A Perfect Rhythmic Tiling Of QUADRUPLETS

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(With help from [Dr. Z's Experimental Mathematics Class](#))

Written: Nov. 30, 2004.

A (k,n)-*Perfect Rhythmic Tiling* (a concept due to composer Tom JOHNSON) is a partition(!) of the set of integers 0 through nk-1 into n mutually-disjoint arithmetical progressions of length k, with *all different spacings*. In Jean-Paul DELAHAYE's fascinating article in *Pour La SCIENCE* (Novembre 2004, p. 93), it is mentioned that none are known with k=4. Well, in yesterday's class, the class (together!) wrote a Maple program, [TomJohnson](#) , that computes such Perfect Rhythmic Tilings. That night, Lara Pudwell ran *Tom(15,4);*, and after 776 seconds of CPU time, got the following beautiful tiling.

[0, 16, 32, 48], [1, 3, 5, 7], [2, 13, 24, 35], [4, 22, 40, 58], [6, 21, 36, 51], [8, 14, 20, 26], [9, 10, 11, 12], [15, 29, 43, 57], [17, 25, 33, 41], [18, 30, 42, 54], [19, 23, 27, 31], [28, 37, 46, 55], [34, 39, 44, 49], [38, 45, 52, 59], [47, 50, 53, 56] .

Added Dec. 1, 2004: We were scooped!, see [Tom Johnson's message to Lara Pudwell](#) .

[Personal Journal of Ekhad and Zeilberger](#) .