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88-04-08

Neil J.A. Sloane,
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600 Mountain Avenue,
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Dear Neil,

Must be two or three days since I wrote!

This is to enclose a table of Auluck's penny partitions (ref. is in S.222; I was alerted to it by Jim Propp) and to make some observations thereon:

An entry, in Auluck's notation, is $R(n,r)$, the number of arrangements of n pennies in rows, r contiguous pennies in the bottom row, and each higher row consisting of contiguous pennies, each touching two pennies in the row below. By contrast, Propp's penny partitions don't require the second contiguity condition, so they are somewhat more numerous.

S.222 is the sums of the rows of the table.

I claim that the 19th & 20th entries in Auluck are in error, and that S.253 should be replaced and extended by

(1), 1, 2, 3, 5, 8, 12, 18, 26, 38, 53, 75, 103, 142, 192, 260, 346, 461, 607 (not 605), 797 (not 796), 1038, 1348, 1738, 2234, 2856, 3638, 4614, 5832, 7342, 9214, 11525, 14369, ... (E.& O.E.)

Sums of *columns* are alternate Fibs.(S.569)but that requires proof (easy!), I'm copying this to Jim Propp, who, hopefully, will check. I think he knows most of it already.

A sequence worthy of note is $R(2r,r)$:

~~0,0~~, 1, 2, 5, 11, 20, 37, 63, 110, 174, 283, 435, 671, 1001 ~~(no array is complete without it)~~, 1492 ~~(a very good year)~~, 2160, 3127, 4442, 6269, 8739, 12109, 16597, 22618, 30576, ...

It must be connected with S.542, and the difference between the two sequences,

(...,0,0,0,0), 1, 2, 10, 19, 52, 105, 224, 429, 820, ...

may be worth looking at. The particular interest though, is that $R(2r+s,r+s)$, $s = 0,1,2,...$, is an AP with common difference, $R(r) = \sum R(r,i)$, the members of S.253., which observation simplifies the calculations.

Another sequence which may be of interest, but which is not in Sloane (whenever I say that, I must admit I'm too lazy to look in the Supplement) is

1, 1, 2, 3, 4, 7, 9, 13, 17, 25, 32, 43, ...

It is $R(T_r-s,r)$, $s = 0,1,2,...,r-1$, extended as far as you want by taking r big enough. Except for its length, the sequence is independent of r; T_r is the rth triangular number, $\frac{1}{2}r(r+1)$.

Best wishes,

Yours sincerely,

Richard

RKG:1 Richard K. Guy.

P.S. A sequence that you might like, which takes its time at the beginning, is that of the maxima of the rows

1, 1, 2, 3, 4, 5, 6, 7, 9, 11, 15, 20, 27, 35, 44, 56, 73, 91, 115, 148, 186, 227, 283, 358, 435, 538, 671, 813, 1001, 1233, 1492, 1815, 2223, 2673, 3247, 3933, 4713, 5683, 6850, 8170, 9785, 11725, 13948, 16587, 19783, 23468, ...

P.P.S. A nice red herring, swimming between the Capell-Narayana and Wedderburn-Etherington numbers, is the sequence generated by $(1-x-x^2)/(1-2x)(1-x^2)$:

1, 1, 2, 3, 6, 11, 22, 43, 86, 171, 342, 683, 1366, 2731, 5462, 10923, 21846, 43691, 87382, 174763, 349526, 699051, 1398102, 2796203, 5592406, 11184811, 22369622, ... (compare S.983).

P.P.P.S. Could the Sloane Inter-Library Loan Service (SILLS, as in "sea-lions and sills") provide a photocopy of the relevant portion of R.A. Fisher, Contributions to Mathematical Statistics, Wiley, New York, 1950, to enable me to check S.455? or at least explain

the reference 41.399 (p.399 of what?). NO!!! Forget it! I found it in MR, which guided me to the *Ann. Eugenics* paper, excellently reviewed by Coxeter (MR 4,183e).

P.P.P.P.S. (88-04-11) Just heard from you. Will be happy to include the lost footnote (p.474) in the review of Conway-Sloane.

encl: 2 sheets

φc: Jim Propp

n \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	1																			1											
2		1																		1											
3		1	1																	2											
5			2	1																3											
6			1	3	1															5											
7			1	2	4	1														8											
8				3	3	5	1													12											
9				2	5	4	6	1												18											
10				1	5	7	5	7	1											26											
11				1	5	8	9	6	8	1										38											
12					4	10	11	11	7	9	1									53											
13					3	11	15	14	13	8	10	1								75											
14					2	9	19	20	17	15	9	11	1							103											
15					1	9	20	27	25	26	17	10	12	1						142											
16					1	7	20	32	35	30	23	19	11	13	1					192											
17						7	20	37	44	43	35	26	21	12	14	1				260											
18						4	21	38	55	56	51	40	29	23	13	15	1			346											
19						3	18	44	63	73	68	59	45	32	25	14	16	1		461											
20		607				2	17	43	73	89	91	80	67	50	35	27	15	17	1												
22		797				1	13	46	79	110	115	109	92	75	55	38	29	16	18	1											
23		1038				1	11	43	89	122	148	141	127	104	83	60	41	31	17	19	1										
24		1348					9	41	93	145	174	186	167	135	116	91	65	44	33	18	20	1									
25		1738					7	37	98	160	209	227	224	193	163	128	99	70	47	35	19	21	1								
26		2234					4	34	97	180	242	283	280	262	219	181	140	107	75	50	37	20	22	1							
27		2856					3	29	98	192	282	333	358	333	300	245	199	152	115	80	53	39	21	23	1						
28		3638					2	24	92	206	317	401	435	433	386	338	271	217	164	123	85	56	41	22	24	1					
29		4614					1	21	89	210	357	465	530	538	508	439	376	297	235	176	131	90	59	43	23	25	1				
30		5832					1	15	83	217	384	543	632	671	641	583	492	414	323	253	188	139	95	62	45	24	26	1			
31		7342						13	75	218	418	609	755	810	813	744	658	545	452	349	271	200	147	100	65	47	25	27	1		
32		9214						9	68	214	442	687	874	988	1001	955	847	733	598	490	375	289	212	155	105	68	49	26	28	1	
		11525						7	59	212	463	757	1015	1168	1233	1193	1097	950	808	651	528	401	307	224	163	110	71	51	27	29	1
		14369						4	51	202	478	827	1152	1388	1485	1492	1385	1239	1053	883	704	566	427	325	236	171	115	74	53	28	30
								3																							
								2																							
								1																							
								1																							

A1524

done

arrangements of pennies in rows

1 2 5 13 34 89 233 610

	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
30	9	68	214	442	687	874	988	(1001)	955	847	733	598	490	375	289	212	155	105	68	49	26	28	1				
31	7	59	212	463	757	1015	1168	1233	1193	1097	950	808	651	528	401	307	224	163	110	71	51	27	29	1			
32	4	51	202	478	827	1152	1388	1485	(1492)	1385	1239	1053	883	704	566	427	325	236	171	115	74	53	28	30	1		
33	3	42	193	490	892	1300	1614	1793	1815	1752	1577	1381	1156	958	757	604	453	343	248	179	120	77	55	29	31	1	
34	2	35	177	495	952	1447	1865	2126	2223	(2160)	2012	1769	1523	1259	1033	810	642	479	361	260	187	125	80	57	30	32	1
35	1	28	165	492	1006	1600	2129	2505	2673	2667	2506	2272	1961	1665	1362	1108	863	680	505	379	272	195	130	83	59	31	33	1
36	1	23	146	487	1049	1744	2415	2915	3200	3247	(3127)	2852	2532	2153	1807	1465	1183	916	718	531	397	284	203	135	86	61	32	34	1
37		17	134	470	1031	1889	2706	3375	3784	3933	3836	3588	3198	2792	2345	1949	1568	1258	969	756	557	415	296	211	140	89	63	33	35	1
38		13	116	455	1111	2028	3009	3861	4451	4713	4695	(4442)	4049	3544	3052	2537	2091	1671	1333	1022	794	583	433	308	219	145	92	65	34	36	1
39		9	101	432	1133	2153	3323	4391	5179	5618	5683	5473	5049	4510	3890	3312	2729	2233	1774	1408	1075	832	609	451	320	227	150	95	67	35	37	1
40		7	86	407	1132	2276	3630	4951	5992	6625	6856	6684	(6269)	5656	4971	4236	3572	2921	2375	1877	1483	1128	870	635	469	332	235	155	98	69	36	38	1
41		4	73	377	1131	2375	3949	5540	6879	7774	8170	8126	7702	7066	6263	5432	4582	3832	3113	2517	1980	1558	1181	908	661	487	344	243	160	101	71	37	39	1
42		3	59	351	1110	2464	4250	6160	7836	9052	9696	9785	9435	(8739)	7883	6870	5893	4928	4092	3305	2659	2083	1633	1234	946	687	505	356	251	165	104	73	38	40	1
43		2	48	316	1092	2530	4552	6796	8879	10470	11424	11725	11447	10762	9777	8660	7477	6354	5274	4352	3497	2801	2186	1708	1287	984	713	523	368	259	170	107	75	39	41	1
44		1	39	286	1051	2590	4824	7450	9983	12037	13375	13948	13829	13444	(12109)	10815	9457	8084	6815	5620	4612	3689	2943	2289	1783	1340	1022	739	541	380	267	175	110	77	40	42	1
45		1	30	253	1014	2613	5103	8106	11161	13753	15571	16497	16587	15983	14860	13457	11853	10254	8691	7276	5966	4872	3881	3085	2392	1858	1393	1060	765	559	392	275	180	113	79	41	43	1
46			25	224	962	2634	5327		12389		18015	19462	19783	19306	18174	(16597)																						
47			17	195								22691	23468	23192	22078	20385	18335																				
48			13	168												24889	(22618)																			
49			9	142													24721	24852																	
50			7	119														(30576)																	
51			4	100																															
52			3	81																															
53			2	65																															
54			1	52																															
55			1	41																															
56				32																															
57				25																															
58				17																															
59				13																															
60				9																															