

Exercise 4: Force and motion

Equipment:

1-meter pieces of butcher paper

wood blocks with hooks

string

masses to put on top of the blocks to increase friction

spring scales (preferably calibrated in Newtons)

Suppose a person pushes a crate, sliding it across the floor at a certain speed, and then repeats the same thing but at a higher speed. This is essentially the situation you will act out in this exercise. What do you think is different about her force on the crate in the two situations? Discuss this with your group and write down your hypothesis:

1. First you will measure the amount of friction between the wood block and the butcher paper when the wood and paper surfaces are slipping over each other. The idea is to attach a spring scale to the block and then slide the butcher paper under the block while using the scale to keep the block from moving with it. Depending on the amount of force your spring scale was designed to measure, you may need to put an extra mass on top of the block in order to increase the amount of friction. It is a good idea to use long piece of string to attach the block to the spring scale, since otherwise one tends to pull at an angle instead of directly horizontally.

First measure the amount of friction force when sliding the butcher paper as slowly as possible:-----

Now measure the amount of friction force at a significantly higher speed, say 1 meter per second. (If you try to go too fast, the motion is jerky, and it is impossible to get an accurate reading.)

Discuss your results. Why are we justified in assuming that the string's force on the block (i.e., the scale reading) is the same amount as the paper's frictional force on the block?

2. Now try the same thing but with the block moving and the paper standing still. Try two different speeds.

Do your results agree with your original hypothesis? If not, discuss what's going on. How does the block "know" how fast to go?



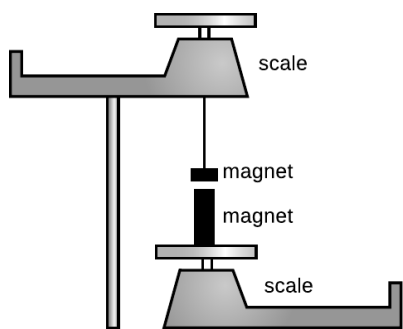
What forces act on the girl?

Chapter 5

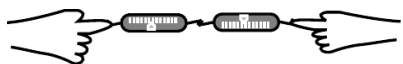
Analysis of Forces

5.1 Newton's third law

Newton created the modern concept of force starting from his insight that all the effects that govern motion are interactions between two objects: unlike the Aristotelian theory, Newtonian physics has no phenomena in which an object changes its own motion.



a / Two magnets exert forces on each other.



b / Two people's hands exert forces on each other.



c / Rockets work by pushing exhaust gases out the back. Newton's third law says that if the rocket exerts a backward force on the gases, the gases must make an equal forward force on the rocket. Rocket engines can function above the atmosphere, unlike propellers and jets, which work by pushing against the surrounding air.

Is one object always the “order-giver” and the other the “order-follower”? As an example, consider a batter hitting a baseball. The bat definitely exerts a large force on the ball, because the ball accelerates drastically. But if you have ever hit a baseball, you also know that the ball makes a force on the bat — often with painful results if your technique is as bad as mine!

How does the ball's force on the bat compare with the bat's force on the ball? The bat's acceleration is not as spectacular as the ball's, but maybe we shouldn't expect it to be, since the bat's mass is much greater. In fact, careful measurements of both objects' masses and accelerations would show that $m_{ball}a_{ball}$ is very nearly equal to $-m_{bat}a_{bat}$, which suggests that the ball's force on the bat is of the same magnitude as the bat's force on the ball, but in the opposite direction.

Figures a and b show two somewhat more practical laboratory experiments for investigating this issue accurately and without too much interference from extraneous forces.

In experiment a, a large magnet and a small magnet are weighed separately, and then one magnet is hung from the pan of the top balance so that it is directly above the other magnet. There is an attraction between the two magnets, causing the reading on the top scale to increase and the reading on the bottom scale to decrease. The large magnet is more “powerful” in the sense that it can pick up a heavier paperclip from the same distance, so many people have a strong expectation that one scale's reading will change by a far different amount than the other. Instead, we find that the two changes are equal in magnitude but opposite in direction: the force of the bottom magnet pulling down on the top one has the same strength as the force of the top one pulling up on the bottom one.

In experiment b, two people pull on two spring scales. Regardless of who tries to pull harder, the two forces as measured on the spring scales are equal. Interposing the two spring scales is necessary in order to measure the forces, but the outcome is not some artificial result of the scales' interactions with each other. If one person slaps another hard on the hand, the slapper's hand hurts just as much as the slapped's, and it doesn't matter if the recipient of the slap tries to be inactive. (Punching someone in the mouth causes just as much force on the fist as on the lips. It's just that the lips are more delicate. The forces are equal, but not the levels of pain and injury.)

Newton, after observing a series of results such as these, decided that there must be a fundamental law of nature at work:

Newton's third law

Forces occur in equal and opposite pairs: whenever object A exerts a force on object B, object B must also be exerting a force on object A. The two forces are equal in magnitude and opposite in direction.

Two modern, high-precision tests of the third law are described on p. 806.

In one-dimensional situations, we can use plus and minus signs to indicate the directions of forces, and Newton's third law can be written succinctly as $F_{A \text{ on } B} = -F_{B \text{ on } A}$.

self-check A

Figure d analyzes swimming using Newton's third law. Do a similar analysis for a sprinter leaving the starting line. ▷ Answer, p. 565

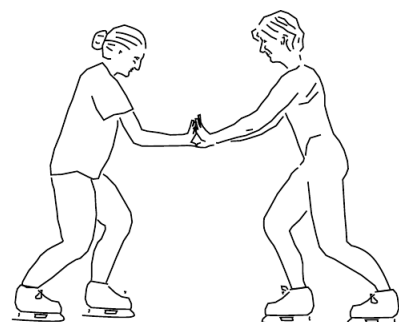
There is no cause and effect relationship between the two forces in Newton's third law. There is no "original" force, and neither one is a response to the other. The pair of forces is a relationship, like marriage, not a back-and-forth process like a tennis match. Newton came up with the third law as a generalization about all the types of forces with which he was familiar, such as frictional and gravitational forces. When later physicists discovered a new type of force, such as the force that holds atomic nuclei together, they had to check whether it obeyed Newton's third law. So far, no violation of the third law has ever been discovered, whereas the first and second laws were shown to have limitations by Einstein and the pioneers of atomic physics.

The English vocabulary for describing forces is unfortunately rooted in Aristotelianism, and often implies incorrectly that forces are one-way relationships. It is unfortunate that a half-truth such as "the table exerts an upward force on the book" is so easily expressed, while a more complete and correct description ends up sounding awkward or strange: "the table and the book interact via a force," or "the table and book participate in a force."

To students, it often sounds as though Newton's third law implies nothing could ever change its motion, since the two equal and opposite forces would always cancel. The two forces, however, are always on two different objects, so it doesn't make sense to add them in the first place — we only add forces that are acting on the same object. If two objects are interacting via a force and no other forces are involved, then *both* objects will accelerate — in opposite directions!

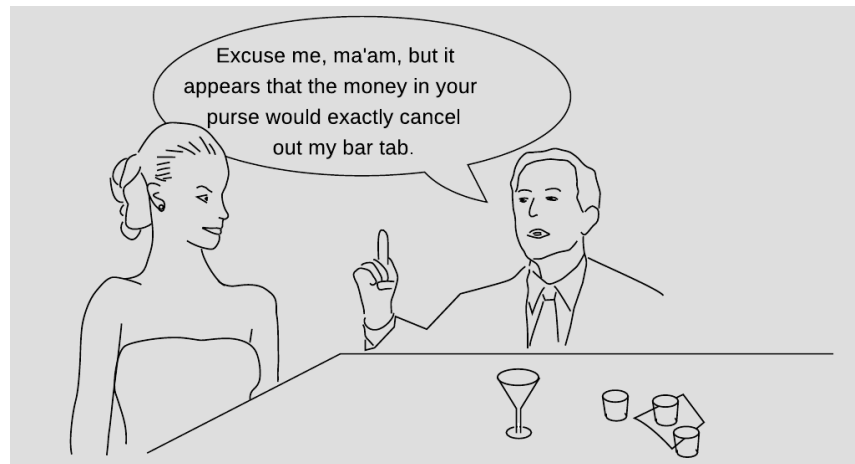


d / A swimmer doing the breast stroke pushes backward against the water. By Newton's third law, the water pushes forward on him.



e / Newton's third law does not mean that forces always cancel out so that nothing can ever move. If these two ice skaters, initially at rest, push against each other, they will both move.

f / It doesn't make sense for the man to talk about using the woman's money to cancel out his bar tab, because there is no good reason to combine his debts and her assets. Similarly, it doesn't make sense to refer to the equal and opposite forces of Newton's third law as canceling. It only makes sense to add up forces that are acting on the *same* object, whereas two forces related to each other by Newton's third law are always acting on two *different* objects.



A mnemonic for using Newton's third law correctly

Mnemonics are tricks for memorizing things. For instance, the musical notes that lie between the lines on the treble clef spell the word FACE, which is easy to remember. Many people use the mnemonic "SOHCAHTOA" to remember the definitions of the sine, cosine, and tangent in trigonometry. I have my own modest offering, POFOSTITO, which I hope will make it into the mnemonics hall of fame. It's a way to avoid some of the most common problems with applying Newton's third law correctly:

Pair of
Opposite
Forces
Of the
Same
Type
Involving
Two
Objects

A book lying on a table *example 1*

▷ A book is lying on a table. What force is the Newton's-third-law partner of the earth's gravitational force on the book?

Answer: Newton's third law works like "B on A, A on B," so the partner must be the book's gravitational force pulling upward on the planet earth. Yes, there is such a force! No, it does not cause the earth to do anything noticeable.

Incorrect answer: The table's upward force on the book is the Newton's-third-law partner of the earth's gravitational force on the book.

X This answer violates two out of three of the commandments of POFOSTITO. The forces are not of the same type, because the table's upward force on the book is not gravitational. Also, three

objects are involved instead of two: the book, the table, and the planet earth.

Pushing a box up a hill *example 2*

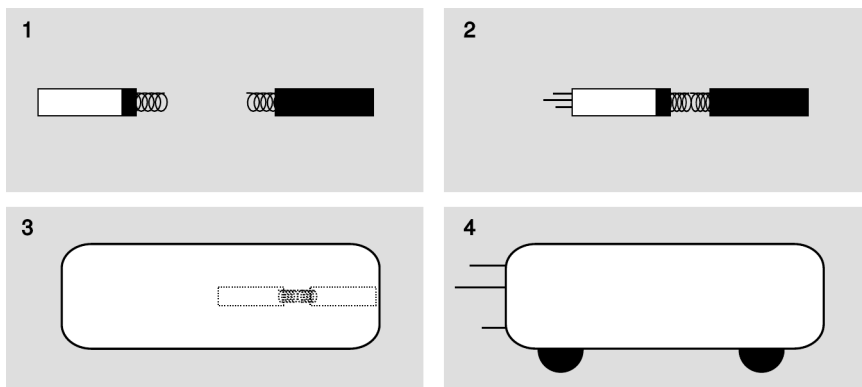
- ▷ A person is pushing a box up a hill. What force is related by Newton's third law to the person's force on the box?
- ▷ The box's force on the person.

Incorrect answer: The person's force on the box is opposed by friction, and also by gravity.

X This answer fails all three parts of the POFOSTITO test, the most obvious of which is that three forces are referred to instead of a pair.

If we could violate Newton's third law... *example 3*

If we could violate Newton's third law, we could do strange and wonderful things. Newton's third laws says that the unequal magnets in figure a on p. 154 should exert equal forces on each other, and this is what we actually find when we do the experiment shown in that figure. But suppose instead that it worked as most people intuitively expect. What if the third law was violated, so that the big magnet made more force on the small one than the small one made on the big one? To make the analysis simple, we add some extra nonmagnetic material to the small magnet in figure g/1, so that it has the same mass and size as the big one. We also attach springs. When we release the magnets, g/2, the weak one is accelerated strongly, while the strong one barely moves. If we put them inside a box, g/3, the recoiling strong magnet bangs hard against the side of the box, and the box mysteriously accelerates itself. The process can be repeated indefinitely for free, so we have a magic box that propels itself without needing fuel. We can make it into a perpetual-motion car, g/4. If Newton's third law was violated, we'd never have to pay for gas!



g / Example 3. This doesn't actually happen!

Optional topic: Newton's third law and action at a distance

Newton's third law is completely symmetric in the sense that neither force constitutes a delayed response to the other. Newton's third law does not even mention time, and the forces are supposed to agree at any given instant. This creates an interesting situation when it comes to noncontact forces. Suppose two people are holding magnets, and when one person waves or wiggles her magnet, the other person feels an effect on his. In this way they can send signals to each other from opposite sides of a wall, and if Newton's third law is correct, it would seem that the signals are transmitted instantly, with no time lag. The signals are indeed transmitted quite quickly, but experiments with electrically controlled magnets show that the signals do not leap the gap instantly: they travel at the same speed as light, which is an extremely high speed but not an infinite one.

Is this a contradiction to Newton's third law? Not really. According to current theories, there are no true noncontact forces. Action at a distance does not exist. Although it appears that the wiggling of one magnet affects the other with no need for anything to be in contact with anything, what really happens is that wiggling a magnet creates a ripple in the magnetic field pattern that exists even in empty space. The magnet shoves the ripples out with a kick and receives a kick in return, in strict obedience to Newton's third law. The ripples spread out in all directions, and the ones that hit the other magnet then interact with it, again obeying Newton's third law.

▷ *Solved problem: More about example 2* page 183, problem 20

▷ *Solved problem: Why did it accelerate?* page 183, problem 18

Discussion questions

A When you fire a gun, the exploding gases push outward in all directions, causing the bullet to accelerate down the barrel. What third-law pairs are involved? [Hint: Remember that the gases themselves are an object.]

B Tam Anh grabs Sarah by the hand and tries to pull her. She tries to remain standing without moving. A student analyzes the situation as follows. “If Tam Anh’s force on Sarah is greater than her force on him, he can get her to move. Otherwise, she’ll be able to stay where she is.” What’s wrong with this analysis?

C You hit a tennis ball against a wall. Explain any and all incorrect ideas in the following description of the physics involved: “According to Newton’s third law, there has to be a force opposite to your force on the ball. The opposite force is the ball’s mass, which resists acceleration, and also air resistance.”

5.2 Classification and behavior of forces

One of the most basic and important tasks of physics is to classify the forces of nature. I have already referred informally to “types” of forces such as friction, magnetism, gravitational forces, and so on. Classification systems are creations of the human mind, so there is always some degree of arbitrariness in them. For one thing, the level of detail that is appropriate for a classification system depends on what you’re trying to find out. Some linguists, the “lumpers,” like to emphasize the similarities among languages, and a few extremists have even tried to find signs of similarities between words in languages as different as English and Chinese, lumping the world’s languages into only a few large groups. Other linguists, the “splitters,” might be more interested in studying the differences in pronunciation between English speakers in New York and Connecticut. The splitters call the lumpers sloppy, but the lumpers say that science isn’t worthwhile unless it can find broad, simple patterns within the seemingly complex universe.

Scientific classification systems are also usually compromises between practicality and naturalness. An example is the question of how to classify flowering plants. Most people think that biological classification is about discovering new species, naming them, and classifying them in the class-order-family-genus-species system according to guidelines set long ago. In reality, the whole system is in a constant state of flux and controversy. One very practical way of classifying flowering plants is according to whether their petals are separate or joined into a tube or cone — the criterion is so clear that it can be applied to a plant seen from across the street. But here practicality conflicts with naturalness. For instance, the begonia has

separate petals and the pumpkin has joined petals, but they are so similar in so many other ways that they are usually placed within the same order. Some taxonomists have come up with classification criteria that they claim correspond more naturally to the apparent relationships among plants, without having to make special exceptions, but these may be far less practical, requiring for instance the examination of pollen grains under an electron microscope.

In physics, there are two main systems of classification for forces. At this point in the course, you are going to learn one that is very practical and easy to use, and that splits the forces up into a relatively large number of types: seven very common ones that we'll discuss explicitly in this chapter, plus perhaps ten less important ones such as surface tension, which we will not bother with right now.

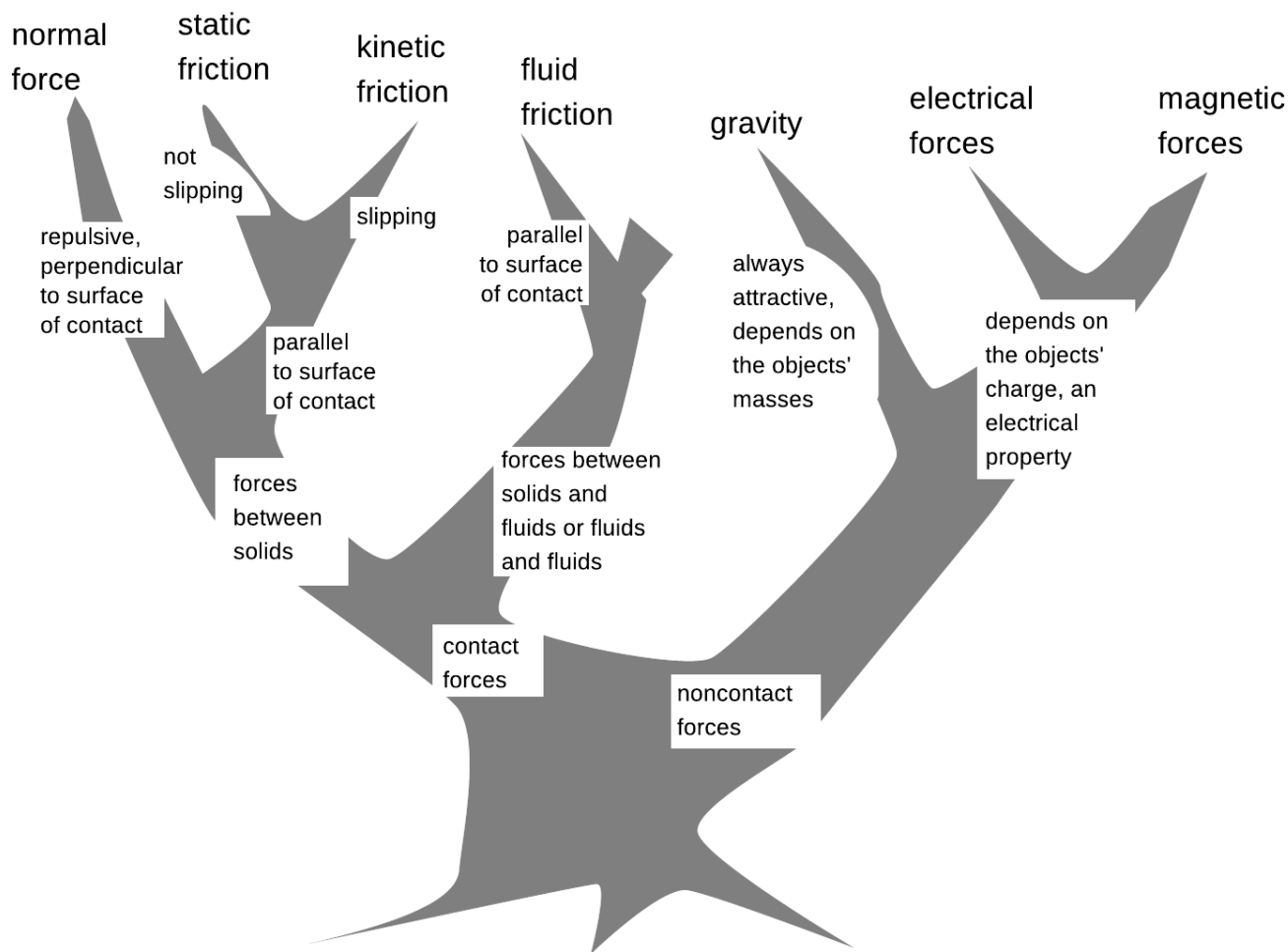
Physicists, however, are obsessed with finding simple patterns, so recognizing as many as fifteen or twenty types of forces strikes them as distasteful and overly complex. Since about the year 1900, physics has been on an aggressive program to discover ways in which these many seemingly different types of forces arise from a smaller number of fundamental ones. For instance, when you press your hands together, the force that keeps them from passing through each other may seem to have nothing to do with electricity, but at the atomic level, it actually does arise from electrical repulsion between atoms. By about 1950, all the forces of nature had been explained as arising from four fundamental types of forces at the atomic and nuclear level, and the lumping-together process didn't stop there. By the 1960's the length of the list had been reduced to three, and some theorists even believe that they may be able to reduce it to two or one. Although the unification of the forces of nature is one of the most beautiful and important achievements of physics, it makes much more sense to start this course with the more practical and easy system of classification. The unified system of four forces will be one of the highlights of the end of your introductory physics sequence.

The practical classification scheme which concerns us now can be laid out in the form of the tree shown in figure i. The most specific types of forces are shown at the tips of the branches, and it is these types of forces that are referred to in the POFOSTITO mnemonic. For example, electrical and magnetic forces belong to the same general group, but Newton's third law would never relate an electrical force to a magnetic force.

The broadest distinction is that between contact and noncontact forces, which has been discussed in ch. 4. Among the contact forces, we distinguish between those that involve solids only and those that have to do with fluids, a term used in physics to include both gases and liquids.



h / A scientific classification system.



i / A practical classification scheme for forces.

It should not be necessary to memorize this diagram by rote. It is better to reinforce your memory of this system by calling to mind your commonsense knowledge of certain ordinary phenomena. For instance, we know that the gravitational attraction between us and the planet earth will act even if our feet momentarily leave the ground, and that although magnets have mass and are affected by gravity, most objects that have mass are nonmagnetic.

Hitting a wall

example 4

▷ A bullet, flying horizontally, hits a steel wall. What type of force is there between the bullet and the wall?

▷ Starting at the bottom of the tree, we determine that the force is a contact force, because it only occurs once the bullet touches the wall. Both objects are solid. The wall forms a vertical plane. If the nose of the bullet was some shape like a sphere, you might imagine that it would only touch the wall at one point. Realistically, however, we know that a lead bullet will flatten out a lot on impact, so there is a surface of contact between the two, and its

orientation is vertical. The effect of the force on the bullet is to stop the horizontal motion of the bullet, and this horizontal acceleration must be produced by a horizontal force. The force is therefore perpendicular to the surface of contact, and it's also repulsive (tending to keep the bullet from entering the wall), so it must be a normal force.

Diagram i is meant to be as simple as possible while including most of the forces we deal with in everyday life. If you were an insect, you would be much more interested in the force of surface tension, which allowed you to walk on water. I have not included the nuclear forces, which are responsible for holding the nuclei of atoms, because they are not evident in everyday life.

You should not be afraid to invent your own names for types of forces that do not fit into the diagram. For instance, the force that holds a piece of tape to the wall has been left off of the tree, and if you were analyzing a situation involving scotch tape, you would be absolutely right to refer to it by some commonsense name such as “sticky force.”

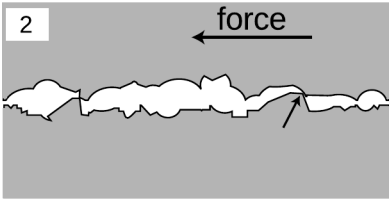
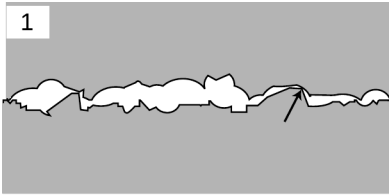
On the other hand, if you are having trouble classifying a certain force, you should also consider whether it is a force at all. For instance, if someone asks you to classify the force that the earth has because of its rotation, you would have great difficulty creating a place for it on the diagram. That's because it's a type of motion, not a type of force!

Normal forces

A normal force, F_N , is a force that keeps one solid object from passing through another. “Normal” is simply a fancy word for “perpendicular,” meaning that the force is perpendicular to the surface of contact. Intuitively, it seems the normal force magically adjusts itself to provide whatever force is needed to keep the objects from occupying the same space. If your muscles press your hands together gently, there is a gentle normal force. Press harder, and the normal force gets stronger. How does the normal force know how strong to be? The answer is that the harder you jam your hands together, the more compressed your flesh becomes. Your flesh is acting like a spring: more force is required to compress it more. The same is true when you push on a wall. The wall flexes imperceptibly in proportion to your force on it. If you exerted enough force, would it be possible for two objects to pass through each other? No, typically the result is simply to strain the objects so much that one of them breaks.

Gravitational forces

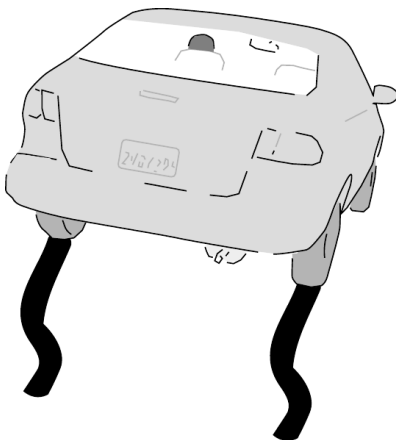
As we'll discuss in more detail later in the course, a gravitational force exists between any two things that have mass. In everyday life,



j / A model that correctly explains many properties of friction. The microscopic bumps and holes in two surfaces dig into each other.



k / Static friction: the tray doesn't slip on the waiter's fingers.



l / Kinetic friction: the car skids.

the gravitational force between two cars or two people is negligible, so the only noticeable gravitational forces are the ones between the earth and various human-scale objects. We refer to these planet-earth-induced gravitational forces as weight forces, and as we have already seen, their magnitude is given by $|F_W| = mg$.

▷ Solved problem: *Weight and mass*

page 184, problem 26

Static and kinetic friction

If you have pushed a refrigerator across a kitchen floor, you have felt a certain series of sensations. At first, you gradually increased your force on the refrigerator, but it didn't move. Finally, you supplied enough force to unstick the fridge, and there was a sudden jerk as the fridge started moving. Once the fridge was unstuck, you could reduce your force significantly and still keep it moving.

While you were gradually increasing your force, the floor's frictional force on the fridge increased in response. The two forces on the fridge canceled, and the fridge didn't accelerate. How did the floor know how to respond with just the right amount of force? Figure j shows one possible *model* of friction that explains this behavior. (A scientific model is a description that we expect to be incomplete, approximate, or unrealistic in some ways, but that nevertheless succeeds in explaining a variety of phenomena.) Figure j/1 shows a microscopic view of the tiny bumps and holes in the surfaces of the floor and the refrigerator. The weight of the fridge presses the two surfaces together, and some of the bumps in one surface will settle as deeply as possible into some of the holes in the other surface. In j/2, your leftward force on the fridge has caused it to ride up a little higher on the bump in the floor labeled with a small arrow. Still more force is needed to get the fridge over the bump and allow it to start moving. Of course, this is occurring simultaneously at millions of places on the two surfaces.

Once you had gotten the fridge moving at constant speed, you found that you needed to exert less force on it. Since zero total force is needed to make an object move with constant velocity, the floor's rightward frictional force on the fridge has apparently decreased somewhat, making it easier for you to cancel it out. Our model also gives a plausible explanation for this fact: as the surfaces slide past each other, they don't have time to settle down and mesh with one another, so there is less friction.

Even though this model is intuitively appealing and fairly successful, it should not be taken too seriously, and in some situations it is misleading. For instance, fancy racing bikes these days are made with smooth tires that have no tread — contrary to what we'd expect from our model, this does not cause any decrease in friction. Machinists know that two very smooth and clean metal

surfaces may stick to each other firmly and be very difficult to slide apart. This cannot be explained in our model, but makes more sense in terms of a model in which friction is described as arising from chemical bonds between the atoms of the two surfaces at their points of contact: very flat surfaces allow more atoms to come in contact.

Since friction changes its behavior dramatically once the surfaces come unstuck, we define two separate types of frictional forces. *Static friction* is friction that occurs between surfaces that are not slipping over each other. Slipping surfaces experience *kinetic friction*. The forces of static and kinetic friction, notated F_s and F_k , are always parallel to the surface of contact between the two objects.

self-check B

1. When a baseball player slides in to a base, is the friction static, or kinetic?
2. A mattress stays on the roof of a slowly accelerating car. Is the friction static, or kinetic?
3. Does static friction create heat? Kinetic friction? ▷ Answer, p. 565

The maximum possible force of static friction depends on what kinds of surfaces they are, and also on how hard they are being pressed together. The approximate mathematical relationships can be expressed as follows:

$$F_{s,max} = \mu_s F_N,$$

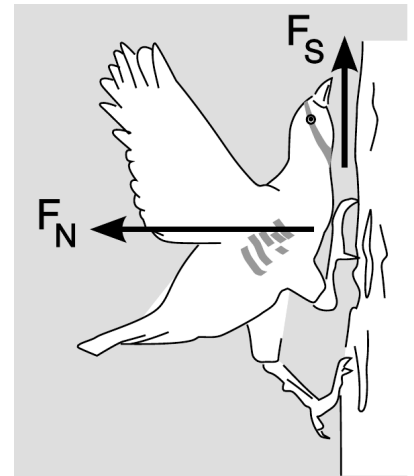
where μ_s is a unitless number, called the coefficient of static friction, which depends on what kinds of surfaces they are. The maximum force that static friction can supply, $\mu_s F_N$, represents the boundary between static and kinetic friction. It depends on the normal force, which is numerically equal to whatever force is pressing the two surfaces together. In terms of our model, if the two surfaces are being pressed together more firmly, a greater sideways force will be required in order to make the irregularities in the surfaces ride up and over each other.

Note that just because we use an adjective such as “applied” to refer to a force, that doesn’t mean that there is some special type of force called the “applied force.” The applied force could be any type of force, or it could be the sum of more than one force trying to make an object move.

self-check C

The arrows in figure m show the forces of the tree trunk on the partridge. Describe the forces the bird makes on the tree. ▷ Answer, p. 565

The force of kinetic friction on each of the two objects is in the direction that resists the slippage of the surfaces. Its magnitude is



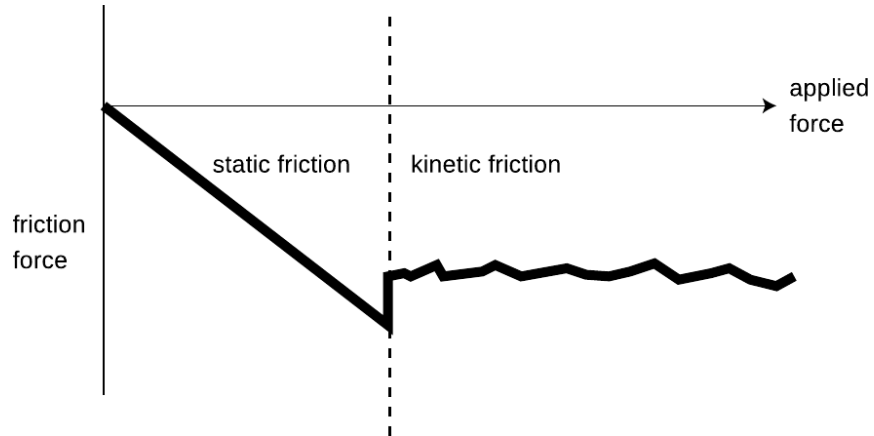
m / Many landfowl, even those that are competent fliers, prefer to escape from a predator by running upward rather than by flying. This partridge is running up a vertical tree trunk. Humans can’t walk up walls because there is no normal force and therefore no frictional force; when $F_N = 0$, the maximum force of static friction $F_{s,max} = \mu_s F_N$ is also zero. The partridge, however, has wings that it can flap in order to create a force between it and the air. Typically when a bird flaps its wings, the resulting force from the air is in the direction that would tend to lift the bird up. In this situation, however, the partridge changes its style of flapping so that the direction is reversed. The normal force between the feet and the tree allows a nonzero static frictional force. The mechanism is similar to that of a spoiler fin on a racing car. Some evolutionary biologists believe that when vertebrate flight first evolved, in dinosaurs, there was first a stage in which the wings were used only as an aid in running up steep inclines, and only later a transition to flight. (Redrawn from a figure by K.P. Dial.)

usually well approximated as

$$F_k = \mu_k F_N$$

where μ_k is the coefficient of kinetic friction. Kinetic friction is usually more or less independent of velocity.

n / We choose a coordinate system in which the applied force, i.e., the force trying to move the objects, is positive. The friction force is then negative, since it is in the opposite direction. As you increase the applied force, the force of static friction increases to match it and cancel it out, until the maximum force of static friction is surpassed. The surfaces then begin slipping past each other, and the friction force becomes smaller in absolute value.



self-check D

Can a frictionless surface exert a normal force? Can a frictional force exist without a normal force? ▷ Answer, p. 565

If you try to accelerate or decelerate your car too quickly, the forces between your wheels and the road become too great, and they begin slipping. This is not good, because kinetic friction is weaker than static friction, resulting in less control. Also, if this occurs while you are turning, the car's handling changes abruptly because the kinetic friction force is in a different direction than the static friction force had been: contrary to the car's direction of motion, rather than contrary to the forces applied to the tire.

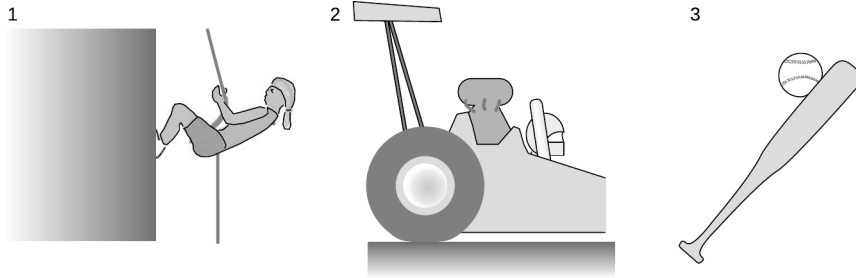
Most people respond with disbelief when told of the experimental evidence that both static and kinetic friction are approximately independent of the amount of surface area in contact. Even after doing a hands-on exercise with spring scales to show that it is true, many students are unwilling to believe their own observations, and insist that bigger tires “give more traction.” In fact, the main reason why you would not want to put small tires on a big heavy car is that the tires would burst!

Although many people expect that friction would be proportional to surface area, such a proportionality would make predictions contrary to many everyday observations. A dog's feet, for example, have very little surface area in contact with the ground compared to a human's feet, and yet we know that a dog can often win a tug-of-war with a person.

The reason a smaller surface area does not lead to less friction is that the force between the two surfaces is more concentrated, causing their bumps and holes to dig into each other more deeply.

self-check E

Find the direction of each of the forces in figure o. ▷ Answer, p. 565



o / 1. The cliff's normal force on the climber's feet. 2. The track's static frictional force on the wheel of the accelerating dragster. 3. The ball's normal force on the bat.

Locomotives

example 5

Looking at a picture of a locomotive, p, we notice two obvious things that are different from an automobile. Where a car typically has two drive wheels, a locomotive normally has many — ten in this example. (Some also have smaller, unpowered wheels in front of and behind the drive wheels, but this example doesn't.) Also, cars these days are generally built to be as light as possible for their size, whereas locomotives are very massive, and no effort seems to be made to keep their weight low. (The steam locomotive in the photo is from about 1900, but this is true even for modern diesel and electric trains.)



p / Example 5.

The reason locomotives are built to be so heavy is for traction. The upward normal force of the rails on the wheels, F_N , cancels the downward force of gravity, F_W , so ignoring plus and minus signs, these two forces are equal in absolute value, $F_N = F_W$. Given this amount of normal force, the maximum force of static friction is $F_s = \mu_s F_N = \mu_s F_W$. This static frictional force, of the rails pushing forward on the wheels, is the only force that can accelerate the train, pull it uphill, or cancel out the force of air resistance while cruising at constant speed. The coefficient of static friction for steel on steel is about 1/4, so no locomotive can pull with a force greater than about 1/4 of its own weight. If the



q / Fluid friction depends on the fluid's pattern of flow, so it is more complicated than friction between solids, and there are no simple, universally applicable formulas to calculate it. From top to bottom: supersonic wind tunnel, vortex created by a crop duster, series of vortices created by a single object, turbulence.

engine is capable of supplying more than that amount of force, the result will be simply to break static friction and spin the wheels.

The reason this is all so different from the situation with a car is that a car isn't pulling something else. If you put extra weight in a car, you improve the traction, but you also increase the inertia of the car, and make it just as hard to accelerate. In a train, the inertia is almost all in the cars being pulled, not in the locomotive.

The other fact we have to explain is the large number of driving wheels. First, we have to realize that increasing the number of driving wheels neither increases nor decreases the total amount of static friction, because static friction is independent of the amount of surface area in contact. (The reason four-wheel-drive is good in a car is that if one or more of the wheels is slipping on ice or in mud, the other wheels may still have traction. This isn't typically an issue for a train, since all the wheels experience the same conditions.) The advantage of having more driving wheels on a train is that it allows us to increase the weight of the locomotive without crushing the rails, or damaging bridges.

Fluid friction

Try to drive a nail into a waterfall and you will be confronted with the main difference between solid friction and fluid friction. Fluid friction is purely kinetic; there is no static fluid friction. The nail in the waterfall may tend to get dragged along by the water flowing past it, but it does not stick in the water. The same is true for gases such as air: recall that we are using the word "fluid" to include both gases and liquids.

Unlike kinetic friction between solids, fluid friction increases rapidly with velocity. It also depends on the shape of the object, which is why a fighter jet is more streamlined than a Model T. For objects of the same shape but different sizes, fluid friction typically scales up with the cross-sectional area of the object, which is one of the main reasons that an SUV gets worse mileage on the freeway than a compact car.

Discussion questions

A A student states that when he tries to push his refrigerator, the reason it won't move is because Newton's third law says there's an equal and opposite frictional force pushing back. After all, the static friction force is equal and opposite to the applied force. How would you convince him he is wrong?

B Kinetic friction is usually more or less independent of velocity. However, inexperienced drivers tend to produce a jerk at the last moment of deceleration when they stop at a stop light. What does this tell you about the kinetic friction between the brake shoes and the brake drums?

C Some of the following are correct descriptions of types of forces that could be added on as new branches of the classification tree. Others are not really types of forces, and still others are not force phenomena at all. In each case, decide what's going on, and if appropriate, figure out how you would incorporate them into the tree.

sticky force	makes tape stick to things
opposite force	the force that Newton's third law says relates to every force you make
flowing force	the force that water carries with it as it flows out of a hose
surface tension	lets insects walk on water
horizontal force	a force that is horizontal
motor force	the force that a motor makes on the thing it is turning
canceled force	a force that is being canceled out by some other force

5.3 Analysis of forces

Newton's first and second laws deal with the total of all the forces exerted on a specific object, so it is very important to be able to figure out what forces there are. Once you have focused your attention on one object and listed the forces on it, it is also helpful to describe all the corresponding forces that must exist according to Newton's third law. We refer to this as "analyzing the forces" in which the object participates.



r / What do the golf ball and the shark have in common? Both use the same trick to reduce fluid friction. The dimples on the golf ball modify the pattern of flow of the air around it, counterintuitively *reducing* friction. Recent studies have shown that sharks can accomplish the same thing by raising, or "bristling," the scales on their skin at high speeds.



s / The wheelbases of the Hummer H3 and the Toyota Prius are surprisingly similar, differing by only 10%. The main difference in shape is that the Hummer is much taller and wider. It presents a much greater cross-sectional area to the wind, and this is the main reason that it uses about 2.5 times more gas on the freeway.

A barge

example 6

A barge is being pulled to the right along a canal by teams of horses on the shores. Analyze all the forces in which the barge participates.

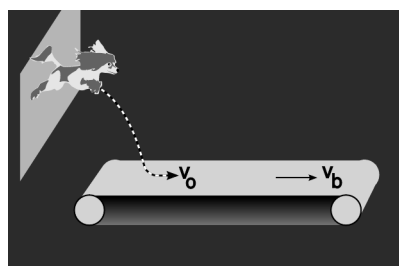
<i>force acting on barge</i>	<i>force related to it by Newton's third law</i>
ropes' normal forces on barge, \rightarrow	barge's normal force on ropes, \leftarrow
water's fluid friction force on barge, \leftarrow	barge's fluid friction force on water, \rightarrow
planet earth's gravitational force on barge, \downarrow	barge's gravitational force on earth, \uparrow
water's "floating" force on barge, \uparrow	barge's "floating" force on water, \downarrow

Here I've used the word "floating" force as an example of a sensible invented term for a type of force not classified on the tree on p. 160. A more formal technical term would be "hydrostatic force."

Note how the pairs of forces are all structured as "A's force on B, B's force on A": ropes on barge and barge on ropes; water on barge and barge on water. Because all the forces in the left column are forces acting on the barge, all the forces in the right column are forces being exerted by the barge, which is why each entry in the column begins with "barge."

Often you may be unsure whether you have forgotten one of the forces. Here are three strategies for checking your list:

1. See what physical result would come from the forces you've found so far. Suppose, for instance, that you'd forgotten the "floating" force on the barge in the example above. Looking at the forces you'd found, you would have found that there was a downward gravitational force on the barge which was not canceled by any upward force. The barge isn't supposed to sink, so you know you need to find a fourth, upward force.
2. Another technique for finding missing forces is simply to go through the list of all the common types of forces and see if any of them apply.
3. Make a drawing of the object, and draw a dashed boundary line around it that separates it from its environment. Look for points on the boundary where other objects come in contact with your object. This strategy guarantees that you'll find every contact force that acts on the object, although it won't help you to find non-contact forces.



t / Example 7.

Fifi

example 7

▷ Fifi is an industrial espionage dog who loves doing her job and looks great doing it. She leaps through a window and lands at initial horizontal speed v_0 on a conveyor belt which is itself moving at the greater speed v_b . Unfortunately the coefficient of kinetic friction μ_k between her foot-pads and the belt is fairly low, so she skids for a time Δt , during which the effect on her coiffure is *un désastre*. Find Δt .

▷ We analyze the forces:

<i>force acting on Fifi</i>	<i>force related to it by Newton's third law</i>
planet earth's gravitational force $F_W = mg$ on Fifi, ↓	Fifi's gravitational force on earth, ↑
belt's kinetic frictional force F_k on Fifi, →	Fifi's kinetic frictional force on belt, ←
belt's normal force F_N on Fifi, ↑	Fifi's normal force on belt, ↓

Checking the analysis of the forces as described on p. 168:

(1) The physical result makes sense. The left-hand column consists of forces $\downarrow \rightarrow \uparrow$. We're describing the time when she's moving horizontally on the belt, so it makes sense that we have two vertical forces that could cancel. The rightward force is what will accelerate her until her speed matches that of the belt.

(2) We've included every relevant type of force from the tree on p. 160.

(3) We've included forces from the belt, which is the only object in contact with Fifi.

The purpose of the analysis is to let us set up equations containing enough information to solve the problem. Using the generalization of Newton's second law given on p. 137, we use the horizontal force to determine the horizontal acceleration, and separately require the vertical forces to cancel out.

Let positive x be to the right. Newton's second law gives

$$(\rightarrow) \quad a = F_k/m$$

Although it's the horizontal motion we care about, the only way to find F_k is via the relation $F_k = \mu_k F_N$, and the only way to find F_N is from the $\uparrow \downarrow$ forces. The two vertical forces must cancel, which means they have to be of equal strength:

$$(\uparrow \downarrow) \quad F_N - mg = 0.$$

Using the constant-acceleration equation $a = \Delta v/\Delta t$, we have

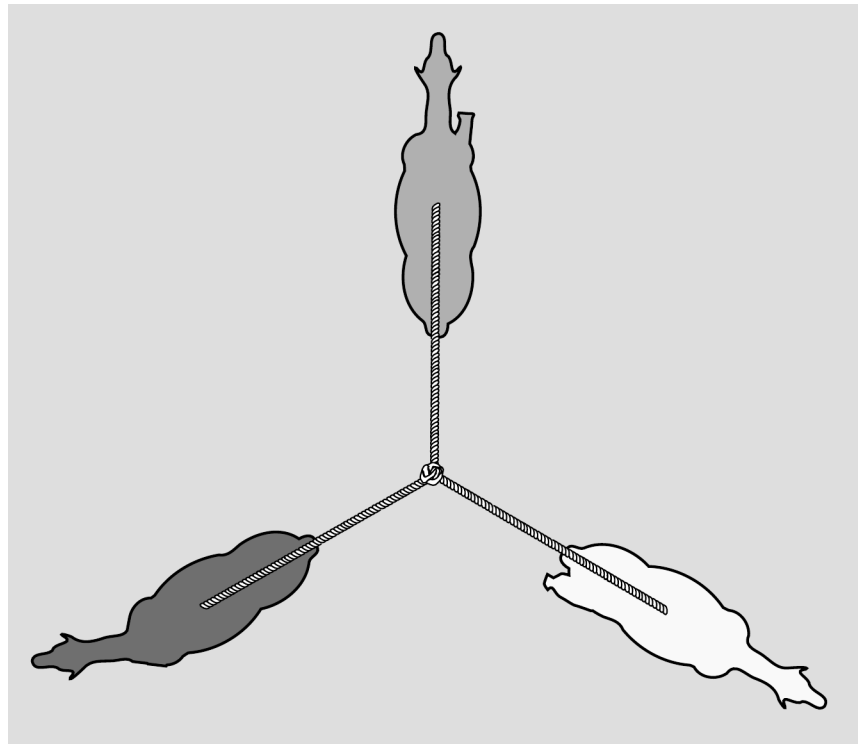
$$\begin{aligned} \Delta t &= \frac{\Delta v}{a} \\ &= \frac{v_b - v_o}{\mu_k mg/m} \\ &= \frac{v_b - v_o}{\mu_k g}. \end{aligned}$$

The units check out:

$$s = \frac{m/s}{m/s^2},$$

where μ_k is omitted as a factor because it's unitless.

We should also check that the dependence on the variables makes sense. If Fifi puts on her rubber ninja booties, increasing μ_k , then dividing by a larger number gives a smaller result for Δt ; this makes sense physically, because the greater friction will cause her to come up to the belt's speed more quickly. The dependence on g is similar; more gravity would press her harder against the belt, improving her traction. Increasing v_b increases Δt , which makes sense because it will take her longer to get up to a bigger speed. Since v_o is subtracted, the dependence of Δt on it is the other way around, and that makes sense too, because if she can land with a greater speed, she has less speeding up left to do.



u / Example 8.

Forces don't have to be in pairs or at right angles *example 8*
In figure u, the three horses are arranged symmetrically at 120 degree intervals, and are all pulling on the central knot. Let's say the knot is at rest and at least momentarily in equilibrium. The analysis of forces on the knot is as follows.

<i>force acting on knot</i>	<i>force related to it by Newton's third law</i>
top rope's normal force on knot, \uparrow	knot's normal force on top rope, \downarrow
left rope's normal force on knot, \leftarrow	knot's normal force on left rope, \rightarrow
right rope's normal force on knot, \rightarrow	knot's normal force on right rope, \leftarrow

In our previous examples, the forces have all run along two perpendicular lines, and they often canceled in pairs. This example shows that neither of these always happens. Later in the book we'll see how to handle forces that are at arbitrary angles, using mathematical objects called vectors. But even without knowing about vectors, we already know what directions to draw the arrows in the table, since a rope can only pull parallel to itself at its ends. And furthermore, we can say something about the forces: by symmetry, we expect them all to be equal in strength. (If the knot was not in equilibrium, then this symmetry would be broken.)

This analysis also demonstrates that it's all right to leave out details if they aren't of interest and we don't intend to include them in our model. We called the forces normal forces, but we can't actually tell whether they are normal forces or frictional forces. They are probably some combination of those, but we don't include such details in this model, since aren't interested in describing the internal physics of the knot. This is an example of a more general fact about science, which is that science doesn't describe reality. It describes simplified *models* of reality, because reality is always too complex to model exactly.

Discussion questions

A In the example of the barge going down the canal, I referred to a “floating” or “hydrostatic” force that keeps the boat from sinking. If you were adding a new branch on the force-classification tree to represent this force, where would it go?

B The earth’s gravitational force on you, i.e., your weight, is always equal to mg , where m is your mass. So why can you get a shovel to go deeper into the ground by jumping onto it? Just because you’re jumping, that doesn’t mean your mass or weight is any greater, does it?

5.4 Transmission of forces by low-mass objects

You’re walking your dog. The dog wants to go faster than you do, and the leash is taut. Does Newton’s third law guarantee that your force on your end of the leash is equal and opposite to the dog’s force on its end? If they’re not exactly equal, is there any reason why they should be approximately equal?

If there was no leash between you, and you were in direct contact with the dog, then Newton’s third law would apply, but Newton’s third law cannot relate your force on the leash to the dog’s force on the leash, because that would involve three separate objects. Newton’s third law only says that your force on the leash is equal and opposite to the leash’s force on you,

$$F_{yL} = -F_{Ly},$$

and that the dog’s force on the leash is equal and opposite to its force on the dog

$$F_{dL} = -F_{Ld}.$$

Still, we have a strong intuitive expectation that whatever force we make on our end of the leash is transmitted to the dog, and vice-versa. We can analyze the situation by concentrating on the forces that act on the leash, F_{dL} and F_{yL} . According to Newton’s second law, these relate to the leash’s mass and acceleration:

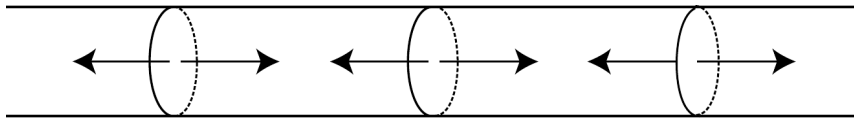
$$F_{dL} + F_{yL} = m_L a_L.$$

The leash is far less massive than any of the other objects involved, and if m_L is very small, then apparently the total force on the leash is also very small, $F_{dL} + F_{yL} \approx 0$, and therefore

$$F_{dL} \approx -F_{yL}.$$

Thus even though Newton’s third law does not apply directly to these two forces, we can approximate the low-mass leash as if it was not intervening between you and the dog. It’s at least approximately as if you and the dog were acting directly on each other, in which case Newton’s third law would have applied.

In general, low-mass objects can be treated approximately as if they simply transmitted forces from one object to another. This can be true for strings, ropes, and cords, and also for rigid objects such as rods and sticks.



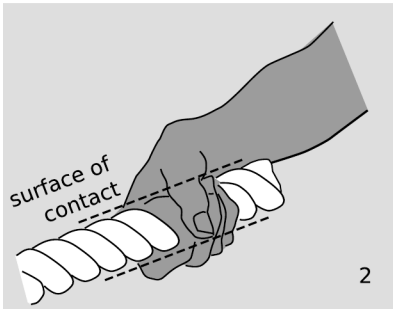
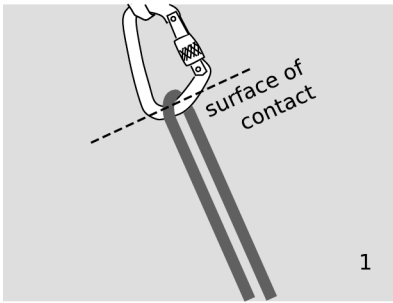
v / If we imagine dividing a taut rope up into small segments, then any segment has forces pulling outward on it at each end. If the rope is of negligible mass, then all the forces equal $+T$ or $-T$, where T , the tension, is a single number.

If you look at a piece of string under a magnifying glass as you pull on the ends more and more strongly, you will see the fibers straightening and becoming taut. Different parts of the string are apparently exerting forces on each other. For instance, if we think of the two halves of the string as two objects, then each half is exerting a force on the other half. If we imagine the string as consisting of many small parts, then each segment is transmitting a force to the next segment, and if the string has very little mass, then all the forces are equal in magnitude. We refer to the magnitude of the forces as the tension in the string, T .

The term “tension” refers only to internal forces within the string. If the string makes forces on objects at its ends, then those forces are typically normal or frictional forces (example 9).



w / The Golden Gate Bridge's roadway is held up by the tension in the vertical cables.



x / Example 9. The forces between the rope and other objects are normal and frictional forces.

Types of force made by ropes

example 9

▷ Analyze the forces in figures x/1 and x/2.

▷ In all cases, a rope can only make “pulling” forces, i.e., forces that are parallel to its own length and that are toward itself, not away from itself. You can’t push with a rope!

In x/1, the rope passes through a type of hook, called a carabiner, used in rock climbing and mountaineering. Since the rope can only pull along its own length, the direction of its force on the carabiner must be down and to the right. This is perpendicular to the surface of contact, so the force is a normal force.

<i>force acting on carabiner</i>	<i>force related to it by Newton’s third law</i>
rope’s normal force on carabiner ↘	carabiner’s normal force on rope ↖

(There are presumably other forces acting on the carabiner from other hardware above it.)

In figure x/2, the rope can only exert a net force at its end that is parallel to itself and in the pulling direction, so its force on the hand is down and to the left. This is parallel to the surface of contact, so it must be a frictional force. If the rope isn’t slipping through the hand, we have static friction. Friction can’t exist without normal forces. These forces are perpendicular to the surface of contact. For simplicity, we show only two pairs of these normal forces, as if the hand were a pair of pliers.

<i>force acting on person</i>	<i>force related to it by Newton’s third law</i>
rope’s static frictional force on person ↙	person’s static frictional force on rope ↗
rope’s normal force on person ↖	person’s normal force on rope ↘
rope’s normal force on person ↘	person’s normal force on rope ↖

(There are presumably other forces acting on the person as well, such as gravity.)

If a rope goes over a pulley or around some other object, then the tension throughout the rope is approximately equal so long as the pulley has negligible mass and there is not too much friction. A rod or stick can be treated in much the same way as a string, but it is possible to have either compression or tension.

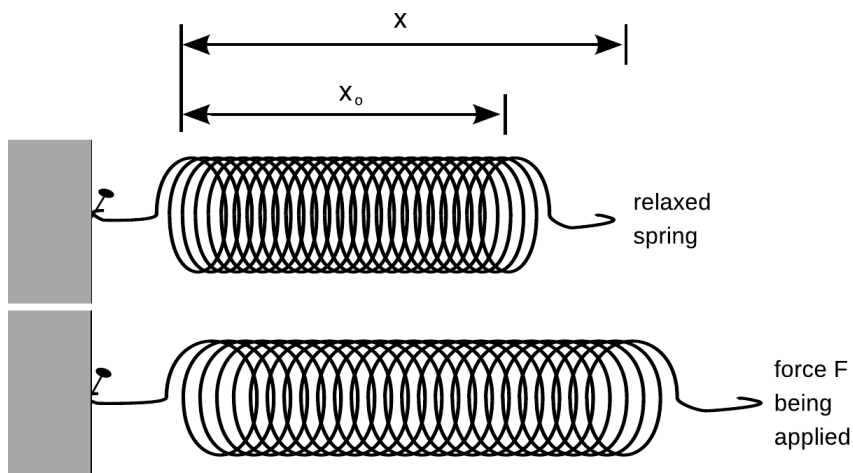
Discussion question

A When you step on the gas pedal, is your foot’s force being transmitted in the sense of the word used in this section?

5.5 Objects under strain

A string lengthens slightly when you stretch it. Similarly, we have already discussed how an apparently rigid object such as a wall is actually flexing when it participates in a normal force. In other cases, the effect is more obvious. A spring or a rubber band visibly elongates when stretched.

Common to all these examples is a change in shape of some kind: lengthening, bending, compressing, etc. The change in shape can be measured by picking some part of the object and measuring its position, x . For concreteness, let's imagine a spring with one end attached to a wall. When no force is exerted, the unfixed end of the spring is at some position x_o . If a force acts at the unfixed end, its position will change to some new value of x . The more force, the greater the departure of x from x_o .



y / Defining the quantities F , x , and x_o in Hooke's law.

Back in Newton's time, experiments like this were considered cutting-edge research, and his contemporary Hooke is remembered today for doing them and for coming up with a simple mathematical generalization called Hooke's law:

$$F \approx k(x - x_o). \quad \text{[force required to stretch a spring; valid for small forces only]}$$

Here k is a constant, called the spring constant, that depends on how stiff the object is. If too much force is applied, the spring exhibits more complicated behavior, so the equation is only a good approximation if the force is sufficiently small. Usually when the force is so large that Hooke's law is a bad approximation, the force ends up permanently bending or breaking the spring.

Although Hooke's law may seem like a piece of trivia about springs, it is actually far more important than that, because all

solid objects exert Hooke's-law behavior over some range of sufficiently small forces. For example, if you push down on the hood of a car, it dips by an amount that is directly proportional to the force. (But the car's behavior would not be as mathematically simple if you dropped a boulder on the hood!)

▷ *Solved problem: Combining springs* page 182, problem 14

▷ *Solved problem: Young's modulus* page 182, problem 16

Discussion question

A A car is connected to its axles through big, stiff springs called shock absorbers, or "shocks." Although we've discussed Hooke's law above only in the case of stretching a spring, a car's shocks are continually going through both stretching and compression. In this situation, how would you interpret the positive and negative signs in Hooke's law?

5.6 Simple Machines: the pulley

Even the most complex machines, such as cars or pianos, are built out of certain basic units called *simple machines*. The following are some of the main functions of simple machines:

transmitting a force: The chain on a bicycle transmits a force from the crank set to the rear wheel.

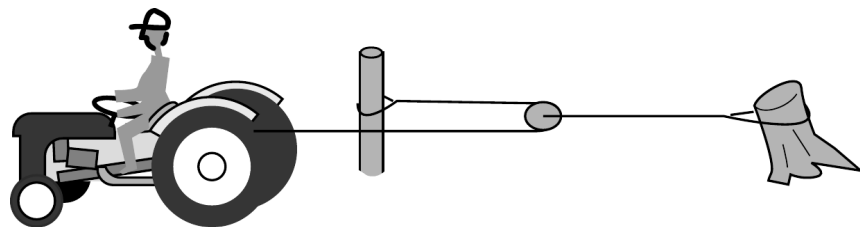
changing the direction of a force: If you push down on a see-saw, the other end goes up.

changing the speed and precision of motion: When you make the "come here" motion, your biceps only moves a couple of centimeters where it attaches to your forearm, but your arm moves much farther and more rapidly.

changing the amount of force: A lever or pulley can be used to increase or decrease the amount of force.

You are now prepared to understand one-dimensional simple machines, of which the pulley is the main example.

z / Example 10.



A pulley

example 10

▷ Farmer Bill says this pulley arrangement doubles the force of his tractor. Is he just a dumb hayseed, or does he know what he's doing?

▷ To use Newton's first law, we need to pick an object and consider the sum of the forces on it. Since our goal is to relate the tension in the part of the cable attached to the stump to the tension in the part attached to the tractor, we should pick an object to which both those cables are attached, i.e., the pulley itself. The tension in a string or cable remains approximately constant as it passes around an idealized pulley.¹ There are therefore two leftward forces acting on the pulley, each equal to the force exerted by the tractor. Since the acceleration of the pulley is essentially zero, the forces on it must be canceling out, so the rightward force of the pulley-stump cable on the pulley must be double the force exerted by the tractor. Yes, Farmer Bill knows what he's talking about.

¹This was asserted in section 5.4 without proof. Essentially it holds because of symmetry. E.g., if the U-shaped piece of rope in figure z had unequal tension in its two legs, then this would have to be caused by some asymmetry between clockwise and counterclockwise rotation. But such an asymmetry can only be caused by friction or inertia, which we assume don't exist.

Summary

Selected vocabulary

repulsive	describes a force that tends to push the two participating objects apart
attractive	describes a force that tends to pull the two participating objects together
oblique	describes a force that acts at some other angle, one that is not a direct repulsion or attraction
normal force	the force that keeps two objects from occupying the same space
static friction	a friction force between surfaces that are not slipping past each other
kinetic friction	a friction force between surfaces that are slipping past each other
fluid	a gas or a liquid
fluid friction	a friction force in which at least one of the object is is a fluid
spring constant	the constant of proportionality between force and elongation of a spring or other object under strain

Notation

F_N	a normal force
F_s	a static frictional force
F_k	a kinetic frictional force
μ_s	the coefficient of static friction; the constant of proportionality between the maximum static frictional force and the normal force; depends on what types of surfaces are involved
μ_k	the coefficient of kinetic friction; the constant of proportionality between the kinetic frictional force and the normal force; depends on what types of surfaces are involved
k	the spring constant; the constant of proportionality between the force exerted on an object and the amount by which the object is lengthened or compressed

Summary

Newton's third law states that forces occur in equal and opposite pairs. If object A exerts a force on object B, then object B must simultaneously be exerting an equal and opposite force on object A. Each instance of Newton's third law involves exactly two objects, and exactly two forces, which are of the same type.

There are two systems for classifying forces. We are presently using the more practical but less fundamental one. In this system, forces are classified by whether they are repulsive, attractive, or oblique; whether they are contact or noncontact forces; and whether

the two objects involved are solids or fluids.

Static friction adjusts itself to match the force that is trying to make the surfaces slide past each other, until the maximum value is reached,

$$F_{s,max} = \mu_s F_N.$$

Once this force is exceeded, the surfaces slip past one another, and kinetic friction applies,

$$F_k = \mu_k F_N.$$

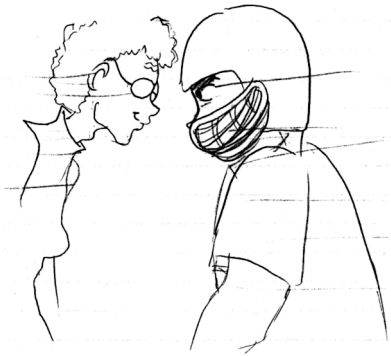
Both types of frictional force are nearly independent of surface area, and kinetic friction is usually approximately independent of the speed at which the surfaces are slipping. The direction of the force is in the direction that would tend to stop or prevent slipping.

A good first step in applying Newton's laws of motion to any physical situation is to pick an object of interest, and then to list all the forces acting on that object. We classify each force by its type, and find its Newton's-third-law partner, which is exerted by the object on some other object.

When two objects are connected by a third low-mass object, their forces are transmitted to each other nearly unchanged.

Objects under strain always obey Hooke's law to a good approximation, as long as the force is small. Hooke's law states that the stretching or compression of the object is proportional to the force exerted on it,

$$F \approx k(x - x_o).$$



Problem 1.



Problem 6.



Problem 7.

Problems

Key

- ✓ A computerized answer check is available online.
- ∫ A problem that requires calculus.
- ★ A difficult problem.

1 A little old lady and a pro football player collide head-on. Compare their forces on each other, and compare their accelerations. Explain.

2 The earth is attracted to an object with a force equal and opposite to the force of the earth on the object. If this is true, why is it that when you drop an object, the earth does not have an acceleration equal and opposite to that of the object?

3 When you stand still, there are two forces acting on you, the force of gravity (your weight) and the normal force of the floor pushing up on your feet. Are these forces equal and opposite? Does Newton's third law relate them to each other? Explain.

In problems 4-8, analyze the forces using a table in the format shown in section 5.3. Analyze the forces in which the italicized object participates.

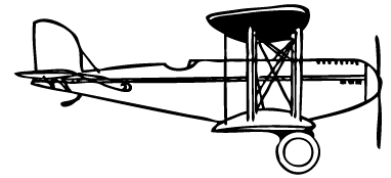
4 Some people put a spare car key in a little magnetic *box* that they stick under the chassis of their car. Let's say that the box is stuck directly underneath a horizontal surface, and the car is parked. (See instructions above.)

5 Analyze two examples of *objects* at rest relative to the earth that are being kept from falling by forces other than the normal force. Do not use objects in outer space, and do not duplicate problem 4 or 8. (See instructions above.)

6 A *person* is rowing a boat, with her feet braced. She is doing the part of the stroke that propels the boat, with the ends of the oars in the water (not the part where the oars are out of the water). (See instructions above.)

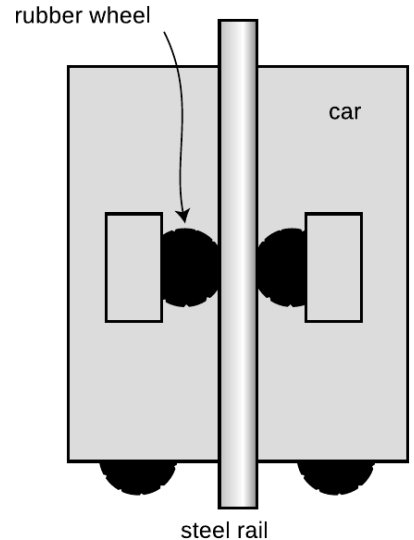
7 A *farmer* is in a stall with a cow when the cow decides to press him against the wall, pinning him with his feet off the ground. Analyze the forces in which the farmer participates. (See instructions above.)

8 A propeller *plane* is cruising east at constant speed and altitude. (See instructions above.)



Problem 8.

9 Today's tallest buildings are really not that much taller than the tallest buildings of the 1940's. One big problem with making an even taller skyscraper is that every elevator needs its own shaft running the whole height of the building. So many elevators are needed to serve the building's thousands of occupants that the elevator shafts start taking up too much of the space within the building. An alternative is to have elevators that can move both horizontally and vertically: with such a design, many elevator cars can share a few shafts, and they don't get in each other's way too much because they can detour around each other. In this design, it becomes impossible to hang the cars from cables, so they would instead have to ride on rails which they grab onto with wheels. Friction would keep them from slipping. The figure shows such a frictional elevator in its vertical travel mode. (The wheels on the bottom are for when it needs to switch to horizontal motion.)

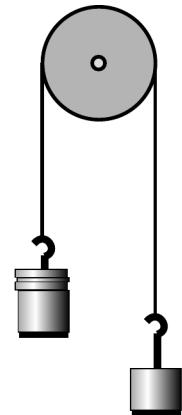


Problem 9.

(a) If the coefficient of static friction between rubber and steel is μ_s , and the maximum mass of the car plus its passengers is M , how much force must there be pressing each wheel against the rail in order to keep the car from slipping? (Assume the car is not accelerating.) \checkmark

(b) Show that your result has physically reasonable behavior with respect to μ_s . In other words, if there was less friction, would the wheels need to be pressed more firmly or less firmly? Does your equation behave that way?

10 Unequal masses M and m are suspended from a pulley as shown in the figure.



Problem 10.

(a) Analyze the forces in which mass m participates, using a table in the format shown in section 5.3. [The forces in which the other mass participates will of course be similar, but not numerically the same.]

(b) Find the magnitude of the accelerations of the two masses. [Hints: (1) Pick a coordinate system, and use positive and negative signs consistently to indicate the directions of the forces and accelerations. (2) The two accelerations of the two masses have to be equal in magnitude but of opposite signs, since one side eats up rope at the same rate at which the other side pays it out. (3) You need to apply Newton's second law twice, once to each mass, and then solve the two equations for the unknowns: the acceleration, a , and the tension in the rope, T .] \checkmark

(c) Many people expect that in the special case of $M = m$, the two masses will naturally settle down to an equilibrium position side by side. Based on your answer from part b, is this correct?

(d) Find the tension in the rope, T . \checkmark

(e) Interpret your equation from part d in the special case where one of the masses is zero. Here "interpret" means to figure out what happens mathematically, figure out what should happen physically, and

connect the two.

11 A tugboat of mass m pulls a ship of mass M , accelerating it. The speeds are low enough that you can ignore fluid friction acting on their hulls, although there will of course need to be fluid friction acting on the tug's propellers.

(a) Analyze the forces in which the tugboat participates, using a table in the format shown in section 5.3. Don't worry about vertical forces.

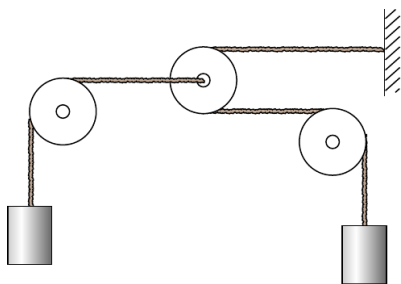
(b) Do the same for the ship.

(c) If the force acting on the tug's propeller is F , what is the tension, T , in the cable connecting the two ships? [Hint: Write down two equations, one for Newton's second law applied to each object. Solve these for the two unknowns T and a .] \checkmark

(d) Interpret your answer in the special cases of $M = 0$ and $M = \infty$.

12 Someone tells you she knows of a certain type of Central American earthworm whose skin, when rubbed on polished diamond, has $\mu_k > \mu_s$. Why is this not just empirically unlikely but logically suspect?

13 In the system shown in the figure, the pulleys on the left and right are fixed, but the pulley in the center can move to the left or right. The two masses are identical. Show that the mass on the left will have an upward acceleration equal to $g/5$. Assume all the ropes and pulleys are massless and frictionless.

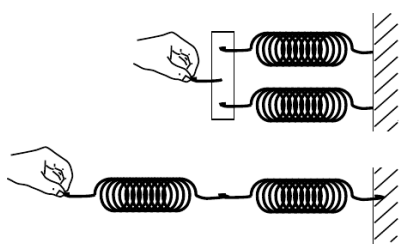


Problem 13.

14 The figure shows two different ways of combining a pair of identical springs, each with spring constant k . We refer to the top setup as parallel, and the bottom one as a series arrangement.

(a) For the parallel arrangement, analyze the forces acting on the connector piece on the left, and then use this analysis to determine the equivalent spring constant of the whole setup. Explain whether the combined spring constant should be interpreted as being stiffer or less stiff.

(b) For the series arrangement, analyze the forces acting on each spring and figure out the same things. \triangleright Solution, p. 552



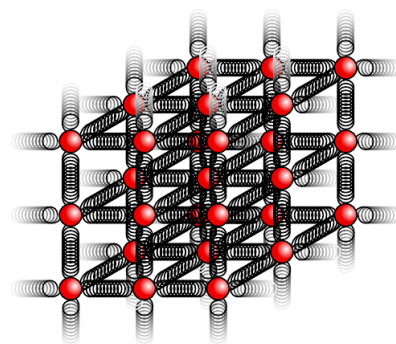
Problem 14.

15 Generalize the results of problem 14 to the case where the two spring constants are unequal.

16 (a) Using the solution of problem 14, which is given in the back of the book, predict how the spring constant of a fiber will depend on its length and cross-sectional area.

(b) The constant of proportionality is called the Young's modulus, E , and typical values of the Young's modulus are about 10^{10} to 10^{11} . What units would the Young's modulus have in the SI (meter-kilogram-second) system? \triangleright Solution, p. 552

17 This problem depends on the results of problems 14 and 16, whose solutions are in the back of the book. When atoms form chemical bonds, it makes sense to talk about the spring constant of the bond as a measure of how “stiff” it is. Of course, there aren’t really little springs — this is just a mechanical model. The purpose of this problem is to estimate the spring constant, k , for a single bond in a typical piece of solid matter. Suppose we have a fiber, like a hair or a piece of fishing line, and imagine for simplicity that it is made of atoms of a single element stacked in a cubical manner, as shown in the figure, with a center-to-center spacing b . A typical value for b would be about 10^{-10} m.



Problem 17.

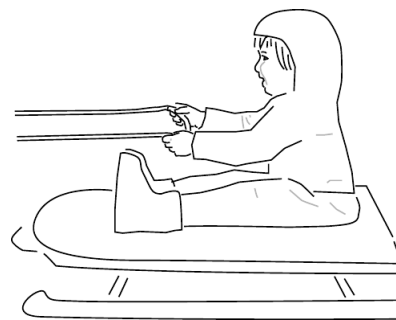
(a) Find an equation for k in terms of b , and in terms of the Young’s modulus, E , defined in problem 16 and its solution.

(b) Estimate k using the numerical data given in problem 16.

(c) Suppose you could grab one of the atoms in a diatomic molecule like H_2 or O_2 , and let the other atom hang vertically below it. Does the bond stretch by any appreciable fraction due to gravity?

18 In each case, identify the force that causes the acceleration, and give its Newton’s-third-law partner. Describe the effect of the partner force. (a) A swimmer speeds up. (b) A golfer hits the ball off of the tee. (c) An archer fires an arrow. (d) A locomotive slows down.
 ▷ Solution, p. 552

19 Ginny has a plan. She is going to ride her sled while her dog Foo pulls her, and she holds on to his leash. However, Ginny hasn’t taken physics, so there may be a problem: she may slide right off the sled when Foo starts pulling.



Problem 19.

(a) Analyze all the forces in which Ginny participates, making a table as in section 5.3.

(b) Analyze all the forces in which the sled participates.

(c) The sled has mass m , and Ginny has mass M . The coefficient of static friction between the sled and the snow is μ_1 , and μ_2 is the corresponding quantity for static friction between the sled and her snow pants. Ginny must have a certain minimum mass so that she will not slip off the sled. Find this in terms of the other three variables. ✓

(d) Interpreting your equation from part c, under what conditions will there be no physically realistic solution for M ? Discuss what this means physically.

20 Example 2 on page 157 involves a person pushing a box up a hill. The incorrect answer describes three forces. For each of these three forces, give the force that it is related to by Newton’s third law, and state the type of force.
 ▷ Solution, p. 553

21 Example 10 on page 176 describes a force-doubling setup involving a pulley. Make up a more complicated arrangement, using two pulleys, that would multiply the force by four. The basic idea is to take the output of one force doubler and feed it into the input

of a second one.

22 Pick up a heavy object such as a backpack or a chair, and stand on a bathroom scale. Shake the object up and down. What do you observe? Interpret your observations in terms of Newton's third law.

23 A cop investigating the scene of an accident measures the length L of a car's skid marks in order to find out its speed v at the beginning of the skid. Express v in terms of L and any other relevant variables. ✓

24 The following reasoning leads to an apparent paradox; explain what's wrong with the logic. A baseball player hits a ball. The ball and the bat spend a fraction of a second in contact. During that time they're moving together, so their accelerations must be equal. Newton's third law says that their forces on each other are also equal. But $a = F/m$, so how can this be, since their masses are unequal? (Note that the paradox isn't resolved by considering the force of the batter's hands on the bat. Not only is this force very small compared to the ball-bat force, but the batter could have just thrown the bat at the ball.)

25 This problem has been deleted.

26 (a) Compare the mass of a one-liter water bottle on earth, on the moon, and in interstellar space. ▷ Solution, p. 553
(b) Do the same for its weight.

27 An ice skater builds up some speed, and then coasts across the ice passively in a straight line. (a) Analyze the forces, using a table in the format shown in section 5.3.

(b) If his initial speed is v , and the coefficient of kinetic friction is μ_k , find the maximum theoretical distance he can glide before coming to a stop. Ignore air resistance. ✓

(c) Show that your answer to part b has the right units.

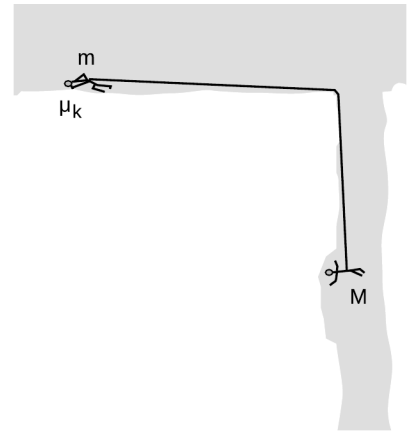
(d) Show that your answer to part b depends on the variables in a way that makes sense physically.

(e) Evaluate your answer numerically for $\mu_k = 0.0046$, and a world-record speed of 14.58 m/s. (The coefficient of friction was measured by De Koning et al., using special skates worn by real speed skaters.) ✓

(f) Comment on whether your answer in part e seems realistic. If it doesn't, suggest possible reasons why.

28 Mountain climbers with masses m and M are roped together while crossing a horizontal glacier when a vertical crevasse opens up under the climber with mass M . The climber with mass m drops down on the snow and tries to stop by digging into the snow with the pick of an ice ax. Alas, this story does not have a happy ending, because this doesn't provide enough friction to stop. Both m and M continue accelerating, with M dropping down into the crevasse and m being dragged across the snow, slowed only by the kinetic friction with coefficient μ_k acting between the ax and the snow. There is no significant friction between the rope and the lip of the crevasse.

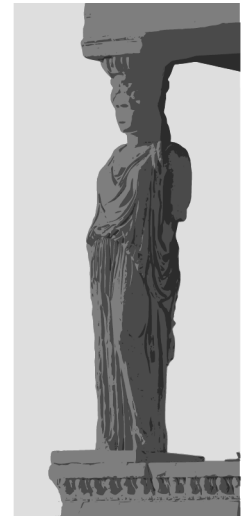
- (a) Find the acceleration a . ✓
- (b) Check the units of your result.
- (c) Check the dependence of your equation on the variables. That means that for each variable, you should determine what its effect on a should be physically, and then what your answer from part a says its effect would be mathematically.



Problem 28.

29 The figure shows a column in the shape of a woman, holding up the roof of part of the Parthenon. Analyze the forces in which she participates, using a table in the format shown in section 5.3.

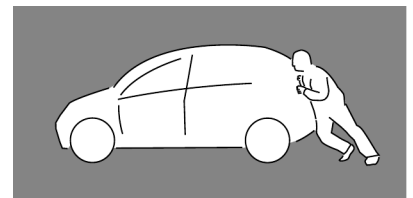
▷ Solution, p. 553



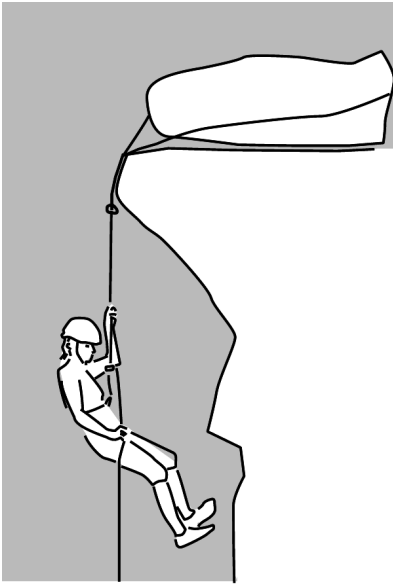
Problem 29.

30 Problem 15, p. 150, which has a solution in the back of the book, was an analysis of the forces acting on a rock climber being lowered back down on the rope. Expand that analysis into a table in the format shown in section 5.3, which includes the types of the forces and their Newton's-third-law partners.

31 The figure shows a man trying to push his car out of the mud. (a) Suppose that he isn't able to move the car. Analyze the forces in which the *car* participates, using a table in the format shown in section 5.3. (b) In the situation described above, consider the forces that act on the car, and compare their strengths. (c) The man takes a nap, eats some chocolate, and now feels stronger. Now he is able to move the car, and the car is currently moving at constant speed. Discuss the strengths of the forces at this time, in relation to one another. (d) The man gets tired again. He is still pushing, but the car, although still moving, begins to decelerate. Again, discuss the strengths of the forces in relation to one another.



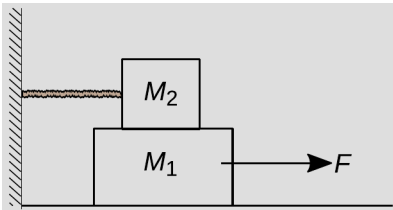
Problem 31.



Problem 32.

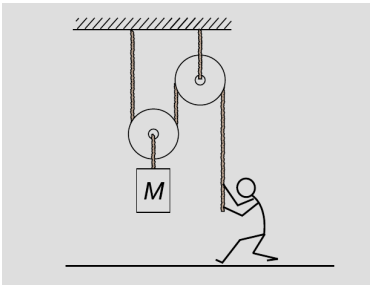
32 The figure shows a mountaineer doing a vertical rappel. Her anchor is a big boulder. The American Mountain Guides Association suggests as a rule of thumb that in this situation, the boulder should be at least as big as a refrigerator, and should be sitting on a surface that is horizontal rather than sloping. The goal of this problem is to estimate what coefficient of static friction μ_s between the boulder and the ledge is required if this setup is to hold the person's body weight. For comparison, reference books meant for civil engineers building walls out of granite blocks state that granite on granite typically has a $\mu_s \approx 0.6$. We expect the result of our calculation to be much less than this, both because a large margin of safety is desired and because the coefficient could be much lower if, for example, the surface was sandy rather than clean. We will assume that there is no friction where the rope goes over the lip of the cliff, although in reality this friction significantly reduces the load on the boulder.

- (a) Let m be the mass of the climber, V the volume of the boulder, ρ its density, and g the strength of the gravitational field. Find the minimum value of μ_s . ✓
- (b) Show that the units of your answer make sense.
- (c) Check that its dependence on the variables makes sense.
- (d) Evaluate your result numerically. The volume of my refrigerator is about 0.7 m^3 , the density of granite is about 2.7 g/cm^3 , and standards bodies use a body mass of 80 kg for testing climbing equipment. ✓



Problem 34.

33 A toy manufacturer is playtesting teflon booties that slip on over your shoes. In the parking lot, giggling engineers find that when they start with an initial speed of 1.2 m/s , they glide for 2.0 m before coming to a stop. What is the coefficient of friction between the asphalt and the booties? ✓ [problem by B. Shotwell]



Problem 35.

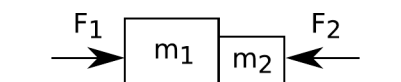
34 Blocks M_1 and M_2 are stacked as shown, with M_2 on top. M_2 is connected by a string to the wall, and M_1 is pulled to the right with a force F big enough to get M_1 to move. The coefficient of kinetic friction has the same value μ_k among all surfaces (i.e., the block-block and ground-block interfaces).

- (a) Analyze the forces in which each block participates, as in section 5.3.
- (b) Determine the tension in the string. ✓
- (c) Find the acceleration of the block of mass M_1 . ✓ [problem by B. Shotwell]

35 A person can pull with a maximum force F . What is the maximum mass that the person can lift with the pulley setup shown in the figure? ✓ [problem by B. Shotwell]

36 Blocks of mass m_1 and m_2 rest, as shown in the figure, on a frictionless plane, and are squeezed by forces of magnitude F_1 and F_2 .

- (a) Find the force f that acts between the two blocks. ✓
- (b) Check that your answer makes sense in the symmetric case where $F_1 = F_2$ and $m_1 = m_2$.
- (c) Find the conditions under which your answer to part a gives $f = 0$, and check that it makes sense.



Problem 36.

Motion in Three Dimensions



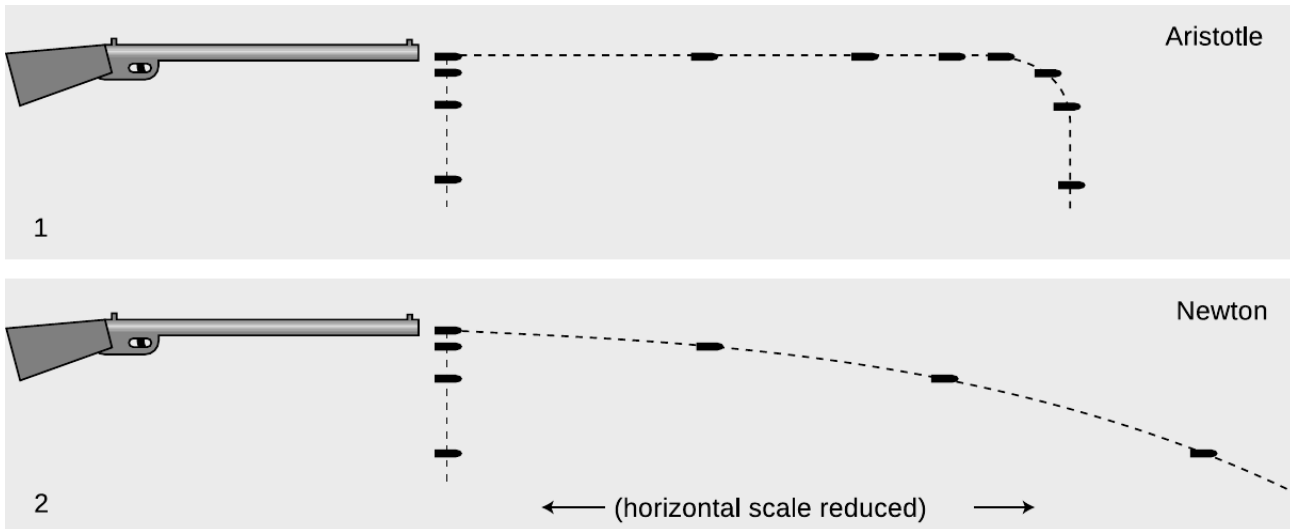
Chapter 6

Newton's Laws in Three Dimensions

6.1 Forces have no perpendicular effects

Suppose you could shoot a rifle and arrange for a second bullet to be dropped from the same height at the exact moment when the first left the barrel. Which would hit the ground first? Nearly everyone expects that the dropped bullet will reach the dirt first,

and Aristotle would have agreed. Aristotle would have described it like this. The shot bullet receives some forced motion from the gun. It travels forward for a split second, slowing down rapidly because there is no longer any force to make it continue in motion. Once it is done with its forced motion, it changes to natural motion, i.e. falling straight down. While the shot bullet is slowing down, the dropped bullet gets on with the business of falling, so according to Aristotle it will hit the ground first.



a / A bullet is shot from a gun, and another bullet is simultaneously dropped from the same height. 1. Aristotelian physics says that the horizontal motion of the shot bullet delays the onset of falling, so the dropped bullet hits the ground first. 2. Newtonian physics says the two bullets have the same vertical motion, regardless of their different horizontal motions.

Luckily, nature isn't as complicated as Aristotle thought! To convince yourself that Aristotle's ideas were wrong and needlessly complex, stand up now and try this experiment. Take your keys out of your pocket, and begin walking briskly forward. Without speeding up or slowing down, release your keys and let them fall while you continue walking at the same pace.

You have found that your keys hit the ground right next to your feet. Their horizontal motion never slowed down at all, and the whole time they were dropping, they were right next to you. The horizontal motion and the vertical motion happen at the same time, and they are independent of each other. Your experiment proves that the horizontal motion is unaffected by the vertical motion, but it's also true that the vertical motion is not changed in any way by the horizontal motion. The keys take exactly the same amount of time to get to the ground as they would have if you simply dropped them, and the same is true of the bullets: both bullets hit the ground

simultaneously.

These have been our first examples of motion in more than one dimension, and they illustrate the most important new idea that is required to understand the three-dimensional generalization of Newtonian physics:

Forces have no perpendicular effects.

When a force acts on an object, it has no effect on the part of the object's motion that is perpendicular to the force.

In the examples above, the vertical force of gravity had no effect on the horizontal motions of the objects. These were examples of projectile motion, which interested people like Galileo because of its military applications. The principle is more general than that, however. For instance, if a rolling ball is initially heading straight for a wall, but a steady wind begins blowing from the side, the ball does not take any longer to get to the wall. In the case of projectile motion, the force involved is gravity, so we can say more specifically that the vertical acceleration is 9.8 m/s^2 , regardless of the horizontal motion.

self-check A

In the example of the ball being blown sideways, why doesn't the ball take longer to get there, since it has to travel a greater distance? ▷

Answer, p. 566

Relationship to relative motion

These concepts are directly related to the idea that motion is relative. Galileo's opponents argued that the earth could not possibly be rotating as he claimed, because then if you jumped straight up in the air you wouldn't be able to come down in the same place. Their argument was based on their incorrect Aristotelian assumption that once the force of gravity began to act on you and bring you back down, your horizontal motion would stop. In the correct Newtonian theory, the earth's downward gravitational force is acting before, during, and after your jump, but has no effect on your motion in the perpendicular (horizontal) direction.

If Aristotle had been correct, then we would have a handy way to determine absolute motion and absolute rest: jump straight up in the air, and if you land back where you started, the surface from which you jumped must have been in a state of rest. In reality, this test gives the same result as long as the surface under you is an inertial frame. If you try this in a jet plane, you land back on the same spot on the deck from which you started, regardless of whether the plane is flying at 500 miles per hour or parked on the runway. The method would in fact only be good for detecting whether the

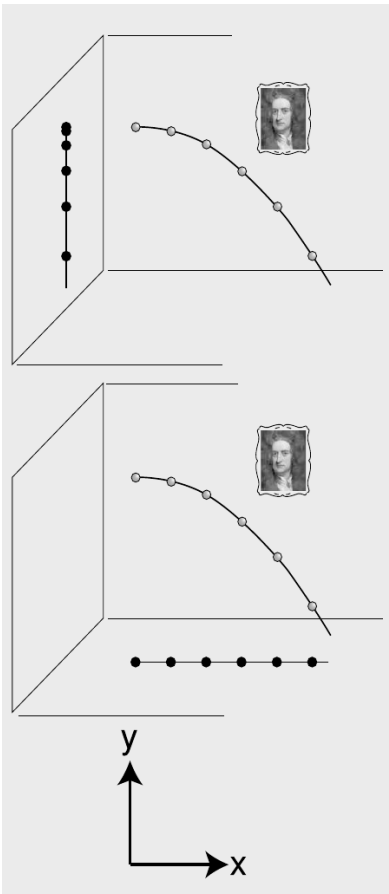
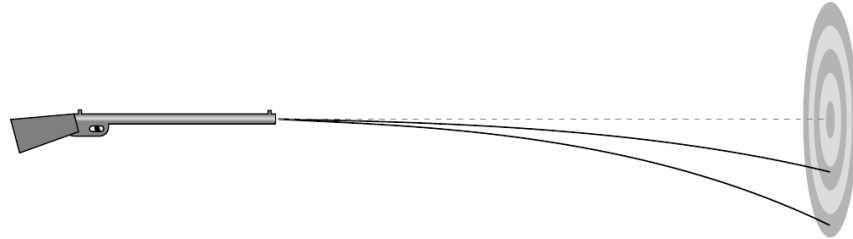
plane was accelerating.

Discussion questions

A The following is an incorrect explanation of a fact about target shooting:

“Shooting a high-powered rifle with a high muzzle velocity is different from shooting a less powerful gun. With a less powerful gun, you have to aim quite a bit above your target, but with a more powerful one you don’t have to aim so high because the bullet doesn’t drop as fast.”

Explain why it’s incorrect. What is the correct explanation?



c / The shadow on the wall shows the ball’s y motion, the shadow on the floor its x motion.

B You have thrown a rock, and it is flying through the air in an arc. If the earth’s gravitational force on it is always straight down, why doesn’t it just go straight down once it leaves your hand?

C Consider the example of the bullet that is dropped at the same moment another bullet is fired from a gun. What would the motion of the two bullets look like to a jet pilot flying alongside in the same direction as the shot bullet and at the same horizontal speed?

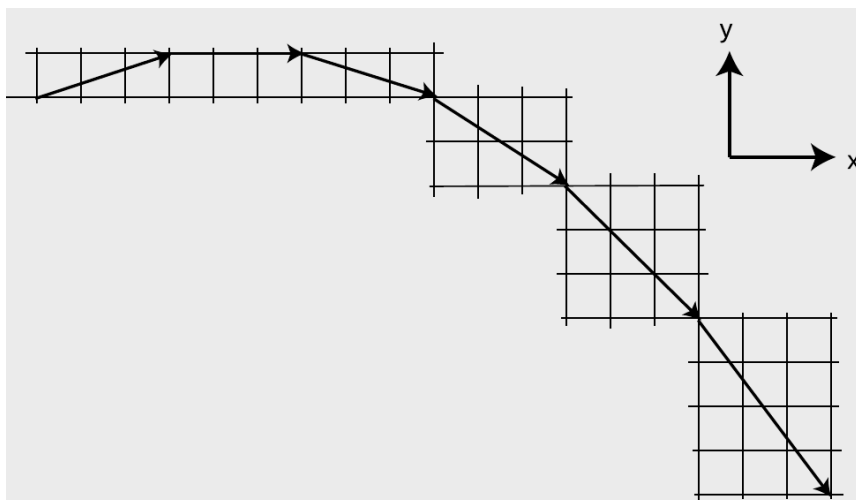
6.2 Coordinates and components

'Cause we're all
Bold as love,
Just ask the axis.

Jimi Hendrix

How do we convert these ideas into mathematics? Figure b shows a good way of connecting the intuitive ideas to the numbers. In one dimension, we impose a number line with an x coordinate on a certain stretch of space. In two dimensions, we imagine a grid of squares which we label with x and y values, as shown in figure b.

But of course motion doesn’t really occur in a series of discrete hops like in chess or checkers. Figure c shows a way of conceptualizing the smooth variation of the x and y coordinates. The ball’s shadow on the wall moves along a line, and we describe its position with a single coordinate, y , its height above the floor. The wall shadow has a constant acceleration of -9.8 m/s^2 . A shadow on the floor, made by a second light source, also moves along a line, and we describe its motion with an x coordinate, measured from the wall.



b / This object experiences a force that pulls it down toward the bottom of the page. In each equal time interval, it moves three units to the right. At the same time, its vertical motion is making a simple pattern of $+1, 0, -1, -2, -3, -4, \dots$ units. Its motion can be described by an x coordinate that has zero acceleration and a y coordinate with constant acceleration. The arrows labeled x and y serve to explain that we are defining increasing x to the right and increasing y as upward.

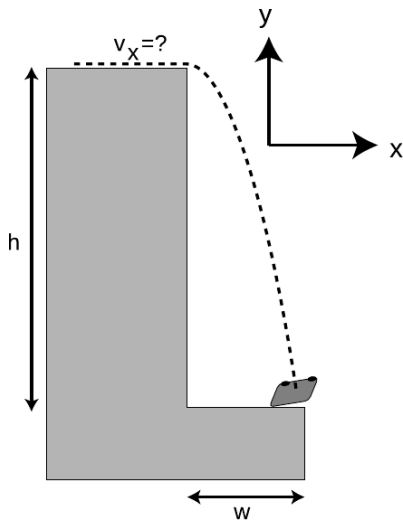
The velocity of the floor shadow is referred to as the x component of the velocity, written v_x . Similarly we can notate the acceleration of the floor shadow as a_x . Since v_x is constant, a_x is zero.

Similarly, the velocity of the wall shadow is called v_y , its acceleration a_y . This example has $a_y = -9.8 \text{ m/s}^2$.

Because the earth's gravitational force on the ball is acting along the y axis, we say that the force has a negative y component, F_y , but $F_x = F_z = 0$.

The general idea is that we imagine two observers, each of whom perceives the entire universe as if it was flattened down to a single line. The y -observer, for instance, perceives y , v_y , and a_y , and will infer that there is a force, F_y , acting downward on the ball. That is, a y component means the aspect of a physical phenomenon, such as velocity, acceleration, or force, that is observable to someone who can only see motion along the y axis.

All of this can easily be generalized to three dimensions. In the example above, there could be a z -observer who only sees motion toward or away from the back wall of the room.



d / Example 1.

A car going over a cliff

example 1

▷ The police find a car at a distance $w = 20$ m from the base of a cliff of height $h = 100$ m. How fast was the car going when it went over the edge? Solve the problem symbolically first, then plug in the numbers.

▷ Let's choose y pointing up and x pointing away from the cliff. The car's vertical motion was independent of its horizontal motion, so we know it had a constant vertical acceleration of $a = -g = -9.8 \text{ m/s}^2$. The time it spent in the air is therefore related to the vertical distance it fell by the constant-acceleration equation

$$\Delta y = \frac{1}{2} a_y \Delta t^2,$$

or

$$-h = \frac{1}{2} (-g) \Delta t^2.$$

Solving for Δt gives

$$\Delta t = \sqrt{\frac{2h}{g}}.$$

Since the vertical force had no effect on the car's horizontal motion, it had $a_x = 0$, i.e., constant horizontal velocity. We can apply the constant-velocity equation

$$v_x = \frac{\Delta x}{\Delta t},$$

i.e.,

$$v_x = \frac{w}{\Delta t}.$$

We now substitute for Δt to find

$$v_x = w / \sqrt{\frac{2h}{g}},$$

which simplifies to

$$v_x = w \sqrt{\frac{g}{2h}}.$$

Plugging in numbers, we find that the car's speed when it went over the edge was 4 m/s, or about 10 mi/hr.

Projectiles move along parabolas.

What type of mathematical curve does a projectile follow through space? To find out, we must relate x to y , eliminating t . The reasoning is very similar to that used in the example above. Arbitrarily

choosing $x = y = t = 0$ to be at the top of the arc, we conveniently have $x = \Delta x$, $y = \Delta y$, and $t = \Delta t$, so

$$y = \frac{1}{2}a_y t^2 \quad (a_y < 0)$$

$$x = v_x t$$

We solve the second equation for $t = x/v_x$ and eliminate t in the first equation:

$$y = \frac{1}{2}a_y \left(\frac{x}{v_x}\right)^2.$$

Since everything in this equation is a constant except for x and y , we conclude that y is proportional to the square of x . As you may or may not recall from a math class, $y \propto x^2$ describes a parabola.

▷ Solved problem: A cannon

page 200, problem 5

Discussion question

A At the beginning of this section I represented the motion of a projectile on graph paper, breaking its motion into equal time intervals. Suppose instead that there is no force on the object at all. It obeys Newton's first law and continues without changing its state of motion. What would the corresponding graph-paper diagram look like? If the time interval represented by each arrow was 1 second, how would you relate the graph-paper diagram to the velocity components v_x and v_y ?

B Make up several different coordinate systems oriented in different ways, and describe the a_x and a_y of a falling object in each one.

6.3 Newton's laws in three dimensions

It is now fairly straightforward to extend Newton's laws to three dimensions:

Newton's first law

If all three components of the total force on an object are zero, then it will continue in the same state of motion.

Newton's second law

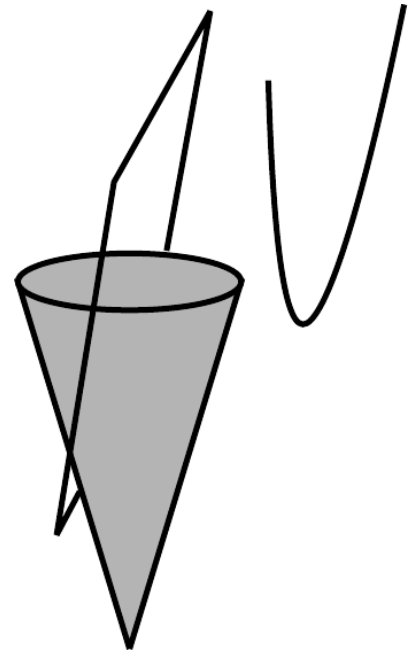
The components of an object's acceleration are predicted by the equations

$$\begin{aligned} a_x &= F_{x,\text{total}}/m, \\ a_y &= F_{y,\text{total}}/m, \quad \text{and} \\ a_z &= F_{z,\text{total}}/m. \end{aligned}$$

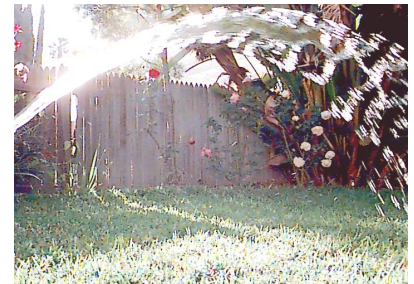
Newton's third law

If two objects A and B interact via forces, then the components of their forces on each other are equal and opposite:

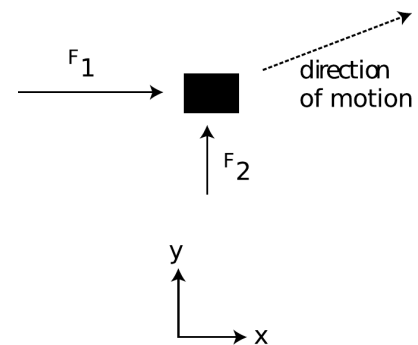
$$\begin{aligned} F_{A \text{ on } B,x} &= -F_{B \text{ on } A,x}, \\ F_{A \text{ on } B,y} &= -F_{B \text{ on } A,y}, \quad \text{and} \\ F_{A \text{ on } B,z} &= -F_{B \text{ on } A,z}. \end{aligned}$$



e / A parabola can be defined as the shape made by cutting a cone parallel to its side. A parabola is also the graph of an equation of the form $y \propto x^2$.



f / Each water droplet follows a parabola. The faster drops' parabolas are bigger.



g / Example 2.

Forces in perpendicular directions on the same object example 2

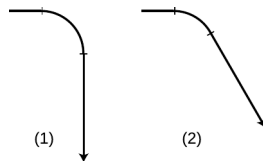
▷ An object is initially at rest. Two constant forces begin acting on it, and continue acting on it for a while. As suggested by the two arrows, the forces are perpendicular, and the rightward force is stronger. What happens?

▷ Aristotle believed, and many students still do, that only one force can “give orders” to an object at one time. They therefore think that the object will begin speeding up and moving in the direction of the stronger force. In fact the object will move along a diagonal. In the example shown in the figure, the object will respond to the large rightward force with a large acceleration component to the right, and the small upward force will give it a small acceleration component upward. The stronger force does not overwhelm the weaker force, or have any effect on the upward motion at all. The force components simply add together:

$$F_{x,total} = F_{1,x} + F_{2,x}$$
$$F_{y,total} = F_{1,y} + F_{2,y}$$

Discussion question

A The figure shows two trajectories, made by splicing together lines and circular arcs, which are unphysical for an object that is only being acted on by gravity. Prove that they are impossible based on Newton’s laws.



Summary

Selected vocabulary

- component the part of a velocity, acceleration, or force that would be perceptible to an observer who could only see the universe projected along a certain one-dimensional axis
- parabola the mathematical curve whose graph has y proportional to x^2

Notation

- x, y, z an object's positions along the $x, y,$ and z axes
- v_x, v_y, v_z the $x, y,$ and z components of an object's velocity; the rates of change of the object's $x, y,$ and z coordinates
- a_x, a_y, a_z the $x, y,$ and z components of an object's acceleration; the rates of change of $v_x, v_y,$ and v_z

Summary

A force does not produce any effect on the motion of an object in a perpendicular direction. The most important application of this principle is that the horizontal motion of a projectile has zero acceleration, while the vertical motion has an acceleration equal to g . That is, an object's horizontal and vertical motions are independent. The arc of a projectile is a parabola.

Motion in three dimensions is measured using three coordinates, $x, y,$ and z . Each of these coordinates has its own corresponding velocity and acceleration. We say that the velocity and acceleration both have $x, y,$ and z components

Newton's second law is readily extended to three dimensions by rewriting it as three equations predicting the three components of the acceleration,

$$a_x = F_{x,total}/m,$$

$$a_y = F_{y,total}/m,$$

$$a_z = F_{z,total}/m,$$

and likewise for the first and third laws.

Problems

Key

- ✓ A computerized answer check is available online.
- ∫ A problem that requires calculus.
- ★ A difficult problem.

1 (a) A ball is thrown straight up with velocity v . Find an equation for the height to which it rises. ✓

(b) Generalize your equation for a ball thrown at an angle θ above horizontal, in which case its initial velocity components are $v_x = v \cos \theta$ and $v_y = v \sin \theta$. ✓

2 At the 2010 Salinas Lettuce Festival Parade, the Lettuce Queen drops her bouquet while riding on a float moving toward the right. Sketch the shape of its trajectory in her frame of reference, and compare with the shape seen by one of her admirers standing on the sidewalk.

3 Two daredevils, Wendy and Bill, go over Niagara Falls. Wendy sits in an inner tube, and lets the 30 km/hr velocity of the river throw her out horizontally over the falls. Bill paddles a kayak, adding an extra 10 km/hr to his velocity. They go over the edge of the falls at the same moment, side by side. Ignore air friction. Explain your reasoning.

(a) Who hits the bottom first?

(b) What is the horizontal component of Wendy's velocity on impact?

(c) What is the horizontal component of Bill's velocity on impact?

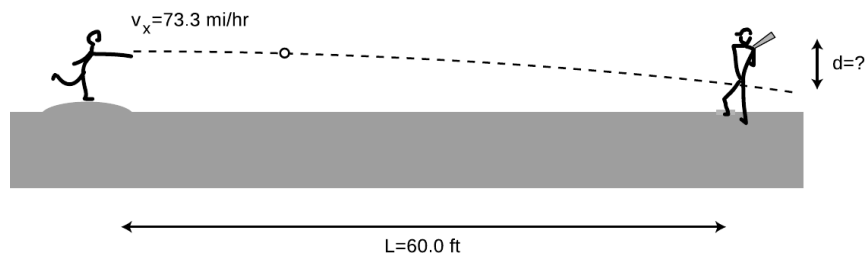
(d) Who is going faster on impact?

4 A baseball pitcher throws a pitch clocked at $v_x = 73.3$ miles/hour. He throws horizontally. By what amount, d , does the ball drop by the time it reaches home plate, $L = 60.0$ feet away?

(a) First find a symbolic answer in terms of L , v_x , and g . ✓

(b) Plug in and find a numerical answer. Express your answer in units of ft. (Note: 1 foot=12 inches, 1 mile=5280 feet, and 1 inch=2.54 cm) ✓

Problem 4.



5 A cannon standing on a flat field fires a cannonball with a muzzle velocity v , at an angle θ above horizontal. The cannonball