Applications of signal and image processing to medicine

Article · January 1988

DOI: 10.1109/ISCAS.1988.15362

Daniel Graupe
University of Illinois at Chicago

Proceedings of the 27th Conference
on Decision and Control
Austin, Texas • December 1988

APPLICATIONS OF NEURAL NETWORKS TO MEDICAL SIGNAL PROCESSING

by
DANIEL GRAUPE *
RUEY WEN LIU **
and
GEORGE S. MOSCHYTZ ***

Abstract

The recent progress in neural networks (NN) opens a vast field of opportunities to the processing of medical signals and images. The particular attraction of NN for such applications lies in the nonlinear and often non-analytical nature of biological signals.

To illustrate at least a few aspects of such applications, we discuss two specific applications, both related to the authors' research into electromyographic (EMG) signals. The first concerns the decomposition of surface (transcutaneous) EMG into its inaccessible motor unit action potentials (MUAP) for the purpose of determining the sequence of single (or groups) of MUAP which form the corresponding surface EMG signal. If extended to EMG surface-electrode arrays, this should also yield information on locations of motor units (MU) relative to electrode location in the array. This application is of importance both as a non-invasive diagnostic tool, to yield information on single or group MU innervation during normal- or test-activity of a patient. It can also serve for subsequent control of prostheses, orthoses, or functional neuromuscular stimulation (FNS) of paraplegics.

The second application concerns the discrimination between intended walking functions (to be activated by FNS) of paraplegics from parameters of signature patterns (AR parameters or equivalent) of above-lesion surface EMG. This above-lesion EMG is as generated by natural posture changes of a paraplegic's upper trunk when preparing to perform the respective walking-related function (taking right or left step etc ...), thus controlling FNS to paralyzed limbs.

The two applications above, one being a diagnostic inverse problem and the other being a functional pattern-discrimination problem, are intended to illustrate some of the opportunities provided by medical applications of NN.

1. Introduction

The recent advances in neural network theory [1-5] and its applications now yield a realistic and powerful tool to deal with a wide array of signal and image processing, discrimination and decision problems, especially those that are ill defined, nonlinear and which are even non-analytical, or which defy analytical analysis with conventional Von-Neuman-based computing machines (at least, at reasonable computational power and programming time). Medical data and medical signals or images fit very much the above description of a nonlinear or non-analytical nature. It is thus a natural area for vast opportunties in applying neural nets, both for medical diagnosis and for functional medical applications (control of prostheses, of neuromuscular stimulation, of artificial organs, of implants, pacemakers, etc ...).

*Dept. of Electrical Engineering & Computer Science, University of Illinois at Chicago (UIC), Chicago, IL.
**Dept. of Electrical & Computer Engineering, University of Notre Dame, Notre Dame, IN.
Drs. Graupe and Liu are members of the joint UIC-Notre Dame Research Center on Signal & Image Processing in Medicine which supported this research in part.
***Institute of Signal & Information Processing, ETH (Swiss Federal Institute of Technology), Zurich, Switzerland.

In this paper we describe two applications of NN that are being studied at the authors' laboratories, and which are intended to provide an illustration of some of the capabilities of NN-based medical signal processing. One application is mainly of diagnostic significance, though it has also functional potential, as a possible approach to control of functional neuromuscular stimulation (FNS) in paraplegics. The other application is mainly functional (for controlling FNS), though diagnostic uses are possible. The first application is to an inverse processing problem and the second is to a signature (patern) discrimination problem. Both applications employ surface (EMG) as obtained in vivo from human patients during tests or while performing normal tasks. The first application is an EMG decomposition problem that follows one of the author's work on decomposition of [6,7]. The second is based on previous work of another author on EMG signature discrimination to control artificial limbs in amputees [8,9] and to control walking under FNS in (complete) paraplegics [10,11,12] and which facilitated such walking (while using non-NN pattern discrimination) at that author's lab for the last several years.

2. Decomposition of Surface EMG Signals

2.1 The Nature of the Surface EMG Signal

The surface EMG signal is a signal that is picked up by transcutaneous electrodes attached to the surface of the skin and which consist of the electrical response of a muscle in response to trains of individual electrical signals of a multitude (hundreds in major muscles) of motor neurons [14]. The electrical activity of a motor neuron is in turn a sequence of more-or-less fixed-form signals known as motor-unit action potentials (MUAP) produced and prorogated to the muscle-fiber/neuron interface and which are the motor neuron's (MU's) response to a train of triggering impulses inputted to the MU from the spinal cord and hence from the central nervous system (CNS). The surface EMG is thus a spatial integration of a multitude of such MUAP's. There can be several (tens of such fibers) connected to a single MU. Thousands or tens of thousands of such fibers thus constitute a major muscle, such as the biceps or quadriceps muscle in man. See Fig. 1. The individual MUAP signal (X_{ap} of Figure 1 involves rather complex dynamics, and is of a more-or-less fixed shape, as shown in Fig. 2, where $t_{o,i}$ is the instant of a neural (firing) at the input to the MU, coming from the spinal cord (SC), where also the time scales involved can be considered fixed (as first approximation, at least) for a given muscle. Gain (amplitude) terms and low-pass time constant element terms should contribute to changes of these patterns. These terms will be introduced later, the gain terms being one aspect of what is to be determined for each MUAP by the NN as is $t_{o,i}$ (in fact, a sequence of $t_{o,i}(K)$ for the k'th firing of MU_i). A few hundreds of MU's may be involved

88CH2531-2/88/0000-0343$1.00 © 1988 IEEE

in a major muscle. Hence, if we consider two EMG electrodes of a relatively large surface area (around 1 cm^2) and spaced at about 25 mm (center to center), we record the summation of activities of a 10-20 of MU's. This recording will also consist of some 10-20 branches of each of the MU's involved, where the MUAP is thus synchronized, differing only by the velocity of propagation of the AP, which may be assumed over a short recording interval (say, 1.0 sec) to be constant at a given electrode location [14]. In the method of recording for the purposes of the present paper, cross talk between motor units of adjacent muscles is acceptable. This acceptability is an important additional feature of the proposed decomposition method. The present NN-based decomposion differs from that of [14] in that (a) no grid needle electrode is employed, (b) decomposition is possible over a multitude of trains of MUAP's by a single electrode pair, rather than by measuring each MUAP directly (without decomposition) with point electrodes, where contact problems, short circuits between electrodes and the need to process hundreds of electrodes are eliminated. However, for evaluation and calibration, the approach of [14] can serve as a valuable reference approach. When topographic mapping is performed then still an array of hundreds of point electrodes as in [14] is replaced by a single pair in the present approach.

2.2 The Surface EMG Decompostion Problem

Noting the nature of the surface EMG signal as described in Section 2.1 above, this signal y(t) is considered to be a summation of many MUAP's of almost identical shape (voltage/time pattern) and of different (random-like) arrival times. The decomposition problem thus becomes that of decomposing the random-like [15], [16] signal y(t), as recorded over a fixed time window (of say, 1.0 second) to its component MUAP's. For this purpose we first employ a single pair of broad (approximately 1.0 cm^2) skin surface electrodes located at approximately 25 mm center-to-center inter-electrode distance. Whereas y(t) is as in Fig. 1, the individual MUAP is as in Fig. 2. y(t) above is composed of both near-surface MUAP's and of more distant below surface MUAP's. However the latter's contribution is minor. It is not neglected in our analysis, but only a spatial (2 dimensional) distribution will be performed in the present analysis.

Decomposition of y(t) in terms of its formant MUAP's, is thus in terms of determining the relative amplitude and arrival time of each of a finite set of MUAP's $p_i(t - t_i)$, considering the surface EMG model

$$y(t) = \sum_{i=1}^{N} k_i \, p_i(t-t_i) \qquad (1)$$

t_i denoting arrival time and k_i denoting the relative amplitude of p_i in y(t). Furthermore, for the NN realization k_i and t_i belong to a finite set of possible gains and times of arrival k_i that differs from one MUAP to another due to distance from the electrode, depth and intensity of neural activity, and where synchronous MUAP's contribute to a single AP. We note that several decomposed p_i above with different t_i and k_i may belong to different branches of one MU. The grouping of AP's according to MU's will be considered in Section 2.3 below.

The NN approach for the EMG decomposition problem above, is a parallel to the approach of and Hopfield [17] where a complicated (Gaussian-sums-type) probability density function is decomposed into individual formant Gaussian density functions with different means and different variances. The different means paralell the arrival times in the EMG decomposition problem, whereas the gains K_i parallel the different variances in [17].

The algorithm of eqn. (1) can be extended to cover a finite set of different MUAP patterns and not just a single pattern, say patterns $p_{ij}(t-t_j)$ where j denotes a pattern shape of a set of M possible shapes, such that

$$y(t) = \sum_{i=1}^{M} \sum_{j=1}^{N} K_{ij} p_{ij}(t-t_{ij}) \qquad (2)$$

This modification will account for effects of filtering of MUAP's through the tissue. For practical reasons M should be small as is reasonable to assume noting the known shapes of MUAP's, such as in [14].

2.3 The Neural Network Solution to the Decomposition Problem

2.3 ~~The NN~~ Solution for the Decomposition Problem.

Consider

$$y(t) = \sum_{i=1}^{N} \sum_{j=1}^{M} K_{ij} f_j(t-t_i) \qquad (3)$$

where y(t) is the received EMG signal, $f_j \epsilon F$ belongs to a finite set of stereotype of motor unit action potential waveforms, $t_i \epsilon T$ is the time delay and K_{ij} K. The sets T and K are finite sets of real numbers. The decomposition problem is the determination of constants K_{ij}, time delays t_i and functions f_j from a given EMG signal y(t).

This problem can be solved by the Hopfield neural network using a method suggested by Tank and Hopfield [17].

Let

$$G = \{K_{ij} f_j(t-t_i) \mid K_{ij} \; K, t_i \epsilon T, f_j \epsilon F\} \qquad (4)$$

G is the exhaustive set of all possible waveforms. It is finite because K,T and F are finite. Denote

$$G = \{g_k(t) \mid k=1,2,\ldots,L\} \qquad (5)$$

for some finite L. Eq. (1) can be rewritten in the form

$$y(t) = \sum_{k=1}^{L} V_k g_k(t) \qquad (6)$$

where V_k = 0 or 1. The decomposition problem becomes the determination of V_k from y(t). Consider

$$E = \frac{1}{2} ||y - \sum_k V_k g_k||^2 + \frac{1}{2} \sum_k V_k[1-V_k]||g_k||^2 \qquad (7)$$

where $||\cdot||$ is an L_2-norm in a functional space. Let $V_k \varepsilon [0,1]$. Clearly,

$E \geq 0$. Furthermore, E = 0 if and only if

$$y = \sum_k^N V_k g_k \qquad (8)$$

and V_k = 0 or 1. This is a solution of the decomposition problem. Hopfield neural network is an analog circuit which is designed so that E of (7) becomes minimum at equilibrium points of the neural network. In fact, according to [17], we have the design parameters as follows.

$$T_{kk'} = \begin{cases} -\langle g_k, g_{k'} \rangle, & k \neq k' \\ 0, & k = k' \end{cases} \qquad (9)$$

$$I_k = \langle y, g_k \rangle + \frac{1}{2} ||g_k||^2 \qquad (10)$$

where T_{ij} is the strength (conductance) of the interconnection between the i^{th} and the j^{th} processor, and I_k is the externally supplied input currents. For the construction of the neural networks, see [17].

The complexity of the neural network is determined by the cardinal number L of G, which depends on the cardinal numbers of K,T and F. It should be noted that the neural network may approach a local minimum, instead of the desirable global minimum. On the other hand, its convergence rate is O(n), where n is the complexity of network. This is an important advantage over alternative methods, especially when n is large.

2.4 Spacial Localization of Motor Units via Decomposition from Electrode Arrays

For diagnostic neurological Studies [14] and for using EMG for control of prostheses [8,9] and of electrical stimulation in paraplegics [10-13], not the decomposition itself is of importance but the ability to obtain (in a non-invasive manner) a mapping of innervation across a muscle or a group of adjacent muscles. The present surface electrode approach has also the advantage (vs. implanted electrodes) that the patient is not conscious or constrained in normal movements by the electrodes.

When an array of surface electrodes as in Sections 2.1, 2.2 is considered, to which decomposition is applied as in Sections 2.2, 2.3, approximate spatial mapping can be accomplished. For this purpose eqn. (1) becomes

$$y_h(t) = \sum_{i=1}^{N} K_{ih}\, p_{ih}(t-t_{ih}), \qquad h=1 \ldots R \qquad (11)$$

R being the number of electrodes in the array. Once NN-based decomposition has been performed per each member of the array, i.e., per each $y_h(t)$, localization can be performed via considering relative gains of members of a sequence of one electrode vs. an adjacent electrode, when accounting for times of arrival.

Furthermore, when accounting for propagation velocities, MAUP's of branches of the same MU can be mapped and inter-related [14]. The mapping may be performed by another NN subsystem, though conventional correlation algorithms can also be effectively used for this purpose.

3. Function Discrimination from Surface EMG Signatures for Controlling Neuromuscular Stimulation in Paraplegics

3.1 Functional Neuromuscular Stimulation (FNS) via EMG Signature Discrimination

It was shown in [8,9] that the parameters of a time series model of the surface EMG (electromyographic) signal can serve as parameters to control artificial limbs in amputees and electrical neuromuscular stimulation (ENS) in paraplegics [10-13]. Such control is based on the nature of the surface EMG signal as the electrical response of the muscle at the skin's surface to the firing of motor neurons which cause that muscle to contract. Consequently, changes in modes (patterns) of muscle contraction in major muscles (or at an electrode location which receives an EMG signal generated by neural firing at adjacent muscles) produce changes in the EMG time-series parameters, such as the AR (autoregressive) parameters. Hence contraction or posture changes at chest muscles above the level of a spinal-cord lesion in parapleics, when the patient prepares himself to execute a walking function, is reflected by a set of AR (or ARMA, etc...) parameters that are more or less unique and repeatable for such an intended walking function.

3.2 The Signature Discrimination Problem

The AR model of the discrete-time EMG signal y(k) is describable by:

$$y(t) = \sum_{i=1}^{n} a_i y_{k-i} + w_k \sum_{i=1}^{n} \qquad k=0,1,2, \ldots \quad (12)$$

w_k being inaccessible white noise.

The decision to activate a particular walk function (right step, left step, sit down, etc ...) by ENS is thus based on the parameter vector $\underline{a} \triangleq [a_1 \ldots a_n]^T$ being within a certain subspace of the parameters space. Once it is determined that $\underline{a}$ is within the appropriate region, ENS is applied to the paralyzed limbs to activate the desired walk function corresponding (by precalibration) to that function. See Fig. 3.

The parameters a_i are repeatedly identified by a recursive near-least squares lattice identifier [18].

Our NN decision problem is thus formulated as follows: Given a set of identified time series (say, AR) parameters a_i, which walk function F_j [j=1, ... m] should the FNS controller activate?

3.3 The NN Solution to the Signature Discrimination Problem

Let Y be the set of EMG signals y(t) of a particular patient. Let there exist a partition of Y into a finite number of groups, say 5 groups as follows:

Y_1 - Stand up
Y_2 - Right leg up
Y_3 - Left leg up
Y_4 - Sit down
Y_5 - Don't change status

The discrimination problem is to find a partition which separates each group from others. Furthermore, after the partition is made, the discriminator should identify the correct group for each incoming EMG signal.

The neural network can be used to solve this discrimination problem, provided the following condition is met. Each signal y(t) can be represented by a set of finite number of parameters, say $\{x_1, x_2, \ldots, x_n \; R^n\}$, so that its associated five groups X_1, X_2, ..., X_5 can be partitioned by a set of fintie number of hyperplanes in R^n. To find the suitable set of parameters for a given problem is sometimes the most difficult problem. For example, the sampled value $\{y_k\}$ of y(t) is not suitable for this purpose. Graupe, et al [8-13] have found that if the AR coefficients is used then the five groups can indeed be separated by a set of finite numbers of hyperplanes. Specifically, let the AR time series model for the left and the right EMG signal be given as follows:

$$y_k = \sum_{i=1}^{n} a_i y_{k-i} + w_k \qquad (13)$$

It was found in [8-13] that if the following parameters are used:

$$\left.\begin{aligned} x_1 &= a_1 \text{ (right EMG signal)} \\ x_2 &= a_2 \text{ (right EMG signal)} \\ x_3 &= a_3 \text{ (right EMG signal)} \\ x_4 &= \text{Variance (right EMG signal)} \end{aligned}\right\} \qquad (14)$$

and similarly x_5, x_6, x_7 and x_8 are defined for the left EMG signal, then the corresponding parameters x = $(x_1, x_2, \ldots, x_8)$ of EMG signals can indeed be separated by linear hyperplanes in R^8.

Based on the above, a multi-layer neural net can be constructed. The number of perceptrons at the input layer is determined by the number of parameters needed to represent each signal. The number of perceptrons in the output layer is determined by the number of groups needed to be partitioned into. The number of perceptrons in the hidden (middle) layer is determined by the number of hyperplanes needed to separate these groups. A (8,3,5) multi-layer neural networks is shown in Figure 4.

The x_i's, z_i's and u_i's are the inputs, outputs and intermediate variables respectively. The outputs are computed as follows:

$$u_j = g\Big(\sum_{i=1}^{8} w_{ij} x_i - \phi_j\Big) \qquad (15)$$

$$z_h = g\Big(\sum_{i=1}^{3} v_{ih} u_i - \theta_h\Big) \qquad (16)$$

where g is the sigmoid function

$$g(x) \triangleq \frac{1}{1+e^{-x}} \qquad (17)$$

and v_{ik}, w_{ij}, θ_h and ϕ_j are the parameters to be adjusted.

There exist many different ways to adjust (train) these parameters [1] . The back propagation algorithm by Rumelhart et al [19] will be presented here.

For each input x, there is associated a desired output d. For example, if x belongs to Group 2, then d = (0,1,0,0,0,). We would like to have z = d for every input x. Now, for a set of training inputs $\{x(k) | k = 1,2,\ldots,n\}$, the coefficients are adjusted in the following way.

Step 1: Initially, set all coefficients v(1), w(1), $\theta(1)$ and $\phi(1)$ to small random numbers.

Step 2: Compute u(1) and z(1) from x(1) by (1) and (2).

Step 3: Adjust the coefficients by the following formulae:

$$\delta_j(k) = z_j(k)(1-z_j(k))(d_j(k)), \quad j=1,2,\ldots,5$$

$$\Delta_i(k) = u_i(k)(1-u_i(k)) \sum_{j=1}^{5} \delta_j(k) v_{ij}(k), \quad i=1,2,3$$

$$v_{ij}(k+1) = v_{ij}(k) + \eta\delta_j(k)u_i(k)$$
$$\theta_j(k+1) = \theta(k) + \eta\delta_j(k)$$
$$w_{ni}(k+1) = w_{nj}(k) + \eta\Delta_i(k)x_n, \quad n=1,2,\ldots,8$$
$$\phi_i(k+1) = \phi_i(k) + \eta\Delta_i(k)$$

and η is a constant chosen between 0 and 1.

Step 4: Repeat step 2 until the process is convergent.

If and when the process is convergent then the neural net becomes a *discriminator*, which will yield the correct output for each input, provided that the training input sequence is widely excited, i.e., it covers the five groups extensively.

Note that the neural network discriminator is patient-independent. Each patient can train the discriminator himself by mere pressing of 4 desired knobs d to d_4 ("no pressing" thus implies d_5). No adjustments by experts are needed.

4. Conclusions

We have considered to biomedical problems whose solution via NN was outlined.

The NN solution for the first problem, which is mainly a diagnostic one, facilitates decomposition of surface EMG signals into their formal action potentials. Such decomposition is otherwise not possible in a non-invasive manner but for when using a very large number of point-EMG channels, where both computation is excessive and the number of channels is in the many hundres [14]. Via NN, 1 to 5 channels will suffice to cover a muscle area of many square inches. Furthermore, the NN-based method can be employed without affecting the patient's freedom of movement, and without making him constrained or bothered at all by the electrodes themselves.

The second problem is an ENS control problem whose solution by conventional AI has already been tested [10], but where NN provide a more elegant and convenient solution.

Both problems are mere examples of the power and breath of possible important applications of NN in concrete medical problems. They also illustrate how (artificial) neural nets can be utilized to decode information and structuree in (biologic) neural nets for functional and diagnostic medical purposes.

References

[1] Lippman, R.P., An Introduction to Computing with Neural Nets, IEEE ASSP Magazine, Vol. 4, pp. 4-22, April 1987.

[2] Hopfield, J.J. Neural Networks and Physical Systems with Emergent Collective Computational Abilities, Proc. National Academy of Science, Vol. 79, No. 8, pp. 2554-2558, Washington 1982.

[3] Widrow, B., The Original Adaptive Neural Net Broom Balancer, Proc. IEEE Internat. Symp. of Circuits & System (ISCAS), pp. 351-357, Philadelphia, May 1987.

[4] Kamp, Y., State Reduction in Hidden Markov Chains Used for Speech Recognition, IEEE Trans. on ASSP, Vol. 33, No. 4, pp. 1138-1145, Oct. 1985.

[5] Gorman, R.P. and Sejnowski, T.J., Analysis of Hidden Units in a Layered Netowrk Trained to Classify Sonar Targets Neural Networks, Vol. 1, no. 1, pp. 75-90, 1988.

[6] Haas, W.F., Studer, R.M., Meyer, M. and Moschytz, G.S., EMG Analysis System for Shapes and Innervation Statistics of Motor Units, Electroenaph Clin. Neurol., Vol. 66, p. 42, 1987.

[7] Haas, W.F., Studer, R.M., Moschytz, G.S. and Meyer, M., Computer-Aided Electromyography, Proc. IEEE Internat. Symp on Circuits & Sys. (ISCAS), pp. 2131-2134, Helsinki, June 1988.

[8] Graupe, D. and Cline, W.K., Functional Separation of EMG Signals via ARMA Identification Methods for Prosthesis Control Purposes, IEEE Trans. on Systems, Men & Cybernb., Vol. SMC-5, pp. 252-259, 1975.

[9] Graupe, D., Magnussen, J. and Beex, A.M., A Microcomputer Control of Upper-Limb Prostheses via Myoelectric Signal Identification, IEEE Trans. on Automaic Control, Vol. 23, pp. 538-544, 1978.

[10] Graupe, D., Kralj, A. and Kohn, K.H., Computerized Signal Discrimination of Above-Lesion EMG for Stimulating Peripheral Nerves of Complete Paraplegics, Proc. IFAC Symp. on Prosthesis Control, Columbus, OH, Feb. 1982.

[11] Graupe, D. and Kohn, K.H., Basseas, S. and Naccarato, E., Electromyographic Control of Functional Electrical Stimulation of Selected Paraplegics, Orthopedics, Vol. 7, pp. 1134-1138, 1984.

[12] Graupe, D. and Kohn, K.H., A Critical Review of EMG-Controlled Stimulation for Paraplegics, CRC Crit. Rev. in Biomed. Eng., Vol. 15, No. 3, pp. 187-210, 1988.

[14] Masuda, T. and Sadoyama, T., Topographical Map of Innervation Zones Within Single Motor Units Measured with a Grid Surface Electrode, IEEE Trans. Biomed. Eng., Vol. 35, no. 8, pp. 63-628, 1988.

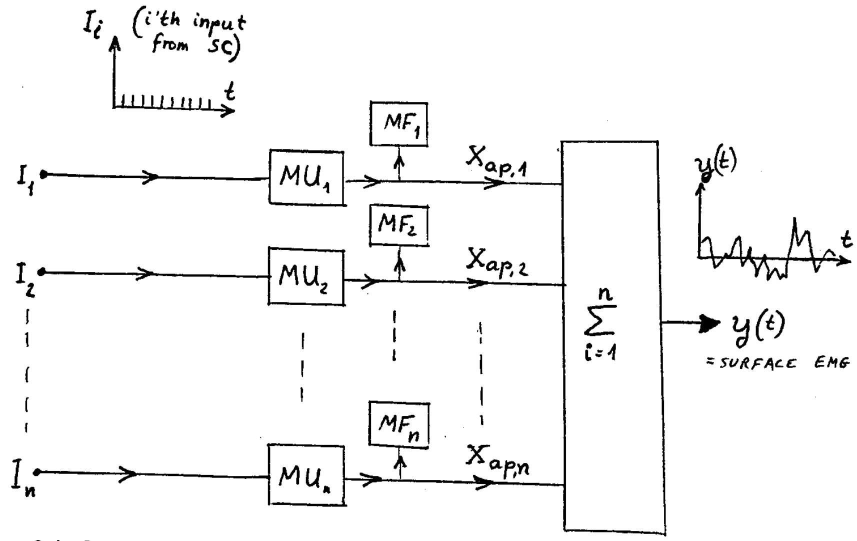
[15] Graupe, D., Salahi, J. and DeSong, Z., Stochastic Analysis of Myoelectric Temporal Signatures for Multifunctional Single Site Activation of Prostheses and Orthoses, J. Biomed. Eng., Vol. 7, pp. 18-29, January 1985.

[16] DeLuca, C.J. Physiology and Mathematics of Myoelectric Signals, IEEE Trans. Biomed. Eng., Vol. 26, pp. 313-325, 1979.

[17] Tank, D.W. and Hopfield, J.J., Simple Neural Optimization Networks: An A/D Converter, Signal Decision Circuit and a Linear Programming Circuit, IEEE Trans. on Circ. and Sys, Vol. 33, No. 5, pp. 533-541, 1986.

[18] Graupe, D., Time Series Analysis, Identification and Adaptive Filtering, 2nd Edition, Krieger Publ. Co., Malabar, FL 1988.

[19] Rumelhart, D.E., Hinton, G.E. and Williams, R.J., Learning Internal Representation by Error P§ropagation, Parallel Distributed Processing, Vol. 1, pp. 318-362, MIT Press, Cambridge, MA 1986.



Key : SC - Spinal Cord

MU_i - i'th motor unit (MU)

MF_i - i'th set of (possibly tens of) muscle fibers that form a single muscle

$X_{ap,i}(t)$ - i'th MUAP (MU Action Potential)

y(t) - surface EMG

The trains of signals I_1 to I_n are not synchronized

Fig. 1. Block Diagram Describing the Surface EMG Signal

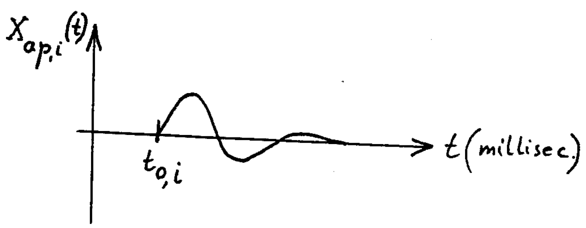


Fig. 2. The MUAP Signal (single MU)

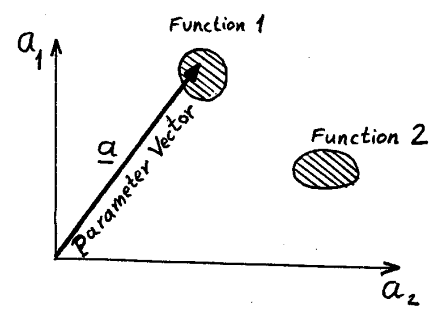


Fig. 3. Parameter Space for $\underline{a} = [a_1, a_2]^T$

Fig. 4. Multi-Layer Neural Net for EMG Signature Discrimination