings - Fifth National Conference on Undergraduate Research, Volume I, pp. 362-366, University of North Carolina Press, University of North Carolina at Asheville (UNCA), 1991.
5. Web pages on twin primes [Link](#) and [Link](#) maintained by Xavier Gourdon and Pascal Sebah [Link](#).
6. Web pages on twin primes [Link](#) and Brun's constant [Link](#) maintained by Thomas Nicely [Link](#), retired [Link](#) from Lynchburg College.
7. The twin prime constant [Link](#), Web page maintained by Gerhard Niklasch [Link](#) at ConSol Software [Link](#), and Pieter Moree [Link](#) and [Link](#) at Korteweg-de Vries Institute for Mathematics (KdV Institute), Amsterdam, the Netherlands [Link](#).
8. Recent history of twin primes: ScienceNews, Twin primes, by Ivars Peterson, June 2, 2001, Vol. 159, No 22 [Link](#).
9. Web page entitled "Computing the number of twin primes up to 10^{14} " [Link](#), maintained by Joerg Richstein [Link](#) at Dalhousie University in Canada.
10. Web page [Link](#) of Sebastian Martmn Ruiz and Azmy Ariff on a characterization of twin primes and generalization.

11. Parallel Computing, Counting Prime Numbers two by two, by Boleslaw Szymanski, New&Ideas, monthly research newspaper published by Rensselaer Polytechnic Institute, issue of April 1998 [Link](#) .
12. Preprints of Marek Wolf on twin primes, cousin primes, and Brun's constant to download from [Link](#) and [Link](#) .

Publications related to twin primes and their applications

Published in 2002

J. V. Leyendekkers and A. G. Shannon, A note on twin primes and the modular ring $\mathbb{Z}/6\mathbb{Z}$, *Internat. J. Math. Ed. Sci. Tech.* 33 (2002), no. 2, 303--306.

V. Yegnanarayanan, On a question concerning prime distance graphs, *Discrete Math.* 245 (2002), no. 1-3, 293--298. Summary [Link](#) .

Published in 2001

Abdullah Aziz Ergin, A relation between prime numbers and twin prime numbers, *Math. Comput. Appl.* 6 (2001), no. 2, 153--154.

José Luis Gómez Pardo, Computational aspects of primes. I. (Spanish) *Gac. R. Soc. Mat. Esp.* 4 (2001), no. 3, 649--673.

This paper contains a survey of computational aspects of analytic number theory with programs in Mathematica. Review by Pedro J. Berrizbeitia [Link](#) .

A. I. Medyanik, A regular simplex inscribed in a cube, half-circulant Hadamard matrices, and Gaussian sums. (Russian), *Mat. Fiz. Anal. Geom.* 8 (2001), no. 1, 58--81. Review by Matti Jutila [Link](#) .

Hugh Montgomery, Harmonic analysis as found in analytic number theory, Twentieth century harmonic analysis---a celebration, NATO Sci. Ser. II, Math., Phys., Chem., 33, *Kluwer Acad.*, 2001, 271--293.

This paper shows that a strong form of the Riemann Hypothesis would follow from the d -twin prime conjecture of Hardy and Littlewood.

Review by George Greaves [Link](#) .

Thomas R. Nicely, A new error analysis for Brun's constant, *Virginia J. Sci.* 52 (1): 45--55, 2001.

Summary [Link](#) .

Mathematical Review number: 2003d:11184

AMS Classification numbers: 11Y60 (11A41 11Y35 11Y70)

John Renze, Stan Wagon, and Brian Wick, The Gaussian zoo, *Experiment. Math.* 10 (2001), no. 2, 161--173. Review by Igor Rivin [Link](#) .

Published in 2000

P. D. T. A. Elliott, The multiplicative group of rationals generated by the shifted primes, II, *J. Reine Angew. Math.* 519 (2000), 59--71.

Review by Gérald Tenenbaum [Link](#) .

D. A. Goldston, S. M. Gonek, A. E. Özlük, C. Snyder, On the pair correlation of zeros of the Riemann zeta-function. *Proc. London Math. Soc.* (3) 80 (2000), no. 1, 31--49. Leads to new questions about twin primes.

Review by D. R. Heath-Brown [Link](#) .

Pieter Moree, Approximation of singular series and automata, with an appendix by Gerhard Niklasch, *Manuscripta Math.* 101 (2000), no. 3, 385--399.

In the appendix, Gerhard Niklasch indicates how one can apply the theorem of this paper to obtain 1000 decimal-place accuracy in the computation of certain constants such as the Artin and the twin prime constants.

Review by Maruti Ram Murty [Link](#) .

Noel Patson, Interesting property observed in the prime numbers, *Austral. Math. Soc. Gaz.* 27, 2000, no. 5, 232--236. Contains an important extension of a new promising approach to Goldbach's conjecture started by Daniel Zwillinger in 1979. Review by Vilius Stakenas [Link](#) .

Laurentiu Panaitopol, Properties of the series of differences of prime numbers, Publications du Centre de Recherches en Mathématiques Pures. Sér. I, Fasc. 31, 21--28, *Publ. Cent. Rech. Math. Pures*, Sér. I, 31, Univ. Neuchâtel, 2000.

Contains some results similar to the convergence of the series of reciprocals of twin primes (Brun's constant). Review by Cécile Dartyge [Link](#) .

Derong Qiu and Xianke Zhang, Elliptic curves of twin-primes over Gauss field and Diophantine equations, *Adv. Math. (China)* 29 (2000), no. 3, 279--281. Mathematical Review 1 789 429 [Link](#) .

Derong Qiu and Xianke Zhang, Selmer groups and Mordell-Weil groups of two types of elliptic curves, *Progr. Natur. Sci.* 10 (2000), no. 12, 946--949. With some applications of twin primes to elliptic curves.

Review by Andrew Bremner [Link](#) .

Peter Sarnak, Some problems in number theory, analysis and mathematical physics, Mathematics: frontiers and perspectives, 261--269, *Amer. Math. Soc.*, 2000. Review by John B. Friedlander [Link](#) .

Maria Suzuki, Alternative formulations of the twin prime problem, *Amer. Math. Monthly* 107 (2000), no. 1, 55--56. Review by H. L. Abbott [Link](#) . Contains an interesting relation with the paper published by Pinhas Naor in 1955.

Shin-ichi Yoshida, On the equation $y^2 = x^3 + pqx$, *Comment. Math. Univ. St. Paul.* 49 (2000), no. 1, 23--42. With some applications of twin primes to elliptic curves. Review by Andrew Bremner [Link](#) .

Published in 1999

Krassimir T. Atanassov, On some of the Smarandache's problems, American Research Press, Lupton, AZ, 1999. Problem 117 contains a new characterization of twin primes. Review by J. Sándor [Link](#) .

Richard Blecksmith, Paul Erdős, and J. Selfridge, L. Cluster

primes, *Amer. Math. Monthly* 106 (1999), no. 1, 43--48.

Review by Michael Filaseta [Link](#) .

H. Gadiyar and R. Gopalkrishna, Padma, Ramanujan-Fourier series, the Wiener-Khintchine formula and the distribution of prime pairs, *Phys. A* 269 (1999), no. 2-4, 503--510.

Review by D. R. Heath-Brown [Link](#) .

Solomon W. Golomb, Cyclic Hadamard difference sets---constructions and applications, Sequences and their applications, Ser. Discrete Math. Theor. Comput. Sci., *Springer*, 1999, 39--48.

Review by Mathematical Review [Link](#) .

Karl-Heinz Indlekofer and Antal Járαι, Largest known twin primes and Sophie Germain primes, *Math. Comp.* 68 (1999), no. 227, 1317--1324.

Review by Oliver Schirokauer [Link](#) .

Enrico Laeng, "Elementary" proofs and distribution of primes (Italian), *Lett. Mat.*, Pristem No. 33-34, (1999), 28--35. Review by Giovanni Coppola [Link](#) .

M. B. S. Laporta, A short intervals result for $2n$ -twin primes in arithmetic progressions, *Tsukuba J. Math.* 23 (1999), no. 2, 201--214. Review by A. Perelli [Link](#) .

Peter Swinnerton-Dyer, Some applications of Schinzel's hypothesis to Diophantine equations, *Number theory in progress*, Vol. 1, 503--530, de Gruyter, 1999. (The Schinzel's hypothesis is a generalization of the twin prime conjecture). Review by Tamás Szamuely [Link](#) .

Published in 1998

Takashi Agoh, Additive representations of integers, *Manuscripta Math.* 95 (1998), no. 3, 311--321. Contains sufficient conditions for the Golbach's and the twin primes conjectures to be true.

Review by Alessandro Languasco [Link](#) .

P. Berrizbeitia and P. D. T. A. Elliott, On products of shifted primes, Paul Erdős (1913--1996). *Ramanujan J.* 2 (1998), no. 1-2, 219--223. Contains an analogue of the twin prime conjecture. See continuation of this work published by the second author in 2000. Review by Gérald Tenenbaum [Link](#) .

Marco Buratti, Small quasimultiple affine and projective planes: some improved bounds, *J. Combin. Des.* 6 (1998), no. 5, 337--345. Contains an application of twin primes to design theory. Summary [Link](#) .

Andrej Dujella, A problem of Diophantus and Dickson's conjecture, *Number theory*, 147--156, de Gruyter, 1998. Summary [Link](#) .

D. A. Goldston, and C. Y. Yildirim, Primes in short segments of arithmetic progressions, *Canad. J. Math.* 50 (1998), no. 3, 563--580.

Review by John B. Friedlander [Link](#) .

J. Nesheiwat and B. K. Szymanski, Computing Twin Primes and Brun's Constant:

A Distributed Approach," Proceedings of the Seventh IEEE International Symposium on High Performance Distributed Computing (HPDC7), pages 42-49, 1998.

K. Ramachandra, Many famous conjectures on primes; meagre but precious progress of a deep nature, *Proc. Indian Nat. Sci. Acad.*, Part A 64 (1998), no. 5, 643--650. Short survey paper. See below reference.

Review by R. C. Baker [Link](#) .

K. Ramachandra, Many famous conjectures on primes; meagre but precious progress of a deep nature, *Math. Student* 67 (1998), no. 1-4, 187--199.

Short survey paper. See above reference. Review by Wen-Bin Zhang [Link](#) .

Simon Wong, Concentration function of additive functions on shifted twin primes, *Acta Arith.* 84 (1998), no. 3, 193--224. Review by Matti Jutila [Link](#) .

Published in 1997

Tony Forbes, A large pair of twin primes, *Math. Comp.*, 66 (1997), no. 217, 451-455.

Y. K. Huen, The twin prime problem revisited, *Internat. J. Math. Ed. Sci. Tech.* 28 (1997), no. 6, 825--834. Review by Marc Deléglise [Link](#) .

Jerzy Kaczorowski, Hilbert's Eighth Problem. (Polish. English summary), *Hilbert's Problems*, 85--118, Polsk. Akad. Nauk, Warsaw, 1997.

Review by Andrzej Madrecki [Link](#) .

Jörg Richstein, Unendliche Geschwisterliebe unter Zahlen oder Die Suche nach den Primzahlzwillingen (German) [Infinite sibling love among numbers or The search for the twin primes], *Überblicke Mathematik* [Surveys of mathematics], Edited by Albrecht Beutelspacher, Norbert Henze, Ulrich Kulisch and Hans Wußing, Vieweg, Braunschweig, 1997, 135--143 . Expository paper on the history of prime number theory, with an emphasis on the twin prime problem conjecture. Review by Matti Jutila [Link](#) .

Published in 1996

K. Indlekofer and A. Jári, "Largest known twin primes," *Math. Comp.*, 65 (1996) 427-428.

T. Nicely, Enumeration to 10^{14} of the Twin Primes and Brun's Constant, *Virginia J. Sci.*, (1996), vol. 46, p. 195-204.

Aldo Peretti, The twin prime problem, according to Hardy-Littlewood, *Notes Number Theory Discrete Math.* 2 (1996), no. 3, 33--41.

Published in 1995

Joseph B. Dence and Thomas P. Dence, A necessary and sufficient condition for twin primes, *Missouri J. Math. Sci.* 7 (1995), no. 3, 129--131.

Thomas R. Nicely, Enumeration to 10^{14} of the twin primes and Brun's constant, *Virginia J. Sci.* 46 (1995), no. 3, 195--204.

Published in 1994

Mojtaba Moniri, Recursive models of open induction of prescribed finite

transcendence degree >1 with cofinal twin primes, *C. R. Acad. Sci. Paris Sér. I Math.* **319** (1994), no. 9, 903--908.

Published in 1992

T. Aaron Gulliver and Vijay K. Bhargava, Self-dual codes based on the twin prime product 35 , *Appl. Math. Lett.* **5** (1992), no. 3, 95--98.

Alberto Perelli and János Pintz, On the exceptional set for the $2k$ -twin primes problem, *Compositio Math.* **82** (1992), no. 3, 355--372.

Published in 1991

Ding Zhu Du and D. Frank Hsu, Partitionable starters for twin prime power type, *Discrete Math.* **87** (1991), no. 1, 23--28.

Published in 1990

Jean-Pierre Bauer, Sur un problème équivalent au problème des nombres premiers jumeaux (French) [On a problem equivalent to that of twin primes], *Ann. Sci. Univ. Franche-Comté Besançon Math.* (4) No. 2 (1990), 3--7.

Liu Hong-Quan, On the prime twins problem, *Sci. China Ser. A.*, **33** (1990), 281-298.

Bodo K. Parady, Joel F. Smith, and Sergio E. Zarantonello, Largest known twin primes, *Math. Comp.*, **55**, no. 191, 1990, pp. 381-382.

Jie Wu, Sur la suite des nombres premiers jumeaux (French) [On the sequence of twin primes], *Acta Arith.* **55** (1990), no. 4, 365--394.

See important review by Wen-Bin Zhang [Link](#)

Published in 1986

Norberto Cuesta Dutari, Arithmetic of the sequences $6n-1$ and $6n+1$ and of twin primes, (Spanish), *Collect. Math.* **37** (1986), no. 3, 211--227.

A. Peretti, On the twin prime problem, *Bull. Number Theory Related Topics* **10** (1986), no. 2, 48--60.

Published in 1983

E. B. Belyakov and V. I. Martcyanov, V. I. The universal theory of the integers and the extended twin prime conjecture (Russian), *Algebra i Logika* **22** (1983), no. 1, 26--34.

Published in 1982

Ding Zhu Du and F. K. Hwang, Balanced Howell rotations of the twin prime power type, *Trans. Amer. Math. Soc.* **271** (1982), no. 2, 415--421.

Rudolf Ondrejka, More very large twin primes, *J. Recreational Math.* **15** (1982/83), no. 1, 7.

Published in 1980

R. F. Faiziev, Two criteria for the theory of k -twin primes (Russian), *Izv. Akad. Nauk Tadzhik. SSR Otdel. Fiz.-Mat. Khim. i Geol. Nauk* **1980**, no. 1(75), 76--77.

Published in 1979

A. O. L. Atkin and N. W. Rickert, On a larger pair of twin primes, Abstract 79T-A132, *Notices Amer. Math. Soc.*, 26 (1979), A-373

Crandall, R. E.; Penk, M. A. A search for large twin prime pairs. *Math. Comp.* **33** (1979), no. 145, 383--388.

Daniel Zwillinger, A Goldbach conjecture using twin primes, *Math. Comp.* **33** (1979), no. 147, 1071. Contains a promising new approach to Goldbach's conjecture. This approach has been extended by Noel Patson in 2000. Summary [Link](#).

Published in 1976

Richard P. Brent, Tables Concerning Irregularities in the Distribution of Primes and Twin Primes Up to 10^{11} , *Math. Comput.*, (1976), vol. 30, p. 379.

Richard P. Brent, Correction to: "Irregularities in the distribution of primes and twin primes", (*Math. Comp.* 29 (1975), 43--56), *Math. Comput.* 30 (1976), no. 133, 198.

J. Mlcek, J. Twin prime problem in an arithmetic without induction, *Comment. Math. Univ. Carolinae* 17 (1976), no. 3, 543--555.

Published in 1975

Richard P. Brent, Irregularities in the distribution of primes and twin primes, Collection of articles dedicated to Derrick Henry Lehmer on the occasion of his seventieth birthday, *Math. Comp.* 29 (1975), 43--56.

Werner Neudecker, On twin "primes" and gaps between successive "primes" for the Hawkins random sieve, *Math. Proc. Cambridge Philos. Soc.* 77 (1975), 365--367.

Published in 1974

D. Shanks and J. W Wrench, Brun's Constant, *Math. Comput.*, 28 (1974), 293-299.

Published in 1972

A. Hedayat, E. Seiden, and W. T. Federer, Some families of designs for multistage experiments: mutually balanced Youden designs when the number of treatments is prime power or twin primes, *I. Ann. Math. Statist.* 43 (1972), 1517--1527.

Published in 1968

C. A. Holben and J. H. Jordan, J. H., The twin prime problem and Goldbach's conjecture in the Gaussian integers, *Fibonacci Quart.* 6 1968 no. 5, 81--85, 92.

Published in 1967

P. Turán, On the twin-prime problem III, *Acta Arith.* 14, 1967/1968, 399--407.

P. Turán, On the twin-prime problem II, *Acta Arith.* 13, 1967/1968, 61--89.

Published in 1964

P. Turán, On the twin-prime problem I, *Magyar Tud. Akad. Mat. Kutató Int. Közl.* 9, 1964, 247--261.

Published in 1961

C.E. Fröberg, On the sum of inverses of primes and twin primes, *Nordisk Tidskr.*, Informationsbehandling (BIT), (1961), vol. 1, p. 15-20.

A. F. Lavrik, The number of k -twin primes lying on an interval of a given length, *Dokl. Akad. Nauk SSSR* 136, 281--283 (Russian); translated as *Soviet Math. Dokl.* 2, 1961, 52--55.

John W. Wrench, Jr., Evaluation of Artin's constant and the twin-prime constant, *Math. Comp.* 15, 1961, 396--398.

Published in 1960

S. W. Golomb, The twin prime constant, *Amer. Math. Monthly* 67 (1960), 767-769.

A. F. Lavrik, On the distribution of k -twin primes (Russian), *Dokl. Akad. Nauk SSSR* 132, 1960, 1258--1260.

A. F. Lavrik, On the twin prime hypothesis of the theory of primes by the method of I. M. Vinogradov, *Dokl. Akad. Nauk SSSR* 132, 1013--1015 (Russian); translated as *Soviet Math. Dokl.* 1, 1960, 700--702.

Daniel Shanks, A note on Gaussian twin primes, *Math. Comput.* 14, 1960, 201--203.

Published in 1955

Pinhas Naor, A property of prime twins (Hebrew. English summary), *Riveon Lematematika* 9 (1955), 36--40. Review by E. G. Straus [Link](#).
A translation into English would be most welcome. Contains an interesting isomorphism which sends ordinary twin primes to primes in a special semigroup. Also contains an interesting relation with the paper published by Maria Suzuki in 2000.

Published in 1923

G.H. Hardy and J.E Littlewood, Some problems of 'Partitio Numerorum' III, On the expression of a number as a sum of primes, *Acta Mathematica*, (1923), vol. 44, p. 1-70. Correction of the date, 1923 instead of 1922, communicated by Noel Patson. The error is wide spread in the mathematical literature.

Published in 1919

V. Brun, La série $1/5+1/7+1/11+1/13+1/17+1/19+1/29+1/31+1/41+1/43+1/59+1/61+...$, où les dénominateurs sont nombres premiers jumeaux est convergente ou finie, *Bulletin des sciences mathématiques*, (1919), vol. 43, p. 100-104 and p. 124-128.

Published in 1878

J.W.L. Glaisher, An enumeration of prime-pairs, *Messenger of Mathematics*, (1878), vol. 8, p. 28-33.

Related Books (In reverse chronological order of publication)

Richard Crandall and Carl Pomerance
 Prime numbers. A computational perspective.
 Springer-Verlag, New York, 2001 . xvi+545 pp.
 Review by: Samuel S. Wagstaff, Jr. [Link](#) .

George Greaves
 Sieves in number theory
 Ergebnisse der Mathematik und ihrer Grenzgebiete (3), 43
 Springer-Verlag, 2001 , xii+304 pp.
 Review by H. G. Diamond [Link](#) .

Melvyn B. Nathanson,
 Additive number theory, The classical bases.
 Graduate Texts in Mathematics, 164
 Springer-Verlag, xiv+342 pp., 1996 .
 Review by D. R. Heath-Brown [Link](#) .

Paulo Ribenboim
 The new Book of Prime Number Records
 Springer-Verlag, xxiv+541 pp., 1996 .
 No review available for this edition.
 Review of the first edition by Don Redmond [Link](#) .

Godfrey Harold Hardy and Edward Maitland Wright
 An Introduction to the Theory of Numbers, fifth edition
 The Clarendon Press, Oxford University Press, xvi+426 pp., 1979 .
 Review by T. M. Apostol [Link](#) .

William J. LeVeque
 Fundamentals of Number Theory
 Reprint of the 1977 original.
 Dover Publications, Inc., viii+280 pp., 1996 .
 Review by R. F. Churchhouse [Link](#) .

H. Halberstam and H.-E. Richert
 Sieve methods
 London Mathematical Society Monographs, No. 4
 Academic Press, xiv+364 pp., 1974 .
 The most cited book in publications about twin primes, which is well deserved.
 Review by J. W. Porter [Link](#) .

Exercise about twin primes

Problem 10844 on Fibonacci twin primes, The American Mathematical Monthly, Volume 108, Number 1, page 77, Proposed by Nick MacKinnon at Winchester College, UK, January 2001.
Solution by Stephen Gagola, The American Mathematical Monthly, Volume 109, Number 1, page 78, January 2002.

Back to my home page [Link](#) .
