ON THE WORK OF PHILLIP A. GRIFFITHS, THE 2008 BROUWER LAUREATE

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When the Executive Council of the Dutch Mathematical Society instructed the selection committee to nominate a Brouwer lecturer in the area of Geometry, it was not so clear what kind it had in mind: algebraic, complex-analytic or differential geometry. Given the size of the area covered by that simple word, it was not to be expected that the committee would find someone who could represent that field in its full glory, so that a further restriction of the domain seemed in order. Fortunately this turned out to be unnecessary, for the committee, whose members were Hans Duistermaat, Jozef Steenbrink, Dirk Siersma and the chairman of the Royal Dutch Mathematical Society, Henk Broer, readily and unanimously agreed on a candidate who has been a towering figure in all these areas, namely tonight's lecturer Phillip A. Griffiths.

A laudatio for a Brouwer lecturer must give a general mathematical audience an idea of the breadth and impact of the work of the medallist and should also explain why this person was singled out for this award. These are clearly not independent duties, but in the present case, I confess that for me, the latter is easier to do than the former.

Let me then, before I begin to say anything about the substance of his work, give you first an idea of its magnitude in published form: five years ago a four volume set of "Selecta" of the laureate was published, comprising about 2600 pages in total. Let me emphasize that this was a selection indeed, as it certainly does not contain all the laureate's published mathematical papers. And given the rate at which he continues to publish research at a high level, it would not surprise me if eventually another volume will be added. In addition he authored or co-authored about a dozen books, some of which have become classics. One of these, *Principles of Algebraic Geometry*, coauthored with Harris and published in 1978, is still generally regarded as the standard text from which to learn complex algebraic geometry. And another, *Geometry of algebraic curves, Volume 1,* dating from

1985 and written with Arbarello, Cornalba and Harris is the only modern comprehensive text of this venerable and beautiful subject. The title indicates that authors had boldly, if not somewhat recklessly, committed themselves to at least a *Volume 2*. I am happy to report that, with such a volume now being virtually finished, they will redeem on this promise. Not quite in the form it had been conceived a quarter of a century ago, I suppose, for the field has spectacularly developed since.

As I have hinted, the impact of this opus has been enormous. But the laureate has not only exerted his influence through his writings: when Griffiths took students, he often attracted the best and as a result many of the prominent algebraic geometers in the US and abroad of a certain age group were formed by him.

A very much larger group has felt his beneficial influence also in other ways. I mean here not a group of mathematicians, but the scientific community as a whole, and even people professionally involved in science politics. This has happened in his capacity of Provost of Duke University and even more so as Scientific Director of the prestigious Institute for Advanced Study and as secretary of the International Mathematical Union. In speeches and essays he formulated his vision and expressed his opinions on the role that mathematics and science plays or should play in society.

I confess that I find the task of giving you an idea of the laureate's work rather daunting and so I hope you will allow me to begin this by recounting a personal experience. Almost forty years ago, in the spring of 1970, he was invited by Frans Oort, then at the Mathematics Department of the University of Amsterdam, to give a series of lectures. At the time I was what we would now call a Master student of Nico Kuiper and my supervisor recommended that I attend these lectures. Since I knew very little about algebraic geometry, I did not expect to get a lot out of these. I was however in for a pleasant surprise: I enjoyed these lectures immensely and found that I could basically follow them to the very end. One reason was surely that the exposition drew heavily on the kind of topological and geometric reasoning that was not completely foreign to me, but it was most of all due to the quality of the lectures. These were not only very clear, making it easy to make notes (so that we could prepare for the next installment), but the material was also great: it was based on his two part paper called *The periods of rational integrals* that had just appeared in the Annals of Mathematics. This is not the place to describe the mathematical content of this work, but in case you know a little about the classical theory of meromorphic differentials on Riemann surfaces and the cohomology classes they can represent: it is essentially a beautiful generalization of that theory to the case smooth complex hypersurfaces in a fixed complex projective space. Griffiths mentions in a commentary in the Selecta that a referee of that paper had berated him for developing this theory not in the most general setting that his methods allowed, but that this criticism was ignored by the Annals' editors. Now these lectures were also attended by two fellow students, who, I am happy to note, are here with us tonight as well, namely Jozef Steenbrink (who provided the musical introduction) and Chris Peters and I daresay that all three of us can testify that from a student's perspective this referee was wrong and the Annals editorial board was right. I might add that these lectures and that paper have had tremendous influence on the direction of our research and for each of us in a different way. As a consequence, Griffiths thus very much influenced the development of algebraic geometry in the Netherlands.

I told you this story also because it reflects a remarkable quality of the laureate to which I alluded earlier, namely his gift of attracting and inspiring students. Until 1983, the year Griffiths left Harvard to become Provost at Duke University, he had had about 20 students, most of whom have become well-known, if not famous, experts in algebraic geometry, differential geometry or Lie groups. That diversity also illustrates that Griffiths is a geometer in the broadest sense of the word: in each of these areas he has made very fundamental contributions making him a true heir of the legacy of some of the great geometers of a century ago, such as Henri Poincaré and Élie Cartan.

The four volumes of the Selecta are titled *Analytic Geometry*, *Algebraic Geometry*, *Variations of Hodge Structure* and *Differential Systems*. Although the laureate's work can be distributed under these headings in more than one way, I find it practical to give you an idea of his accomplishments by picking an item from each of these.

From the first category, *Analytic Geometry*, I mention his work in what we may call complex integral geometry, which you might think of as a kind of fusion of differential and algebraic geometry. In order to have some idea what this is about, let me mention Crofton's formula as the archetypical example. This formula says that there is on the space of lines in a Euclidean plane an (easily described) measure with the property that for any decent curve in the plane, its length is equal to the integral of the (integer valued) function which associates to a line in that plane its number of intersections with the curve. That little gem of integral geometry is generalized by Griffiths in a beautiful manner to a setting that is higher dimensional and also complex-analytic: similarly defined integrals turn out to reproduce interesting complex analytic, metric and topological invariants involving for example Chern and Milnor numbers.

As for the second category, *Algebraic Geometry*, it is hard and admittedly, also unfair, to choose a single paper from his many contributions in this area. My favourite is the one with his erstwhile student Herb Clemens about the *Intermediate Jacobian of the cubic threefold*. I hope you bear with me when I take a minute to be moderately technical, so that I can describe the essence of this paper. Some of you may recall from a course on Riemann surfaces that to a compact Riemann surface, or what is the same, a complex projective nonsingular curve, is associated a complex torus, its so-called *Jacobian*. The standard and easiest way to introduce this Jacobian is in terms of the periods of the holomorphic differentials of the curve. The famous Abel-Jacobi theorem asserts that this torus can also be understood differently, namely as parameterizing the curve's degree zero divisor classes. The Jacobian comes with a little bit of extra linear structure, called a *polarization* and another famous theorem, the Torelli theorem, asserts that no information is lost if we pass from the hard-to-grasp curve (a *curved* object after all) to the *linear* data that are embodied by its polarized Jacobian. Griffiths and Clemens show that the situation is amazingly similar if we replace the curve by a nonsingular cubic hypersurface in complex projective 4-space. They also prove that such a hypersurface is not rational. It is an extraordinary paper and one of the most beautiful I know.

Griffiths' work on the third topic, *Variations of Hodge Structure*, probably had the biggest impact. This concerns an important generalization of Poincaré's results on rational integrals on surfaces and includes his work on the periods of rational integrals on which he lectured in Amsterdam, but as the present occasion is not well suited for trying to explain this to a general mathematical audience in just a few minutes, I will not elaborate. In related work we find also a simple, but very fundamental property of period mappings that is now named after him: *Griffiths transversality*.

It is also hard to make a choice from the fourth and last topic, *Differential Systems*. The subject matter has classical origins, going back Sophus Lie and Élie Cartan. It has become part of classical mechanics, but, as Griffiths showed with his former student Robert Bryant, is also intimately tied to the topic just mentioned, Variations of Hodge Structure. Under that same heading falls the classical isometric embedding problem. The central question here is easy to state: is it possible to embed locally a compact Riemann manifold isometrically in a Euclidean space? In other words, does the abstract approach to Riemannian geometry yield more Riemannian manifolds than the more elementary (and clumsy) approach of studying submanifolds of Euclidean space? This question was answered affirmatively long ago by John Nash (in work that in my opinion belongs to the deepest of this Noble prize winner). In Nash's result there is little control on the embedding dimension, but Bryant and Griffiths were able to prove that a 3-dimensional Riemannian manifold admits a local isometric embedding in Euclidean 6-space. This must be the minimal value for that dimension.

I have tried to give you an idea of the work of the laureate, but I am well aware that my own limitations prevented me from doing it justice. I do hope however to have convinced you that Phillip Griffiths is not just very much worthy of the Brouwer medal, but also that we have reason to be proud that he has accepted and is here tonight to give the accompanying lecture.