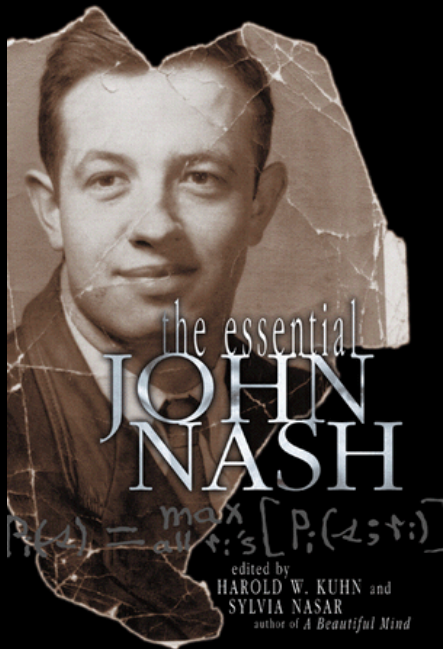


15-396

Science of teh Interwebs

Preliminaries of Game Theory

Lecture 8 (October 4, 2011)



Players

		Roommate 2	
		WiFi	Wired
Roommate 1	WiFi	10,10	15,5
	Wired	5,15	5,5

Payoffs

Strategies for Player 1: $S_1 = \{\text{WiFi}, \text{Wired}\}$

Strategies for Player 2: $S_2 = \{\text{WiFi}, \text{Wired}\}$

$S = \{ (\text{WiFi}, \text{Wired}), (\text{WiFi}, \text{WiFi}), (\text{Wired}, \text{WiFi}), (\text{Wired}, \text{Wired}) \}$

We assume that everything a player cares about is summarized in the player's payoff

We also assume that each player knows everything about the game

Prisoner's Dilemma

Suspect 2

Conf

No Conf

Suspect 1

Conf

-4,-4

0,-10

No Conf

-10,0

-1,-1

		Conf	No Conf
Suspect 1	Conf	-4,-4	0,-10
	No Conf	-10,0	-1,-1

Best Responses

A strategy s_1^* is a **best response** by player 1 to a strategy s_2 for player 2 if

$$\pi_1(s_1^*, s_2) \geq \pi_1(s_1, s_2)$$

for all strategies $s_1 \in S_1$.

		Suspect 2	
		Conf	No Conf
Suspect 1	Conf	-4,-4	0,-10
	No Conf	-10,0	-1,-1

If Suspect 2 does not confess, then confessing is a best response for Suspect 1

Dominant Strategy

A strategy s_1^* is a **Dominant Strategy** for player 1 if s_1^* is a Best Response to every possible strategy for player 2.

		Suspect 2	
		Conf	No Conf
Suspect 1	Conf	-4,-4	0,-10
	No Conf	-10,0	-1,-1

**Confessing is a dominant strategy
for both Suspects!**

		Player 2	
		I	II
Player 1	I	3,3	1,1
	II	1,1	0,0

I is a dominant strategy for both players

Optimal Pricing

		Firm 2	
		H	L
Firm 1	H	2,2	0,3
	L	3,2	5,1

(L,H) will be played

		Player 2		
		L	M	R
Player 1	t	3,3	2,2	2,1
	m	2,2	1,2	3,1
	b	1,2	3,1	2,3

Neither player has a dominant strategy

Nash Equilibrium

A pair of strategies (s_1^*, s_2^*) is in Nash Equilibrium if s_1^* is a Best Response by player 1 to s_2^* , and s_2^* is a Best Response by player 2 to s_1^* .

		Player 2		
		L	M	R
Player 1	t	3,3	2,2	2,1
	m	2,2	1,2	3,1
	b	1,2	3,1	2,3

Coordination Game

		Player 2	
		L	R
Player 1	L	1,1	0,0
	R	0,0	1,1

Nash Equilibria: (L,L), (R,R)

Dove-Hawk

Animal 2

D

H

Animal 1

D

3,3

1,5

H

5,1

0,0

Nash Equilibria: (D,H), (H,D)

Matching Pennies

		Player 2	
		H	T
Player 1	H	-1,+1	+1,-1
	T	+1,-1	-1,+1

No pure Nash Equilibria Exist!

Randomized Strategies

		Player 2	
		H	T
Player 1	H	-1,+1	+1,-1
	T	+1,-1	-1,+1

Player 1 picks H with probability p and
Player 2 picks H with probability q

		Player 2	
		H	T
Player 1	H	-1,+1	+1,-1
	T	+1,-1	-1,+1

**($p=1/2, q=1/2$)
is an
equilibrium!**

$$E[\text{Payoff for P1 doing H}] = (-1)q + (+1)(1-q) = 1-2q$$

$$E[\text{Payoff for P1 doing T}] = (+1)q + (-1)(1-q) = 2q-1$$

Player 1 will choose H if $1-2q > 2q-1$. i.e., if $q < 1/2$

Player 1 will choose T if $1-2q < 2q-1$. i.e., if $q > 1/2$

We say that (p^*, q^*) is a mixed strategy Nash Equilibrium if p^* is a best response by player 1 to q^* and q^* is a best response by player 2 to p^*

		Player 2	
		L	R
Player 1	U	1,1	4,0
	D	2,1	1,3

**($p=2/3, q=3/4$)
is an
equilibrium!**

Player 1 is only willing to randomize if the expected payoffs of U and D are equal:
 $q+4(1-q)=2q+(1-q)$, so $q=3/4$

Goalie

l

r

Kicker

L

.58,.42

.95,.05

R

.93,.07

.70,.30

	<i>l</i>	<i>r</i>
L	.58,.42	.95,.05
R	.93,.07	.70,.30

		Player 2	
		L	R
Player 1	U	1,1	3,0
	D	0,3	2,2

The only Nash Equilibrium is (U,L)

But (U,L) gives each player a payoff of 1, whereas (D,R) gives them 2.

Nash Equilibrium not always socially optimal

g2g

ttyl