Ben Green und Terence Tao

Professor Ben Joseph Green of the University of Cambridge and Professor Terence Tao of the University of California at Los Angeles are the recipients of the Ostrowski prize 2005. The prize consists of 50,000 Swiss Francs for each prize winner and the possibility to nominate a promising young candidate for a Postdoctoral Fellowship of 30,000 Francs.

The Ostrowski prize is conferred to Green and Tao for their exceptional achievements in the area of analytic and combinatorial number theory. In joint work they have obtained a series of impressive results which implies a proof for the old conjecture that there exist arbitrary long arithmetic progressions of primes. Their methods open entirely new perspectives for prime number theory and some other parts of mathematics.

The question as to whether for given k there are infinitely many arithmetic progressions of primes of length k had been around for more than a century. It is easy to construct such quadruples, for example 7, 37, 67, 97, but it is not clear that there are infinitely many of them, nor that such progressions of length 40, say, exist. Before the work of Green and Tao, no-one had succeeded in going beyond the existence of infinitely many three term arithmetic progressions of primes. In a 2005 paper Green gave a new proof of the latter result that was much more combinatorial in nature than previous proofs. The match-up of Green and Tao turned out to be extraordinary fertile. They came up with a very novel plan to prove that there are infinitely many four term arithmetic progressions of primes, based on Green's earlier work. Soon after they had succeeded, they established the full conjecture. As in the case of Szemeredi's celebrated proof of the existence of arbitrarily long arithmetic progressions in sets of positive integers of positive density, the leap from three to four was the most difficult one. Green and Tao next obtained a more useful result in applications: to give an asymptotic estimate for the number of four-term arithmetic progressions of primes up to some point. Their goal is the proof of the k-tuplets conjecture, one of the Holy Grails of prime number theory.

The new methods have their genesis in several older developments: work on sumsets by Freiman (1962), by Roth (1953) on three term arithmetic progressions, by Szemeredi on k-term arithmetic progressions, by Gowers in the late 90s in his new perspective on harmonic analysis, and in more recent work by Bourgain and Konyagin. The work of Green and Tao has given birth to a new subject, "additive combinatorics", which sits at the crossroads of many classical areas:

harmonic analysis, analytic combinatorics, ergodic theory, Ramsey theory, analytic number theory, random graph theory, discrete geometry, ...

Green had already several other excellent earlier works in combinatorial number theory. E.g. in 2004 he had settled one of Paul Erdös' favourite conjectures, viz. the Cameron-Erdös conjecture on the number of sumfree subsets. In 2002 he had proved that sumsets contain long arithmetic progressions. Tao was already one of the world's top mathematicians by the time they got together. He has made major contributions in many subjects in analysis and combinatorics, such as startling work on the matrix multiplication conjecture with Knutson, contributions to the restriction problem and the Kakeya problem in harmonic analysis, and the development of sum-product formulas in finite fields. He was awarded the Fields medal in 2006, at the age of 31!

The Ostrowski prize is granted every two years for the best achievements in pure mathematics and the theoretical foundations of numerical analysis. The prize is awarded by a foundation established by Professor A.M. Ostrowski (1893-1986), a former Professor of Mathematics at the University of Basel, Switzerland. According to the rules the jury consists of representatives of the universities of Basel, Jerusalem, and Waterloo and the academies of sciences of Denmark and The Netherlands. Previous prize winners have been: Louis de Branges (1989); Jean Bourgain (1991); Miklos Laczkovich and Marina Ratner (1993); Andrew Wiles (1995); Yuri Nesterenko and Gilles Pisier (1997); Alexander Beilinson and Helmut Hofer (1999); Henryk Iwaniec, Peter Sarnak and Richard L. Taylor (2001); Paul D. Seymour (2003). The prize ceremony for the Ostrowski prize 2005 will take place in Leiden, The Netherlands, in April 2007.