

# Tiling in My music

Tom Johnson

*Abstract: Rhythmic canons and other one-dimensional tiling techniques dominated my composing for two or three years, and a number of instrumental pieces, both solo and ensemble, evolved during this time. With numerous examples, I have put together a little survey of different ways in which I have done this, often leaving holes and using other procedures not normally considered correct.*

It must have been 1999 when Moreno Andreatta gave me a copy of Dan Tudor Vuza's landmark essay *Supplementary Sets and Regular Complementary Unending Canons (Perspectives of New Music, 1991-92)*, because already in 2000 its influence on me became clear with a *Canon in 3, 6 or 9 voices*, written for an installation of Martin Riches in Berlin. When the MaMuX meetings at IRCAM began in 2001, I was able to hear regularly the mathematical music theories of Guerino Mazzola, Thomas Noll, Emmanuel Amiot, Franck Jedrzejewski and Harald Friepertinger, as well as Andreatta himself and many occasional visitors, and this information stimulated me more and more. My composing time during the year 2002 was devoted almost exclusively to fitting together little rhythmic tiles, and early in 2003 I brought out the edition *Tilework : 14 Pieces for 14 Solo Instruments*. Later that year I wrote *Tilework for String Quartet*, and *Tilework for Piano*, though by the end of that year my concentration was already turning more toward combinatorial designs, which is quite a different topic.

I recently reread the Vuza essay, and it seems all the more clear to me that this text has been important for all the mathematicians and music theorists cited above, not to mention for Jon Wild and other North American music theorists, and for quite a few composers. I now consider this work the most important music theory treatise of the last 20 years, particularly since it is one of those rare cases where music theory has preceded musical practice. Harmony and counterpoint books, essays on serial techniques, manuals for figured bass, and music analysis texts have generally dealt exclusively with musical procedures already practiced by composers. Only in rare cases like Vuza, Leonhard Euler, and Hugo Riemann have theorists preceded composers, though Henry Cowell's 1930 book *New Musical Resources* should also be mentioned. This highly original text never circulated much in Europe, but it was widely read in the U.S. The author never followed up on these theories much in his own music, but John Cage and Conlon Nancarrow both sited this as a seminal influence on their music, and younger American composers such as Kyle Gann, Larry Polanskyn, David First, and John Luther Adams also acknowledge the

influence. Cowell's book has left a significant mark on music history, as Vuza's ideas are just beginning to do.

As I was rereading Vuza, it also became clear that later researchers have tended to pay attention mostly to the stimulating problems in Vuza's maximal category, and to the classical linear tilings, where every point is filled exactly once. We often forget that Vuza also considered many tilings where some points are not filled, or are filled with more than one note. In Part III of the article, for example, there is a case on page 112 where every sixth point is left empty, and another case on pages 109-110 where every fifth and sixth point is silent. In Part II, page 195, there is a lovely example of an eight-beat cycle where a simple two-note rhythm (0,5) enters four times (at points 0, 3, 4 and 7), resulting in an accumulative pattern of two notes, one note, pause, one note, two notes, one note, pause, one note, two notes... I suspect Vuza realized that long streams of eighth notes can make dull music, and that he should consider variations if he wanted his concepts to be useful for composers. In any case, irregular tiling patterns have been compositionally very rich for me, so the first section here is about "Tiling with Holes". The following section, "Tiling in Different Tempos" goes back to the "pavages par augmentation" discussed in chapter V of this book, but with specific examples not discussed there.

## Tiling with Holes

Perhaps the clearest example for showing the advantages of tiling with holes is *Tilework for Oboe*, one of the 14 pieces from the 2003 collection. In the excerpt shown here we see one rhythm moving closer and closer to another rhythm until they finally mesh together. Gradually filling holes is the central idea of this movement.

The image displays six staves of musical notation in treble clef, illustrating a rhythmic process. The notation consists of eighth notes and rests. The first staff shows a sequence of notes: C4, D4, E4, F4, G4, A4, B4, C5, with a half rest following. The second staff is identical to the first. The third staff shows the notes: C4, D4, E4, F4, G4, A4, B4, C5, with a half rest following. The fourth staff shows the notes: C4, D4, E4, F4, G4, A4, B4, C5, with a half rest following. The fifth staff shows the notes: C4, D4, E4, F4, G4, A4, B4, C5, with a half rest following. The sixth staff shows the notes: C4, D4, E4, F4, G4, A4, B4, C5, with a half rest following. The notation is presented in a way that suggests a rhythmic pattern that is gradually filling in or meshing together over time.

One of the more remarkable little discoveries of post-Vuza tiling mathematics is Jon Wild's observation that any three-note rhythm, in combination with its retrograde, can tile a line. Start with some three-note rhythm, fill the first hole with another statement of the basic rhythm if that is possible, and with the retrograde if it is not. *Tilework for Clarinet* made use of this technique, using the rhythm (0,2,5) and its retrograde (0,3,5). With the entrance of the sixth voice, a complete line is tiled, as shown here, with the ascending (0, 2, 5) beginning on beats 1, 2, and 9, and the descending (0, 3, 5) beginning on beats 5, 12, and 13, as you see below. Of course, if you want people to hear the counterpoint, you want to hear the voices enter one at a time, as in normal canons, which means leaving lots of holes within and around the music as it comes together.



A particularly fruitful general truth, which became rather obvious after Vuza's work, is that grouping pitches together and tiling rhythms together is essentially the same thing. I can't imagine that a composer in the 1980s would ever have thought of composing a 12-tone piece in which four combinatorial trichords would also be complemented by four three-note rhythmic tiles, though today this seems like quite a normal procedure. In the excerpt below, from the first movement of *Tilework for Saxophone*, the 12 notes of the octave are distributed into two three-note phrases and their inversions, and the rhythms tile together in a similar way. Even in the last four measures, however, a few holes remain in order to permit the player to breathe, and to help us separate the three-note building blocks.



I want to turn now to the first tiling piece I ever wrote, the *Canon in 3, 6 or 9 Voices*, referred to at the beginning of this article. This was an audience participation installation with a row of 8 tubular bells, and participants, one after the other, could walk from the beginning of the line to the end, playing a simple eight-note melody, following the rhythm of blinking lights. By the time the third participant entered, the complete 24-beat phrase was filled out. Again, it was necessary to begin with lots of holes, so that people could see and hear the basic theme as the layers were added.

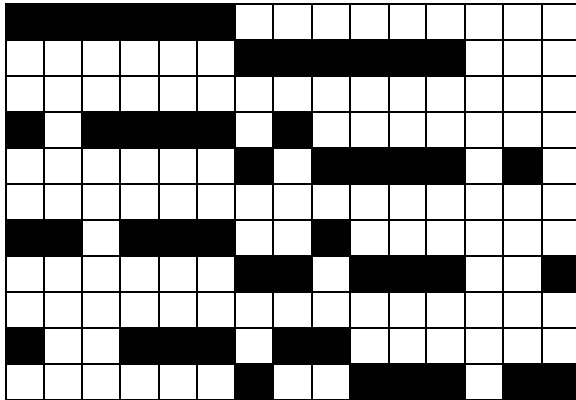


If enough participants wanted to join in, we could have as many as nine voices, shown below, at which point one hears a repeating sequence of eight chords, with a new voice entering every two or three beats. Theoretically one could continue until 27 voices were playing, in which case a new voice would enter on every beat and one would hear a single repeated chord, consisting of all eight notes of the melody.



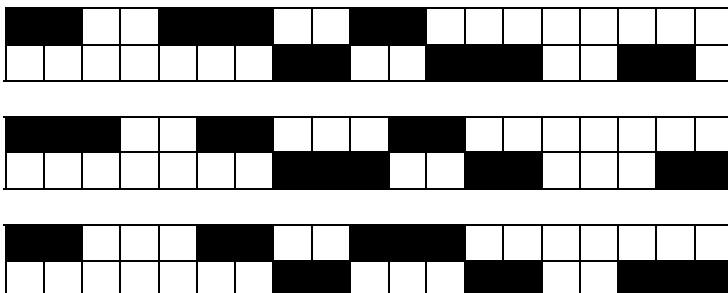
So far the examples I have given can all be worked out with pencil and paper, but in many cases I have found the computer necessary, sometimes because I could not find some solution without one, and sometimes because I knew a few solutions but wanted to have the complete set. A mathematician whose work has been particularly useful for me in such cases is Harald Friperinger, whose web site contains a gigantic list of rhythmic canons. I stopped downloading when I came to the 516 ways of tiling 30 points with six notes in five voices, because I already had 120 pages of output, and I figured this would be enough to solve most of my problems, but I could have gone further. I refer to this document quite often, and three pieces were calculated almost completely from the lists it contains.

For *Tilework for String Quartet*, the problem was to find six-note rhythms that tile the line when the four instruments enter six beats apart. Friperinger's list begins with the four rhythms you see here. I show the second entrances as well as the first, so you can see more easily how the rhythms fit together:

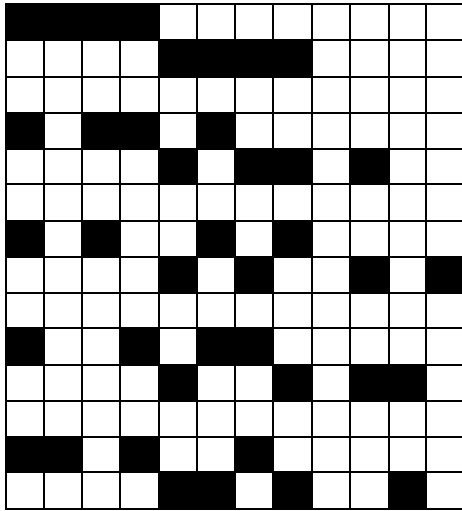


When all four instruments are playing, the music makes a cycle 24 beats long, with entrances of the instruments at four points (0, 6, 12, 18). Friperinger's list contains 39 canons in all, but my string quartet music includes only 29 of these, as the rhythms after that became too long for the counterpoint to be heard clearly.

In *Tilework for Double Bass* I wanted the bass player to play seven-note canons with himself (or herself). Friperinger's list shows exactly nine unique solutions, but I enlarged the list to 14 when I found that I could sometimes form two or three variations from a single official solution. One of Friperinger's rhythms, for example, is the palindrome that begins the diagram below, which I displaced to form the two following canons. All three have quite a different feeling, despite the fact that they are mathematically the same. I respect mathematical discipline enormously, and want my music to have that kind of rigor, but sometimes there is a difference between things that are logically the same and things that are musically the same.



A similar situation occurs in *Tilework for Tuba*, where the fast section at the end turns on rhythmic canons of four notes, turning in a cycle of eight beats. Friperinger's list gives only the first and second rhythms, since the third, fourth, and fifth are simply displacements of the second one, moving the rhythm around a circle of eight points. The rhythms are scarcely the same for the tubist, however, nor for the listener.



## Tiling in Different Tempos

Jean-Paul Davalan goes into much detail about perfect tilings, tilings where each voice is in a different tempo, my own *Tilework for Piano* (2003) being no doubt the first musical realization of a perfect tiling. The tempo ratios in all these cases, however, are in irregular patterns like  $7 : 5 : 4 : 2 : 1$ , which is the case of the shortest perfect tiling, used in *Tilework for Piano*. It would be nice if the ratios were  $n : n-1 : n-2 : n-3 : \dots : 1$ , but I couldn't believe that this would be possible until October 4, 2006, when I received the following email from Jon Wild, explaining that this is quite possible provided you are willing to tile a line with holes, a case of what he calls a "gapped" or "warped" tiling:

*Dear Tom,*

*Just a very quick note to let you know I was messing around with some gapped tilings again, and found several perfect tilings that I did not believe were possible previously. By "perfect" I mean they use every scaled version of the tile from 1 to n, exactly once each. I thought you might be interested...*

(Yes, I was interested, and I think you will be too. Read on.)

*These are of order 8 and they tile this waltz-like rhythm :*

*--/--/--/*

*The tiles I find perfect order-8 tilings for are (0,1,3), (0,2,5), (0,3,7), (0,3,8), and (0,3,11).*

Jon only sent examples of (0,1,3) and (0,2,5), but anyone with a computer can find the solutions for the other rhythms if they wish. Rather than reproduce Jon's schematic diagram, I decided to write out a musical realization of his (0,1,3) example, so what you see below is a new little composition, dedicated to him and published here for the first time. I decided to use the octatonic scale,

skipping every third pitch of the chromatic, just as the rhythm skips every third note in time. The fastest voice is the highest one, beginning on the high C (indicated “1”), the 2 : 1 tempo is the second voice down, beginning on A (indicated “2”), the 3 : 1 is the third voice, beginning on G-flat (indicated “3”), and so on. Of course, this is a loop, rather than a line, but you can hear all the voices at once during the five repeating measures. I called it « Extra perfect », because the requirement that one uses all possible tempos from 1 to n is quite a bit more constraining than the tilings discussed by Davalan, which are only perfect:

for Jon Wild  
**Extra Perfect**

Tom Johnson

1 —                      2                                      3

4 5

6 —                                      7 —

8

Many would have been so pleased to find such a thing that they would have announced it as an important discovery and written a long article, but for Jon this was only a little observation to share with a composer colleague. Without diminishing the originality of Wild's work here, we should remember that the idea of tiling only certain points and leaving some holes is a technique that already has several precedents in Vuza's treatise.

*Tilework for Viola* is in three tempos, and it is full of holes. In this excerpt, consisting of 10 bars of 9/8 time, the six-note theme is played four times in the upper voice, three times more slowly in the second voice, and twice more slowly yet in the lower voice. With only 54 of the 82 beats filled, the surrounding pauses enable us to hear rather clearly the three voices in the three tempos in ratios of 4 : 3 : 2.

The image displays three systems of musical notation for the piece 'Tilework for Viola'. Each system consists of three staves: a treble clef staff (upper voice), a bass clef staff (second voice), and a bass clef staff (lower voice). The music is written in 9/8 time. The first system shows the six-note theme being played four times in the upper voice, three times more slowly in the second voice, and twice more slowly in the lower voice. The second system continues this pattern, and the third system shows the theme being played in the upper voice, with the second and lower voices having rests in the final two bars.

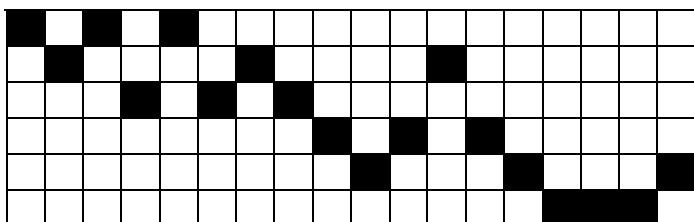
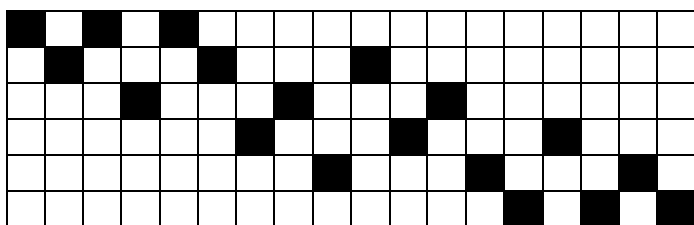


*Tilework for Violin*, shown below, is also a canon in three voices in three tempos. In the upper octave you see the first voice, which leaves a pause after each two-note phrase. Two out of three of these pauses are then filled by the second voice, playing three times slower, so when the second voice in the middle octave is added, we hear notes 8/9 of the time instead of only 2/3 of the time. With the addition in the lower octave of the third voice, three times slower still, we hear notes 26/27 of the time. Of course, we could keep adding voices and multiplying the density by three, but we would never fill the time completely, just as a Peano curve never fills a space completely in the fractals books. *Tilework for Violin* is not a neat process placing exactly one note on each beat, but it is something even more interesting for me.

The image shows a musical score for a violin piece. It consists of six staves of music, each starting with a measure number: 57, 63, 69, 75, 81, and 87. The music is written in treble clef and features a series of two-note phrases with various accidentals (flats and sharps). The first voice (top staff) has a two-note phrase followed by a pause. The second voice (middle staff) fills in the first two of these pauses. The third voice (bottom staff) fills in the first two of the second voice's pauses. The piece ends with a double bar line and the instruction "played three times".

In both the viola and violin pieces the slower voices, filling in the holes, play the same music as the first voice, but this is not necessarily the case. A couple of times I have filled in regular sequences of holes with some completely different idea, resulting in two kinds of music, which are completely different, but which tile together all the same.

At one point I wanted to tile a line of 18 points with a simple three-note rhythm (0,1,2), where the six voices could play in any of five tempos : 5 : 4 : 3 : 2 : 1, leaving no holes, as in these two examples:



This was one of those cases that required a computer search, and I can do this myself up to a certain point, but of course, it is arbitrary to simply stop with the solutions I can find myself, so I discussed the problem with Andranik Tanguian, a Russian mathematician whom I had met at a math and music conference in Bourges. He ran the problem through his computer and sent me a total of seven solutions, assuring me that there were no others. After several revisions these seven tilings became *Tilework for Log Drums* (2005).

As a final example of tiling the line in different tempos, I want to show you how this can happen with a self-replicating melody. I've written quite a few melodic loops that make copies of themselves at slower tempos at the same time as they turn around. The self-replicating technique began with *Rational Melody No. 15* (1983), long before I knew anything about Vuza's work, and the idea was basically quite different. A self-replicating melody, and the copies it makes at slower tempos, must all be in unison with one another, so in principle the notes are simultaneous, whereas the different layers of a tilework construction are encrusted around one another so as to *avoid* simultaneities. The two principles are quite different, but curiously they sometimes do the same things, and I am sure now that the reason I was so stimulated by Vuza's treatise is because what he was doing was similar to things I had already done in this other framework.

Let us take a look at this melody, which is the theme, and really the only material, of *La Vie est si courte*, a 16 minute piece for eight instruments that I wrote in 1998.



The duration of the melodic loop is 10 quarter notes or 20 eighth notes, and it makes copies of itself both three times slower and seven times slower than itself. I won't go into detail here about the construction of self-replicating

melodies, which is explained rather thoroughly in *Self-Similar Melodies* (1996). Let me just go on to show you how this self-replicating melody can also become a tiling construction. Below the theme is written in three voices, each three times slower than the original tempo. The meter is still 10/4, but it now takes three measures for the theme in the slow tempo to make its cycle. When the first voice arrives at the third measure of the theme, the second voice is playing the second measure and the third voice is playing the first measure, and the accumulation of the three voices produces the theme in the original tempo, as indicated by the small notes in the lowest staff. As far as the three slow voices are concerned, there are no simultaneities, and they form a rhythmic canon in the purest Vuza sense, with two supplementary sets: one for the rhythm (0, 6, 12, 15, 18, 24, 30, 36, 42, 45, 48, 54) and one for the onset points (0, 20, 40). Of course, there are some holes in there, but hopefully, by now, the reader is no longer too upset with music that has a few holes.



Tom Johnson, Paris, August, 2008

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