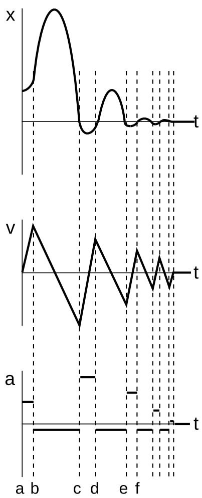
how to draw it more realistically. Since our acceleration graph consists of constant-acceleration segments, the velocity graph must consist of line segments, and the position graph must consist of parabolas. On the x graph, I chose zero to be the height of the center of the ball above the floor when the ball is just lying on the floor. When the ball is touching the floor and compressed, as in interval cd, its center is below this level, so its x is negative.



Page 120, problem 22:
We have $v_f^2 = 2a\Delta x$, so the distance is proportional to the square of the velocity. To get up to half the speed, the ball needs 1/4 the distance, i.e., $L/4$.

Solutions for chapter 4

Page 148, problem 7:
$a = \Delta v/\Delta t$, and also $a = F/m$, so

$$\begin{aligned}\Delta t &= \frac{\Delta v}{a}\\ &= \frac{m\Delta v}{F}\\ &= \frac{(1000\text{ kg})(50\text{ m/s} - 20\text{ m/s})}{3000\text{ N}}\\ &= 10\text{ s}\end{aligned}$$

Page 149, problem 10:
(a) This is a measure of the box's resistance to a change in its state of motion, so it measures the box's mass. The experiment would come out the same in lunar gravity.
(b) This is a measure of how much gravitational force it feels, so it's a measure of weight. In lunar gravity, the box would make a softer sound when it hit.
(c) As in part a, this is a measure of its resistance to a change in its state of motion: its mass. Gravity isn't involved at all.

Page 150, problem 15:
The partner's hands are not touching the climber, so they don't make any force on him. The hands have an indirect effect through the rope, but our concept of force only includes direct effects (section 4.4, p. 141).

The corrected table looks like this:

force of the earth's gravity, ↓
force from the rope, ↑

The student is also wrong to claim that the upward and downward forces are unbalanced. The climber is moving down at constant speed, so his acceleration is zero, and the total force acting on him is zero. The upward and downward forces are of equal strength, and they cancel.

Solutions for chapter 5

Page 182, problem 14:

(a)

top spring's rightward force on connector
...connector's leftward force on top spring
bottom spring's rightward force on connector
...connector's leftward force on bottom spring
hand's leftward force on connector
...connector's rightward force on hand

Looking at the three forces on the connector, we see that the hand's force must be double the force of either spring. The value of $x - x_o$ is the same for both springs and for the arrangement as a whole, so the spring constant must be $2k$. This corresponds to a stiffer spring (more force to produce the same extension).

(b) Forces in which the left spring participates:

hand's leftward force on left spring
...left spring's rightward force on hand
right spring's rightward force on left spring
...left spring's leftward force on right spring

Forces in which the right spring participates:

left spring's leftward force on right spring
...right spring's rightward force on left spring
wall's rightward force on right spring
...right spring's leftward force on wall

Since the left spring isn't accelerating, the total force on it must be zero, so the two forces acting on it must be equal in magnitude. The same applies to the two forces acting on the right spring. The forces between the two springs are connected by Newton's third law, so all eight of these forces must be equal in magnitude. Since the value of $x - x_o$ for the whole setup is double what it is for either spring individually, the spring constant of the whole setup must be $k/2$, which corresponds to a less stiff spring.

Page 182, problem 16:
(a) Spring constants in parallel add, so the spring constant has to be proportional to the cross-sectional area. Two springs in series give half the spring constant, three springs in series give 1/3, and so on, so the spring constant has to be inversely proportional to the length. Summarizing, we have $k \propto A/L$. (b) With the Young's modulus, we have $k = (A/L)E$.The spring constant has units of N/m, so the units of E would have to be N/m^2.

Page 183, problem 18:
(a) The swimmer's acceleration is caused by the water's force on the swimmer, and the swimmer

makes a backward force on the water, which accelerates the water backward. (b) The club's normal force on the ball accelerates the ball, and the ball makes a backward normal force on the club, which decelerates the club. (c) The bowstring's normal force accelerates the arrow, and the arrow also makes a backward normal force on the string. This force on the string causes the string to accelerate less rapidly than it would if the bow's force was the only one acting on it. (d) The tracks' backward frictional force slows the locomotive down. The locomotive's forward frictional force causes the whole planet earth to accelerate by a tiny amount, which is too small to measure because the earth's mass is so great.

Page 183, problem 20:
The person's normal force on the box is paired with the box's normal force on the person. The dirt's frictional force on the box pairs with the box's frictional force on the dirt. The earth's gravitational force on the box matches the box's gravitational force on the earth.

Page 184, problem 26:
(a) A liter of water has a mass of 1.0 kg. The mass is the same in all three locations. Mass indicates how much an object resists a change in its motion. It has nothing to do with gravity. (b) The term "weight" refers to the force of gravity on an object. The bottle's weight on earth is $F_W = mg = 9.8$ N. Its weight on the moon is about one sixth that value, and its weight in interstellar space is zero.

Page 185, problem 29:
First, let's account for every object that's touching her: the floor and the roof. Any time two solid objects are in contact, we expect a normal force, which is the force that keeps them from passing through each other. Normal forces are repulsive, which means here that the roof's force on her head is down (away from itself), and the floor's force on her feet is up (away from itself). There could also be frictional forces, but in this problem there is symmetry between left and right, so it wouldn't make sense for frictional forces to exist here — if they did, there would be no way to decide which way they should point.

In addition to these contact forces, we will have a non-contact force: the earth's gravity.

The physical reasoning above establishes the left-hand column of the table below. Once we've established the left-hand column, the right-hand column can be generated purely by manipulating the words and symbols, without further recourse to physical insight. By Newton's third law, we interchange the two objects and reverse the arrow. The type of the force is the same.

force acting on woman	*force related to it by Newton's third law*
roof's normal force on woman, ↓	woman's normal force on roof, ↑
floor's normal force on woman, ↑	woman's normal force on floor, ↓
planet earth's gravitational force on woman, ↓	woman's gravitational force on earth, ↑

Solutions for chapter 6

Page 200, problem 5:
(a) The easiest strategy is to find the time spent aloft, and then find the range. The vertical motion and the horizontal motion are independent. The vertical motion has acceleration $-g$, and the cannonball spends enough time in the air to reverse its vertical velocity component completely, so we have

$$\begin{aligned} \Delta v_y &= v_{yf} - v_{yo} \\ &= -2v\sin\theta. \end{aligned}$$

The time spent aloft is therefore

$$\begin{aligned}\Delta t &= \Delta v_y / a_y \\ &= 2v \sin\theta / g.\end{aligned}$$

During this time, the horizontal distance traveled is

$$\begin{aligned}R &= v_x \Delta t \\ &= 2v^2 \sin\theta \cos\theta / g.\end{aligned}$$

(b) The range becomes zero at both $\theta = 0$ and at $\theta = 90°$. The $\theta = 0$ case gives zero range because the ball hits the ground as soon as it leaves the mouth of the cannon. A 90-degree angle gives zero range because the cannonball has no horizontal motion.

Solutions for chapter 8

Page 234, problem 8:

We want to find out about the velocity vector v_{BG} of the bullet relative to the ground, so we need to add Annie's velocity relative to the ground v_{AG} to the bullet's velocity vector v_{BA} relative to her. Letting the positive x axis be east and y north, we have

$$\begin{aligned}v_{BA,x} &= (140 \text{ mi/hr}) \cos 45° \\ &= 100 \text{ mi/hr} \\ v_{BA,y} &= (140 \text{ mi/hr}) \sin 45° \\ &= 100 \text{ mi/hr}\end{aligned}$$

and

$$\begin{aligned}v_{AG,x} &= 0 \\ v_{AG,y} &= 30 \text{ mi/hr}.\end{aligned}$$

The bullet's velocity relative to the ground therefore has components

$$\begin{aligned}v_{BG,x} &= 100 \text{ mi/hr} \qquad \text{and} \\ v_{BG,y} &= 130 \text{ mi/hr}.\end{aligned}$$

Its speed on impact with the animal is the magnitude of this vector

$$\begin{aligned}|v_{BG}| &= \sqrt{(100 \text{ mi/hr})^2 + (130 \text{ mi/hr})^2} \\ &= 160 \text{ mi/hr}\end{aligned}$$

(rounded off to 2 significant figures).

Page 234, problem 9:

Since its velocity vector is constant, it has zero acceleration, and the sum of the force vectors acting on it must be zero. There are three forces acting on the plane: thrust, lift, and gravity. We are given the first two, and if we can find the third we can infer its mass. The sum of the y components of the forces is zero, so

$$\begin{aligned}0 &= F_{thrust,y} + F_{lift,y} + F_{W,y} \\ &= |\mathbf{F}_{thrust}| \sin\theta + |\mathbf{F}_{lift}| \cos\theta - mg.\end{aligned}$$

The mass is

$$
\begin{aligned}
m &= (|\mathbf{F}_{thrust}|\sin\theta + |\mathbf{F}_{lift}|\cos\theta)/g \\
&= 7.0\times 10^4\ \text{kg}
\end{aligned}
$$

Page 234, problem 10:
(a) Since the wagon has no acceleration, the total forces in both the x and y directions must be zero. There are three forces acting on the wagon: $\mathbf{F}_T$, $\mathbf{F}_W$, and the normal force from the ground, $\mathbf{F}_N$. If we pick a coordinate system with x being horizontal and y vertical, then the angles of these forces measured counterclockwise from the x axis are $90°-\phi$, $270°$, and $90°+\theta$, respectively. We have

$$
\begin{aligned}
F_{x,total} &= |\mathbf{F}_T|\cos(90°-\phi) + |\mathbf{F}_W|\cos(270°) + |\mathbf{F}_N|\cos(90°+\theta) \\
F_{y,total} &= |\mathbf{F}_T|\sin(90°-\phi) + |\mathbf{F}_W|\sin(270°) + |\mathbf{F}_N|\sin(90°+\theta),
\end{aligned}
$$

which simplifies to

$$
\begin{aligned}
0 &= |\mathbf{F}_T|\sin\phi - |\mathbf{F}_N|\sin\theta \\
0 &= |\mathbf{F}_T|\cos\phi - |\mathbf{F}_W| + |\mathbf{F}_N|\cos\theta.
\end{aligned}
$$

The normal force is a quantity that we are not given and do not wish to find, so we should choose it to eliminate. Solving the first equation for $|\mathbf{F}_N| = (\sin\phi/\sin\theta)|\mathbf{F}_T|$, we eliminate $|\mathbf{F}_N|$ from the second equation,

$$
0 = |\mathbf{F}_T|\cos\phi - |\mathbf{F}_W| + |\mathbf{F}_T|\sin\phi\cos\theta/\sin\theta
$$

and solve for $|\mathbf{F}_T|$, finding

$$
|\mathbf{F}_T| = \frac{|\mathbf{F}_W|}{\cos\phi + \sin\phi\cos\theta/\sin\theta}.
$$

Multiplying both the top and the bottom of the fraction by $\sin\theta$, and using the trig identity for $\sin(\theta+\phi)$ gives the desired result,

$$
|\mathbf{F}_T| = \frac{\sin\theta}{\sin(\theta+\phi)}|\mathbf{F}_W|.
$$

(b) The case of $\phi = 0$, i.e., pulling straight up on the wagon, results in $|\mathbf{F}_T| = |\mathbf{F}_W|$: we simply support the wagon and it glides up the slope like a chair-lift on a ski slope. In the case of $\phi = 180° - \theta$, $|\mathbf{F}_T|$ becomes infinite. Physically this is because we are pulling directly into the ground, so no amount of force will suffice.

Page 235, problem 11:
(a) If there was no friction, the angle of repose would be zero, so the coefficient of static friction, μ_s, will definitely matter. We also make up symbols θ, m and g for the angle of the slope, the mass of the object, and the acceleration of gravity. The forces form a triangle just like the one in example 5 on p. 225, but instead of a force applied by an external object, we have static friction, which is less than $\mu_s|\mathbf{F}_N|$. As in that example, $|\mathbf{F}_s| = mg\sin\theta$, and $|\mathbf{F}_s| < \mu_s|\mathbf{F}_N|$, so

$$
mg\sin\theta < \mu_s|\mathbf{F}_N|.
$$

From the same triangle, we have $|\mathbf{F}_N| = mg\cos\theta$, so

$$
mg\sin\theta < \mu_s mg\cos\theta.
$$

Rearranging,

$$\theta < \tan^{-1} \mu_s.$$

(b) Both m and g canceled out, so the angle of repose would be the same on an asteroid.

Page 238, problem 25:
(a) There is no theoretical limit on how much normal force F_N the climber can make on the walls with each foot, so the frictional force can be made arbitrarily large. This means that with any $\mu > 0$, we can always get the vertical forces to cancel. The theoretical minimum value of μ will be determined by the need for the horizontal forces to cancel, so that the climber doesn't pop out of the corner like a watermelon seed squeezed between two fingertips. The horizontal component of the frictional force is always less than the magnitude of the frictional force, which is turn is less than μF_N. To find the minimum value of μ, we set the static frictional force equal to μF_N.

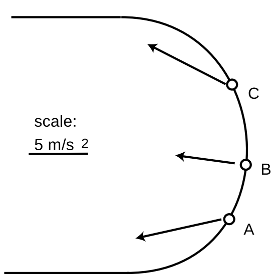
Let the x axis be along the plane that bisects the two walls, let y be the horizontal direction perpendicular to x, and let z be vertical. Then cancellation of the forces in the z direction is not the limiting factor, for the reasons described above, and cancellation in y is guaranteed by symmetry, so the only issue is the cancellation of the x forces. We have $2F_s \cos(\theta/2) - 2F_N \sin(\theta/2) = 0$. Combining this with $F_s = \mu F_N$ results in $\mu = \tan(\theta/2)$.

(b) For $\theta = 0$, μ is very close to zero. That is, we can always theoretically stay stuck between two parallel walls, simply by pressing hard enough, even if the walls are made of ice or polished marble with a coating of WD-40. As θ gets close to 180°, μ blows up to infinity. We need at least some dihedral angle to do this technique, because otherwise we're facing a flat wall, and there is nothing to cancel the wall's normal force on our feet.

(c) The result is 99.0°, i.e., just a little wider than a right angle.

Solutions for chapter 9

Page 255, problem 5:
Each cyclist has a radial acceleration of $v^2/r = 5\ \text{m/s}^2$. The tangential accelerations of cyclists A and B are $375\ \text{N}/75\ \text{kg} = 5\ \text{m/s}^2$.



Page 256, problem 6:
(a) The inward normal force must be sufficient to produce circular motion, so

$$|\mathbf{F}_N| = mv^2/r.$$

We are searching for the minimum speed, which is the speed at which the static friction force is just barely able to cancel out the downward gravitational force. The maximum force of static friction is

$$|\mathbf{F}_s| = \mu_s |\mathbf{F}_N|,$$

and this cancels the gravitational force, so

$$|\mathbf{F}_s| = mg.$$

Solving these three equations for v gives

$$v = \sqrt{\frac{gr}{\mu_s}}.$$

(b) Greater by a factor of $\sqrt{3}$.

Page 256, problem 7:
The inward force must be supplied by the inward component of the normal force,

$$|\mathbf{F}_N| \sin\theta = mv^2/r.$$

The upward component of the normal force must cancel the downward force of gravity,

$$|\mathbf{F}_N| \cos\theta = mg.$$

Eliminating $|\mathbf{F}_N|$ and solving for θ, we find

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right).$$

Solutions for chapter 10

Page 282, problem 10:
Newton's law of gravity is $F = GMm/r^2$. Both G and the astronaut's mass m are the same in the two situations, so $F \propto Mr^{-2}$. In terms of ratios, this is

$$\frac{F_c}{F_e} = \frac{M_c}{M_e}\left(\frac{r_c}{r_e}\right)^{-2}.$$

The result is 11 N.

Page 283, problem 11:
Newton's law of gravity says $F = Gm_1m_2/r^2$, and Newton's second law says $F = m_2a$, so $Gm_1m_2/r^2 = m_2a$. Since m_2 cancels, a is independent of m_2.

Page 283, problem 12:
Newton's second law gives

$$F = m_Da_D,$$

where F is Ida's force on Dactyl. Using Newton's universal law of gravity, F= Gm_Im_D/r^2,and the equation $a = v^2/r$ for circular motion, we find

$$Gm_Im_D/r^2 = m_Dv^2/r.$$

Dactyl's mass cancels out, giving

$$Gm_I/r^2 = v^2/r.$$

Dactyl's velocity equals the circumference of its orbit divided by the time for one orbit: $v = 2\pi r/T$. Inserting this in the above equation and solving for m_I, we find

$$m_I = \frac{4\pi^2r^3}{GT^2},$$

so Ida's density is

$$\begin{aligned}\rho &= m_I/V \\ &= \frac{4\pi^2 r^3}{GVT^2}.\end{aligned}$$

Page 283, problem 15:
Newton's law of gravity depends on the inverse square of the distance, so if the two planets' masses had been equal, then the factor of $0.83/0.059 = 14$ in distance would have caused the force on planet c to be $14^2 = 2.0 \times 10^2$ times weaker. However, planet c's mass is 3.0 times greater, so the force on it is only smaller by a factor of $2.0 \times 10^2/3.0 = 65$.

Page 284, problem 16:
The reasoning is reminiscent of section 10.2. From Newton's second law we have

$$F = ma = mv^2/r = m(2\pi r/T)^2/r = 4\pi^2 mr/T^2,$$

and Newton's law of gravity gives $F = GMm/r^2$, where M is the mass of the earth. Setting these expressions equal to each other, we have

$$4\pi^2 mr/T^2 = GMm/r^2,$$

which gives

$$\begin{aligned}r &= \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \\ &= 4.22 \times 10^4 \text{ km}.\end{aligned}$$

This is the distance from the center of the earth, so to find the altitude, we need to subtract the radius of the earth. The altitude is 3.58×10^4 km.

Page 284, problem 17:
Any fractional change in r results in double that amount of fractional change in $1/r^2$. For example, raising r by 1% causes $1/r^2$ to go down by very nearly 2%. A 27-day orbit is 1/13.5 of a year, so the fractional change in $1/r^2$ is

$$2 \times \frac{(4/13.5) \text{ cm}}{3.84 \times 10^5 \text{ km}} \times \frac{1 \text{ km}}{10^5 \text{ cm}} = 1.5 \times 10^{-11}$$

Page 285, problem 19:
(a) The asteroid's mass depends on the cube of its radius, and for a given mass the surface gravity depends on r^{-2}. The result is that surface gravity is directly proportional to radius. Half the gravity means half the radius, or one eighth the mass. (b) To agree with a, Earth's mass would have to be 1/8 Jupiter's. We assumed spherical shapes and equal density. Both planets are at least roughly spherical, so the only way out of the contradiction is if Jupiter's density is significantly less than Earth's.

Solutions for chapter 11

Page 311, problem 7:
A force is an interaction between two objects, so while the bullet is in the air, there is no force. There is only a force while the bullet is in contact with the book. There is energy the whole

time, and the total amount doesn't change. The bullet has some kinetic energy, and transfers some of it to the book as heat, sound, and the energy required to tear a hole through the book.

Page 311, problem 8:
(a) The energy stored in the gasoline is being changed into heat via frictional heating, and also probably into sound and into energy of water waves. Note that the kinetic energy of the propeller and the boat are not changing, so they are not involved in the energy transformation. (b) The crusing speed would be greater by a factor of the cube root of 2, or about a 26% increase.

Page 311, problem 9:
We don't have actual masses and velocities to plug in to the equation, but that's OK. We just have to reason in terms of ratios and proportionalities. Kinetic energy is proportional to mass and to the square of velocity, so B's kinetic energy equals

$$(13.4\ \text{J})(3.77)/(2.34)^2 = 9.23\ \text{J}$$

Page 311, problem 11:
Room temperature is about 20°C. The fraction of the energy that actually goes into heating the water is

$$\frac{(250\ \text{g})/(0.24\ \text{g}\cdot{}^\circ\text{C/J}) \times (100^\circ\text{C} - 20^\circ\text{C})}{(1.25 \times 10^3\ \text{J/s})\,(126\ \text{s})} = 0.53$$

So roughly half of the energy is wasted. The wasted energy might be in several forms: heating of the cup, heating of the oven itself, or leakage of microwaves from the oven.

Solutions for chapter 12

Page 327, problem 5:

$$\begin{aligned}
E_{total,i} &= E_{total,f} \\
PE_i + \text{heat}_i &= PE_f + KE_f + \text{heat}_f \\
\frac{1}{2}mv^2 &= PE_i - PE_f + \text{heat}_i - \text{heat}_f \\
&= -\Delta PE - \Delta\text{heat} \\
v &= \sqrt{2\left(\frac{-\Delta PE - \Delta\text{heat}}{m}\right)} \\
&= 6.4\ \text{m/s}
\end{aligned}$$

Page 328, problem 7:
Let θ be the angle by which he has progressed around the pipe. Conservation of energy gives

$$\begin{aligned}
E_{total,i} &= E_{total,f} \\
PE_i &= PE_f + KE_f \\
0 &= \Delta PE + KE_f
\end{aligned}$$

$$0 = mgr(\cos\theta - 1) + \frac{1}{2}mv^2.$$

While he is still in contact with the pipe, the radial component of his acceleration is

$$a_r = \frac{v^2}{r},$$

and making use of the previous equation we find

$$a_r = 2g(1 - \cos\theta).$$

There are two forces on him, a normal force from the pipe and a downward gravitational force from the earth. At the moment when he loses contact with the pipe, the normal force is zero, so the radial component, $mg\cos\theta$, of the gravitational force must equal ma_r,

$$mg\cos\theta = 2mg(1 - \cos\theta),$$

which gives

$$\cos\theta = \frac{2}{3}.$$

The amount by which he has dropped is $r(1 - \cos\theta)$, which equals $r/3$ at this moment.

Page 328, problem 9:
(a) Example: As one child goes up on one side of a see-saw, another child on the other side comes down. (b) Example: A pool ball hits another pool ball, and transfers some KE.

Page 328, problem 11:
Suppose the river is 1 m deep, 100 m wide, and flows at a speed of 10 m/s, and that the falls are 100 m tall. In 1 second, the volume of water flowing over the falls is 10^3 m^3, with a mass of 10^6 kg. The potential energy released in one second is $(10^6\text{ kg})(g)(100\text{ m}) = 10^9$ J, so the power is 10^9 W. A typical household might have 10 hundred-watt applicances turned on at any given time, so it consumes about 10^3 watts on the average. The plant could supply a about million households with electricity.

Solutions for chapter 13

Page 357, problem 18:
No. Work describes how energy was transferred by some process. It isn't a measurable property of a system.

Solutions for chapter 14

Page 389, problem 8:
Let m be the mass of the little puck and $M = 2.3m$ be the mass of the big one. All we need to do is find the direction of the total momentum vector before the collision, because the total momentum vector is the same after the collision. Given the two components of the momentum vector $p_x = Mv$ and $p_y = mv$, the direction of the vector is $\tan^{-1}(p_y/p_x) = 23°$ counterclockwise from the big puck's original direction of motion.

Page 390, problem 11:
Momentum is a vector. The total momentum of the molecules is always zero, since the momenta in different directions cancal out on the average. Cooling changes individual molecular momenta, but not the total.

Page 390, problem 14:
By conservation of momentum, the total momenta of the pieces after the explosion is the same as the momentum of the firework before the explosion. However, there is no law of conservation of kinetic energy, only a law of conservation of energy. The chemical energy in the gunpowder is converted into heat and kinetic energy when it explodes. All we can say about the kinetic energy of the pieces is that their total is greater than the kinetic energy before the explosion.

Page 390, problem 15:
(a) Particle i had velocity v_i in the center-of-mass frame, and has velocity $v_i + u$ in the new frame. The total kinetic energy is

$$\frac{1}{2}m_1\left(\mathbf{v}_1+\mathbf{u}\right)^2+\ldots,$$

where "..." indicates that the sum continues for all the particles. Rewriting this in terms of the vector dot product, we have

$$\frac{1}{2}m_1\left(\mathbf{v}_1+\mathbf{u}\right)\cdot\left(\mathbf{v}_1+\mathbf{u}\right)+\ldots=\frac{1}{2}m_1\left(\mathbf{v}_1\cdot\mathbf{v}_1+2\mathbf{u}\cdot\mathbf{v}_1+\mathbf{u}\cdot\mathbf{u}\right)+\ldots.$$

When we add up all the terms like the first one, we get K_{cm}. Adding up all the terms like the third one, we get $M|\mathbf{u}|^2/2$. The terms like the second term cancel out:

$$m_1\mathbf{u}\cdot\mathbf{v}_1+\ldots=\mathbf{u}\cdot\left(m_1\mathbf{v}_1+\ldots\right),$$

where the sum in brackets equals the total momentum in the center-of-mass frame, which is zero by definition.
(b) Changing frames of reference doesn't change the distances between the particles, so the potential energies are all unaffected by the change of frames of reference. Suppose that in a given frame of reference, frame 1, energy is conserved in some process: the initial and final energies add up to be the same. First let's transform to the center-of-mass frame. The potential energies are unaffected by the transformation, and the total kinetic energy is simply reduced by the quantity $M|\mathbf{u}_1|^2/2$, where $\mathbf{u}_1$ is the velocity of frame 1 relative to the center of mass. Subtracting the same constant from the initial and final energies still leaves them equal. Now we transform to frame 2. Again, the effect is simply to change the initial and final energies by adding the same constant.

Page 391, problem 16:
A conservation law is about addition: it says that when you add up a certain thing, the total always stays the same. Funkosity would violate the additive nature of conservation laws, because a two-kilogram mass would have twice as much funkosity as a pair of one-kilogram masses moving at the same speed.

Solutions for chapter 15

Page 427, problem 20:
The pliers are not moving, so their angular momentum remains constant at zero, and the total torque on them must be zero. Not only that, but each half of the pliers must have zero total torque on it. This tells us that the magnitude of the torque at one end must be the same as that at the other end. The distance from the axis to the nut is about 2.5 cm, and the distance from the axis to the centers of the palm and fingers are about 8 cm. The angles are close enough to 90° that we can pretend they're 90 degrees, considering the rough nature of the other assumptions and measurements. The result is (300 N)(2.5 cm) = (F)(8 cm), or $F = 90$ N.

Page 428, problem 28:
The foot of the rod is moving in a circle relative to the center of the rod, with speed $v = \pi b/T$, and acceleration $v^2/(b/2) = (\pi^2/8)g$. This acceleration is initially upward, and is greater in magnitude than g, so the foot of the rod will lift off without dragging. We could also worry about whether the foot of the rod would make contact with the floor again before the rod finishes up flat on its back. This is a question that can be settled by graphing, or simply by

inspection of figure o on page 405. The key here is that the two parts of the acceleration are both independent of m and b, so the result is univeral, and it does suffice to check a graph in a single example. In practical terms, this tells us something about how difficult the trick is to do. Because $\pi^2/8 = 1.23$ isn't much greater than unity, a hit that is just a little too weak (by a factor of $1.23^{1/2} = 1.11$) will cause a fairly obvious qualitative change in the results. This is easily observed if you try it a few times with a pencil.

Solutions for chapter 16

Page 452, problem 11:

(a) We have

$$\begin{aligned} \mathrm{d}P &= \rho g\,\mathrm{d}y \\ \Delta P &= \int \rho g\,\mathrm{d}y, \end{aligned}$$

and since we're taking water to be incompressible, and g doesn't change very much over 11 km of height, we can treat ρ and g as constants and take them outside the integral.

$$\begin{aligned} \Delta P &= \rho g \Delta y \\ &= (1.0\ \mathrm{g/cm^3})(9.8\ \mathrm{m/s^2})(11.0\ \mathrm{km}) \\ &= (1.0\times 10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(1.10\times 10^4\ \mathrm{m}) \\ &= 1.0\times 10^8\ \mathrm{Pa} \\ &= 1.0\times 10^3\ \mathrm{atm}. \end{aligned}$$

The precision of the result is limited to a few percent, due to the compressibility of the water, so we have at most two significant figures. If the change in pressure were exactly a thousand atmospheres, then the pressure at the bottom would be 1001 atmospheres; however, this distinction is not relevant at the level of approximation we're attempting here.

(b) Since the air in the bubble is in thermal contact with the water, it's reasonable to assume that it keeps the same temperature the whole time. The ideal gas law is $PV = nkT$, and rewriting this as a proportionality gives

$$V \propto P^{-1},$$

or

$$\frac{V_f}{V_i} = \left(\frac{P_f}{P_i}\right)^{-1} \approx 10^3.$$

Since the volume is proportional to the cube of the linear dimensions, the growth in radius is about a factor of 10.

Page 452, problem 12:

(a) Roughly speaking, the thermal energy is $\sim k_B T$ (where k_B is the Boltzmann constant), and we need this to be on the same order of magnitude as ke^2/r (where k is the Coulomb constant). For this type of rough estimate it's not especially crucial to get all the factors of two right, but let's do so anyway. Each proton's average kinetic energy due to motion along a particular axis is $(1/2)k_B T$. If two protons are colliding along a certain line in the center-of-mass frame, then their average combined kinetic energy due to motion along that axis is $2(1/2)k_B T = k_B T$. So in fact the factors of 2 cancel. We have $T = ke^2/k_B r$.

(b) The units are $\mathrm{K} = (\mathrm{J\cdot m/C^2})(\mathrm{C^2})/((\mathrm{J/K})\cdot\mathrm{m})$, which does work out.

(c) The numerical result is $\sim 10^{10}$ K, which as suggested is much higher than the temperature at the core of the sun.

Page 453, problem 13:
If the full-sized brick A undergoes some process, such as heating it with a blowtorch, then we want to be able to apply the equation $\Delta S = Q/T$ to either the whole brick or half of it, which would be identical to B. When we redefine the boundary of the system to contain only half of the brick, the quantities ΔS and Q are each half as big, because entropy and energy are additive quantities. T, meanwhile, stays the same, because temperature isn't additive — two cups of coffee aren't twice as hot as one. These changes to the variables leave the equation consistent, since each side has been divided by 2.

Page 453, problem 14:
(a) If the expression $1 + by$ is to make sense, then by has to be unitless, so b has units of m^{-1}. The input to the exponential function also has to be unitless, so k also has of m^{-1}. The only factor with units on the right-hand side is P_o, so P_o must have units of pressure, or Pa.
(b)

$$\begin{aligned} \mathrm{d}P &= \rho g\, \mathrm{d}y \\ \rho &= \frac{1}{g}\frac{\mathrm{d}P}{\mathrm{d}y} \\ &= \frac{P_o}{g} e^{-ky}(-k - kby + b) \end{aligned}$$

(c) The three terms inside the parentheses on the right all have units of m^{-1}, so it makes sense to add them, and the factor in parentheses has those units. The units of the result from b then look like

$$\begin{aligned} \frac{\text{kg}}{\text{m}^3} &= \frac{\text{Pa}}{\text{m/s}^2}\text{m}^{-1} \\ &= \frac{\text{N/m}^2}{\text{m}^2/\text{s}^2} \\ &= \frac{\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}}{\text{m}^2/\text{s}^2}, \end{aligned}$$

which checks out.

Answers to self-checks for volume 1

Answers to self-checks for chapter 0

Page 17, self-check A:
If only he has the special powers, then his results can never be reproduced.

Page 19, self-check B:
They would have had to weigh the rays, or check for a loss of weight in the object from which they were have emitted. (For technical reasons, this was not a measurement they could actually do, hence the opportunity for disagreement.)

Page 25, self-check C:
A dictionary might define "strong" as "possessing powerful muscles," but that's not an operational definition, because it doesn't say how to measure strength numerically. One possible operational definition would be the number of pounds a person can bench press.

Page 28, self-check D:
A microsecond is 1000 times longer than a nanosecond, so it would seem like 1000 seconds, or about 20 minutes.

Page 29, self-check E:
Exponents have to do with multiplication, not addition. The first line should be 100 times longer than the second, not just twice as long.

Page 32, self-check F:
The various estimates differ by 5 to 10 million. The CIA's estimate includes a ridiculous number of gratuitous significant figures. Does the CIA understand that every day, people in are born in, die in, immigrate to, and emigrate from Nigeria?

Page 32, self-check G:
(1) 4; (2) 2; (3) 2

Answers to self-checks for chapter 1

Page 42, self-check A:
$1\ \text{yd}^2 \times (3\ \text{ft}/1\ \text{yd})^2 = 9\ \text{ft}^2$
$1\ \text{yd}^3 \times (3\ \text{ft}/1\ \text{yd})^3 = 27\ \text{ft}^3$

Page 48, self-check B:
$C_1/C_2 = (w_1/w_2)^4$

Answers to self-checks for chapter 2

Page 71, self-check A:
Coasting on a bike and coasting on skates give one-dimensional center-of-mass motion, but running and pedaling require moving body parts up and down, which makes the center of mass move up and down. The only example of rigid-body motion is coasting on skates. (Coasting on a bike is not rigid-body motion, because the wheels twist.)

Page 71, self-check B:
By shifting his weight around, he can cause the center of mass not to coincide with the geometric center of the wheel.

Page 72, self-check C:
(1) a point in time; (2) time in the abstract sense; (3) a time interval

Page 73, self-check D:
Zero, because the "after" and "before" values of x are the same.

Page 81, self-check E:
(1) The effect only occurs during blastoff, when their velocity is changing. Once the rocket engines stop firing, their velocity stops changing, and they no longer feel any effect. (2) It is only an observable effect of your motion relative to the air.

Answers to self-checks for chapter 3

Page 97, self-check A:
Its speed increases at a steady rate, so in the next second it will travel 19 cm.

Answers to self-checks for chapter 4

Page 139, self-check A:
(1) The case of $\rho = 0$ represents an object falling in a vacuum, i.e., there is no density of air.

The terminal velocity would be infinite. Physically, we know that an object falling in a vacuum would never stop speeding up, since there would be no force of air friction to cancel the force of gravity. (2) The 4-cm ball would have a mass that was greater by a factor of $4 \times 4 \times 4$, but its cross-sectional area would be greater by a factor of 4×4. Its terminal velocity would be greater by a factor of $\sqrt{4^3/4^2} = 2$. (3) It isn't of any general importance. It's just an example of one physical situation. You should not memorize it.

Page 142, self-check B:
(1) This is motion, not force. (2) This is a description of how the sub is able to get the water to produce a forward force on it. (3) The sub runs out of energy, not force.

Answers to self-checks for chapter 5

Page 155, self-check A:
The sprinter pushes backward against the ground, and by Newton's third law, the ground pushes forward on her. (Later in the race, she is no longer accelerating, but the ground's forward force is needed in order to cancel out the backward forces, such as air friction.)

Page 163, self-check B:
(1) It's kinetic friction, because her uniform is sliding over the dirt. (2) It's static friction, because even though the two surfaces are moving relative to the landscape, they're not slipping over each other. (3) Only kinetic friction creates heat, as when you rub your hands together. If you move your hands up and down together without sliding them across each other, no heat is produced by the static friction.

Page 163, self-check C:
By the POFOSTITO mnemonic, we know that each of the bird's forces on the trunk will be of the same type as the corresponding force of the tree on the bird, but in the opposite direction. The bird's feet make a normal force on the tree that is to the right and a static frictional force that is downward.

Page 164, self-check D:
Frictionless ice can certainly make a normal force, since otherwise a hockey puck would sink into the ice. Friction is not possible without a normal force, however: we can see this from the equation, or from common sense, e.g., while sliding down a rope you do not get any friction unless you grip the rope.

Page 165, self-check E:
(1) Normal forces are always perpendicular to the surface of contact, which means right or left in this figure. Normal forces are repulsive, so the cliff's force on the feet is to the right, i.e., away from the cliff. (2) Frictional forces are always parallel to the surface of contact, which means right or left in this figure. Static frictional forces are in the direction that would tend to keep the surfaces from slipping over each other. If the wheel was going to slip, its surface would be moving to the left, so the static frictional force on the wheel must be in the direction that would prevent this, i.e., to the right. This makes sense, because it is the static frictional force that accelerates the dragster. (3) Normal forces are always perpendicular to the surface of contact. In this diagram, that means either up and to the left or down and to the right. Normal forces are repulsive, so the ball is pushing the bat away from itself. Therefore the ball's force is down and to the right on this diagram.

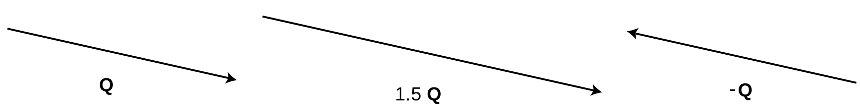
Answers to self-checks for chapter 6

Page 193, self-check A:
The wind increases the ball's overall speed. If you think about it in terms of overall speed, it's not so obvious that the increased speed is exactly sufficient to compensate for the greater distance. However, it becomes much simpler if you think about the forward motion and the sideways motion as two separate things. Suppose the ball is initially moving at one meter per second. Even if it picks up some sideways motion from the wind, it's still getting closer to the wall by one meter every second.

Answers to self-checks for chapter 7

Page 205, self-check A:
$\mathbf{v} = \Delta\mathbf{r}/\Delta t$

Page 206, self-check B:



Page 211, self-check C:
$\mathbf{A} - \mathbf{B}$ is equivalent to $\mathbf{A} + (-\mathbf{B})$, which can be calculated graphically by reversing $\mathbf{B}$ to form $-\mathbf{B}$, and then adding it to $\mathbf{A}$.

Answers to self-checks for chapter 8

Page 221, self-check A:
(1) It is speeding up, because the final velocity vector has the greater magnitude. (2) The result would be zero, which would make sense. (3) Speeding up produced a $\Delta\mathbf{v}$ vector in the same direction as the motion. Slowing down would have given a $\Delta\mathbf{v}$ that pointed backward.

Page 222, self-check B:
As we have already seen, the projectile has $a_x = 0$ and $a_y = -g$, so the acceleration vector is pointing straight down.

Answers to self-checks for chapter 9

Page 245, self-check A:
(1) Uniform. They have the same motion as the drum itself, which is rotating as one solid piece. No part of the drum can be rotating at a different speed from any other part. (2) Nonuniform. Gravity speeds it up on the way down and slows it down on the way up.

Answers to self-checks for chapter 10

Page 264, self-check A:
It would just stay where it was. Plugging $v = 0$ into eq. [1] would give $F = 0$, so it would not accelerate from rest, and would never fall into the sun. No astronomer had ever observed an object that did that!

Page 265, self-check B:

$$F \propto mr/T^2 \propto mr/(r^{3/2})^2 \propto mr/r^3 = m/r^2$$

Page 268, self-check C:
The equal-area law makes equally good sense in the case of a hyperbolic orbit (and observations verify it). The elliptical orbit law had to be generalized by Newton to include hyperbolas. The

law of periods doesn't make sense in the case of a hyperbolic orbit, because a hyperbola never closes back on itself, so the motion never repeats.

Page 273, self-check D:
Above you there is a small part of the shell, comprising only a tiny fraction of the earth's mass. This part pulls you up, while the whole remainder of the shell pulls you down. However, the part above you is extremely close, so it makes sense that its force on you would be far out of proportion to its small mass.

Answers to self-checks for chapter 11

Page 302, self-check A:
(1) A spring-loaded toy gun can cause a bullet to move, so the spring is capable of storing energy and then converting it into kinetic energy. (2) The amount of energy stored in the spring relates to the amount of compression, which can be measured with a ruler.

Answers to self-checks for chapter 12

Page 322, self-check A:
Both balls start from the same height and end at the same height, so they have the same Δy. This implies that their losses in potential energy are the same, so they must both have gained the same amount of kinetic energy.

Answers to self-checks for chapter 13

Page 332, self-check A:
Work is defined as the transfer of energy, so like energy it is a scalar with units of joules.

Page 336, self-check B:
Whenever energy is transferred out of the spring, the same amount has to be transferred into the ball, and vice versa. As the spring compresses, the ball is doing positive work on the spring (giving up its KE and transferring energy into the spring as PE), and as it decompresses the ball is doing negative work (extracting energy).

Page 339, self-check C:
(a) No. The pack is moving at constant velocity, so its kinetic energy is staying the same. It is only moving horizontally, so its gravitational potential energy is also staying the same. No energy transfer is occurring. (b) No. The horse's upward force on the pack forms a 90-degree angle with the direction of motion, so $\cos\theta = 0$, and no work is done.

Page 341, self-check D:
Only in (a) can we use Fd to calculate work. In (b) and (c), the force is changing as the distance changes.

Answers to self-checks for chapter 15

Page 409, self-check A:
1, 2, and 4 all have the same sign, because they are trying to twist the wrench clockwise. The sign of torque 3 is opposite to the signs of the others. The magnitude of torque 3 is the greatest, since it has a large r, and the force is nearly all perpendicular to the wrench. Torques 1 and 2 are the same because they have the same values of r and $F_\perp$. Torque 4 is the smallest, due to its small r.

Answers to self-checks for chapter 16

Page 435, self-check A:
Solids can exert shear forces. A solid could be in an equilibrium in which the shear forces were canceling the forces due to unequal pressures on the sides of the cube.

Answers to self-checks for chapter 18

Page 474, self-check A:
The horizontal axis is a time axis, and the period of the vibrations is independent of amplitude. Shrinking the amplitude does not make the cycles any faster.

Page 475, self-check B:
Energy is proportional to the square of the amplitude, so its energy is four times smaller after every cycle. It loses three quarters of its energy with each cycle.

Page 481, self-check C:
She should tap the wine glasses she finds in the store and look for one with a high Q, i.e., one whose vibrations die out very slowly. The one with the highest Q will have the highest-amplitude response to her driving force, making it more likely to break.

Answers to self-checks for chapter 19

Page 497, self-check A:
The leading edge is moving up, the trailing edge is moving down, and the top of the hump is motionless for one instant.

Page 504, self-check B:
(a) It doesn't have w or h in it. (b) Inertia is measured by μ, tightness by T. (c) Inertia would be measured by the density of the metal, tightness by its resistance to compression. Lead is more dense than aluminum, and this would tend to make the speed of the waves lower in lead. Lead is also softer, so it probably has less resistance to compression, and we would expect this to provide an additional effect in the same direction. Compressional waves will definitely be slower in lead than in aluminum.

Answers to self-checks for chapter 20

Page 523, self-check A:
The energy of a wave is usually proportional to the square of its amplitude. Squaring a negative number gives a positive result, so the energy is the same.

Page 523, self-check B:
A substance is invisible to sonar if the speed of sound waves in it is the same as in water. Reflections only occur at boundaries between media in which the wave speed is different.

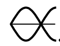
Page 525, self-check C:
No. A material object that loses kinetic energy slows down, but a wave is not a material object. The velocity of a wave ordinarily only depends on the medium, not the amplitude. The speed of a soft sound, for example, is the same as the speed of a loud sound.

Page 533, self-check D:
1. No. To get the best possible interference, the thickness of the coating must be such that the second reflected wave train lags behind the first by an integer number of wavelengths. Optimal performance can therefore only be produced for one specific color of light. The typical greenish color of the coatings shows that they do the worst job for green light.

2. Light can be reflected either from the outer surface of the film or from the inner surface, and

there can be either constructive or destructive interference between the two reflections. We see a pattern that varies across the surface because its thickness isn't constant. We see rainbow colors because the condition for destructive or constructive interference depends on wavelength. White light is a mixture of all the colors of the rainbow, and at a particular place on the soap bubble, part of that mixture, say red, may be reflected strongly, while another part, blue for example, is almost entirely transmitted.

Page 534, self-check E:
The period is the time required to travel a distance $2L$ at speed v, $T = 2L/v$. The frequency is $f = 1/T = v/2L$.

Page 539, self-check F:
The wave pattern will look like this: . Three quarters of a wavelength fit in the tube, so the wavelength is three times shorter than that of the lowest-frequency mode, in which one quarter of a wave fits. Since the wavelength is smaller by a factor of three, the frequency is three times higher. Instead of $f_o, 2f_o, 3f_o, 4f_o, \ldots$, the pattern of wave frequencies of this air column goes $f_o, 3f_o, 5f_o, 7f_o, \ldots$

Answers for volume 1

Answers for chapter 1

Page 61, problem 23:
Check: The actual number of species of lupine occurring in the San Gabriels is 22. You should find that your answer comes out in the same ballpark as this figure, but not exactly the same, of course, because the scaling rule is only a generalization.

Answers for chapter 16

Page 452, problem 10:
(a) $\sim 2 - 10\%$ (b) 5% (c) The high end for the body's actual efficiency is higher than the limit imposed by the laws of thermodynamics. However, the high end of the 1-5 watt range quoted in the problem probably includes large people who aren't just lying around. Still, it's impressive that the human body comes so close to the thermodynamic limit.

Answers for chapter 20

Page 543, problem 3:
Check: The actual length of a flute is about 66 cm.

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Relativity and Electromagnetism

Chapter 21
Electricity and Circuits

Where the telescope ends, the microscope begins. Which of the two has the grander view? *Victor Hugo*

His father died during his mother's pregnancy. Rejected by her as a boy, he was packed off to boarding school when she remarried. He himself never married, but in middle age he formed an intense relationship with a much younger man, a relationship that he terminated when he underwent a psychotic break. Following his early scientific successes, he spent the rest of his professional life mostly in frustration over his inability to unlock the secrets of alchemy.

The man being described is Isaac Newton, but not the triumphant Newton of the standard textbook hagiography. Why dwell on the sad side of his life? To the modern science educator, Newton's lifelong obsession with alchemy may seem an embarrassment, a distraction from his main achievement, the creation of the modern science of mechanics. To Newton, however, his alchemical researches were naturally related to his investigations of force and motion. What was radical about Newton's analysis of motion was its universality: it succeeded in describing both the heavens and the earth with the same equations, whereas previously it had been assumed that the sun, moon, stars, and planets were fundamentally different from earthly objects. But Newton realized that if science was to describe all of nature in a unified way, it was not enough to unite the human scale with the scale of the universe: he would not be satisfied until he fit the microscopic universe into the picture as well.

It should not surprise us that Newton failed. Although he was a firm believer in the existence of atoms, there was no more experimental evidence for their existence than there had been when the ancient Greeks first posited them on purely philosophical grounds. Alchemy labored under a tradition of secrecy and mysticism. Newton had already almost single-handedly transformed the fuzzyheaded field of "natural philosophy" into something we would recognize as the modern science of physics, and it would be unjust to criticize him for failing to change alchemy into modern chemistry as well. The time was not ripe. The microscope was a new invention, and it was cutting-edge science when Newton's contemporary Hooke discovered that living things were made out of cells.

21.1 The quest for the atomic force

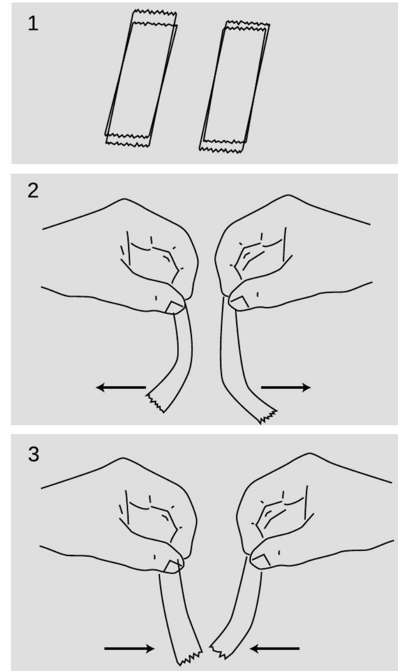
Newton was not the first of the age of reason. He was the last of the magicians. *John Maynard Keynes*

Nevertheless it will be instructive to pick up Newton's train of thought and see where it leads us with the benefit of modern hindsight. In uniting the human and cosmic scales of existence, he had reimagined both as stages on which the actors were objects (trees and houses, planets and stars) that interacted through attractions and repulsions. He was already convinced that the objects inhabiting the microworld were atoms, so it remained only to determine what kinds of forces they exerted on each other.

His next insight was no less brilliant for his inability to bring it to fruition. He realized that the many human-scale forces — friction, sticky forces, the normal forces that keep objects from occupying the same space, and so on — must all simply be expressions of a more fundamental force acting between atoms. Tape sticks to paper because the atoms in the tape attract the atoms in the paper. My house doesn't fall to the center of the earth because its atoms repel the atoms of the dirt under it.

Here he got stuck. It was tempting to think that the atomic force was a form of gravity, which he knew to be universal, fundamental, and mathematically simple. Gravity, however, is always attractive, so how could he use it to explain the existence of both attractive and repulsive atomic forces? The gravitational force between objects of ordinary size is also extremely small, which is why we never notice cars and houses attracting us gravitationally. It would be hard to understand how gravity could be responsible for anything as vigorous as the beating of a heart or the explosion of gunpowder. Newton went on to write a million words of alchemical notes filled with speculation about some other force, perhaps a "divine force" or "vegetative force" that would for example be carried by the sperm to the egg.

Luckily, we now know enough to investigate a different suspect as a candidate for the atomic force: electricity. Electric forces are often observed between objects that have been prepared by rubbing (or other surface interactions), for instance when clothes rub against each other in the dryer. A useful example is shown in figure a/1: stick two pieces of tape on a tabletop, and then put two more pieces on top of them. Lift each pair from the table, and then separate them. The two top pieces will then repel each other, a/2, as will the two bottom pieces. A bottom piece will attract a top piece, however, a/3. Electrical forces like these are similar in certain ways to gravity, the other force that we already know to be fundamental:



a / Four pieces of tape are prepared, 1, as described in the text. Depending on which combination is tested, the interaction can be either repulsive, 2, or attractive, 3.

- Electrical forces are *universal*. Although some substances, such as fur, rubber, and plastic, respond more strongly to electrical preparation than others, all matter participates in electrical forces to some degree. There is no such thing as a "nonelectric" substance. Matter is both inherently gravitational and inherently electrical.

- Experiments show that the electrical force, like the gravitational force, is an *inverse square* force. That is, the electrical force between two spheres is proportional to $1/r^2$, where r is the center-to-center distance between them.

Furthermore, electrical forces make more sense than gravity as candidates for the fundamental force between atoms, because we have observed that they can be either attractive or repulsive.

21.2 Electrical forces

Charge

"Charge" is the technical term used to indicate that an object has been prepared so as to participate in electrical forces. This is to be distinguished from the common usage, in which the term is used indiscriminately for anything electrical. For example, although we speak colloquially of "charging" a battery, you may easily verify that a battery has no charge in the technical sense, e.g., it does not exert any electrical force on a piece of tape that has been prepared as described in the previous section.

Two types of charge

We can easily collect reams of data on electrical forces between different substances that have been charged in different ways. We find for example that cat fur prepared by rubbing against rabbit fur will attract glass that has been rubbed on silk. How can we make any sense of all this information? A vast simplification is achieved by noting that there are really only two types of charge. Suppose we pick cat fur rubbed on rabbit fur as a representative of type A, and glass rubbed on silk for type B. We will now find that there is no "type C." Any object electrified by any method is either A-like, attracting things A attracts and repelling those it repels, or B-like, displaying the same attractions and repulsions as B. The two types, A and B, always display opposite interactions. If A displays an attraction with some charged object, then B is guaranteed to undergo repulsion with it, and vice-versa.

The coulomb

Although there are only two types of charge, each type can come in different amounts. The metric unit of charge is the coulomb (rhymes with "drool on"), defined as follows:

> One Coulomb (C) is the amount of charge such that a force of 9.0×10^9 N occurs between two pointlike objects with charges of 1 C separated by a distance of 1 m.

The notation for an amount of charge is q. The numerical factor in the definition is historical in origin, and is not worth memorizing. The definition is stated for pointlike, i.e., very small, objects, because otherwise different parts of them would be at different distances from each other.

A model of two types of charged particles

Experiments show that all the methods of rubbing or otherwise charging objects involve two objects, and both of them end up getting charged. If one object acquires a certain amount of one type of charge, then the other ends up with an equal amount of the other type. Various interpretations of this are possible, but the simplest is that the basic building blocks of matter come in two flavors, one with each type of charge. Rubbing objects together results in the transfer of some of these particles from one object to the other. In this model, an object that has not been electrically prepared may actually possesses a great deal of *both* types of charge, but the amounts are equal and they are distributed in the same way throughout it. Since type A repels anything that type B attracts, and vice versa, the object will make a total force of zero on any other object. The rest of this chapter fleshes out this model and discusses how these mysterious particles can be understood as being internal parts of atoms.

Use of positive and negative signs for charge

Because the two types of charge tend to cancel out each other's forces, it makes sense to label them using positive and negative signs, and to discuss the *total* charge of an object. It is entirely arbitrary which type of charge to call negative and which to call positive. Benjamin Franklin decided to describe the one we've been calling

"A" as negative, but it really doesn't matter as long as everyone is consistent with everyone else. An object with a total charge of zero (equal amounts of both types) is referred to as electrically *neutral*.

self-check A

Criticize the following statement: "There are two types of charge, attractive and repulsive." ▷ Answer, p. 1043

Coulomb's law

A large body of experimental observations can be summarized as follows:

Coulomb's law: The magnitude of the force acting between pointlike charged objects at a center-to-center distance r is given by the equation

$$|\mathbf{F}| = k\frac{|q_1||q_2|}{r^2},$$

where the constant k equals $9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. The force is attractive if the charges are of different signs, and repulsive if they have the same sign.

Clever modern techniques have allowed the $1/r^2$ form of Coulomb's law to be tested to incredible accuracy, showing that the exponent is in the range from 1.9999999999999998 to 2.0000000000000002.

Note that Coulomb's law is closely analogous to Newton's law of gravity, where the magnitude of the force is Gm_1m_2/r^2, except that there is only one type of mass, not two, and gravitational forces are never repulsive. Because of this close analogy between the two types of forces, we can recycle a great deal of our knowledge of gravitational forces. For instance, there is an electrical equivalent of the shell theorem: the electrical forces exerted externally by a uniformly charged spherical shell are the same as if all the charge was concentrated at its center, and the forces exerted internally are zero.

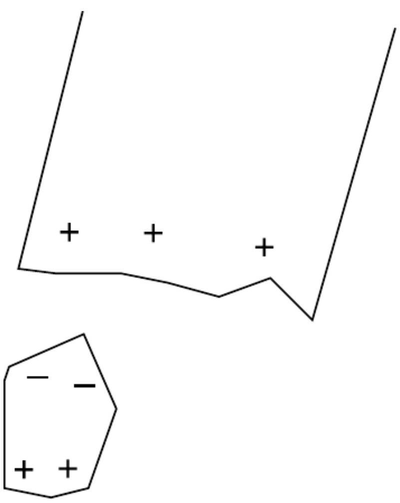
Conservation of charge

An even more fundamental reason for using positive and negative signs for electrical charge is that experiments show that charge is conserved according to this definition: in any closed system, the total amount of charge is a constant. This is why we observe that rubbing initially uncharged substances together always has the result that one gains a certain amount of one type of charge, while the other acquires an equal amount of the other type. Conservation of charge seems natural in our model in which matter is made of positive and negative particles. If the charge on each particle is a fixed property of that type of particle, and if the particles themselves can be neither created nor destroyed, then conservation of charge is inevitable.

Electrical forces involving neutral objects

b / A charged piece of tape attracts uncharged pieces of paper from a distance, and they leap up to it.

As shown in figure b, an electrically charged object can attract objects that are uncharged. How is this possible? The key is that even though each piece of paper has a total charge of zero, it has at least some charged particles in it that have some freedom to move. Suppose that the tape is positively charged, c. Mobile particles in the paper will respond to the tape's forces, causing one end of the paper to become negatively charged and the other to become positive. The attraction between the paper and the tape is now stronger than the repulsion, because the negatively charged end is closer to the tape.



c / The paper has zero total charge, but it does have charged particles in it that can move.

self-check B

What would have happened if the tape was negatively charged? ▷ Answer, p. 1043

Discussion questions

A If the electrical attraction between two pointlike objects at a distance of 1 m is 9×10^9 N, why can't we infer that their charges are +1 and −1 C? What further observations would we need to do in order to prove this?

B An electrically charged piece of tape will be attracted to your hand. Does that allow us to tell whether the mobile charged particles in your hand are positive or negative, or both?

21.3 Current

Unity of all types of electricity

We are surrounded by things we have been *told* are "electrical," but it's far from obvious what they have in common to justify being grouped together. What relationship is there between the way socks cling together and the way a battery lights a lightbulb? We have been told that both an electric eel and our own brains are somehow electrical in nature, but what do they have in common?

British physicist Michael Faraday (1791-1867) set out to address this problem. He investigated electricity from a variety of sources — including electric eels! — to see whether they could all produce the same effects, such as shocks and sparks, attraction and repulsion. "Heating" refers, for example, to the way a lightbulb filament gets hot enough to glow and emit light. Magnetic induction is an effect discovered by Faraday himself that connects electricity and magnetism. We will not study this effect, which is the basis for the electric generator, in detail until later in the book.

source	effect			
	shocks	sparks	attraction and repulsion	heating
rubbing	√	√	√	√
battery	√	√	√	√
animal	√	√	(√)	√
magnetically induced	√	√	√	√

The table shows a summary of some of Faraday's results. Check marks indicate that Faraday or his close contemporaries were able to verify that a particular source of electricity was capable of producing a certain effect. (They evidently failed to demonstrate attraction and repulsion between objects charged by electric eels, although modern workers have studied these species in detail and been able to understand all their electrical characteristics on the same footing as other forms of electricity.)

d / Michael Faraday (1791-1867) was the son of a poor blacksmith.

e / *Gymnotus carapo*, a knifefish, uses electrical signals to sense its environment and to communicate with others of its species.

Faraday's results indicate that there is nothing fundamentally different about the types of electricity supplied by the various sources. They are all able to produce a wide variety of identical effects. Wrote Faraday, "The general conclusion which must be drawn from this collection of facts is that electricity, whatever may be its source, is identical in its nature."

If the types of electricity are the same thing, what thing is that? The answer is provided by the fact that all the sources of electricity can cause objects to repel or attract each other. We use the word "charge" to describe the property of an object that allows it to participate in such electrical forces, and we have learned that charge is present in matter in the form of nuclei and electrons. Evidently all these electrical phenomena boil down to the motion of charged particles in matter.

Electric current

If the fundamental phenomenon is the motion of charged particles, then how can we define a useful numerical measurement of it? We might describe the flow of a river simply by the velocity of the water, but velocity will not be appropriate for electrical purposes because we need to take into account how much charge the moving particles have, and in any case there are no practical devices sold at Radio Shack that can tell us the velocity of charged particles. Experiments show that the intensity of various electrical effects is related to a different quantity: the number of coulombs of charge that pass by a certain point per second. By analogy with the flow of water, this quantity is called the electric *current*, I. Its units

f / André Marie Ampère (1775-1836).

of coulombs/second are more conveniently abbreviated as amperes, 1 A=1 C/s. (In informal speech, one usually says "amps.")

The main subtlety involved in this definition is how to account for the two types of charge. The stream of water coming from a hose is made of atoms containing charged particles, but it produces none of the effects we associate with electric currents. For example, you do not get an electrical shock when you are sprayed by a hose. This type of experiment shows that the effect created by the motion of one type of charged particle can be canceled out by the motion of the opposite type of charge in the same direction. In water, every oxygen atom with a charge of $+8e$ is surrounded by eight electrons with charges of $-e$, and likewise for the hydrogen atoms.

We therefore refine our definition of current as follows:

definition of electric current
When charged particles are exchanged between regions of space A and B, the electric current flowing from A to B is

$$I = \frac{\Delta q}{\Delta t},$$

where Δq is the change in region B's total charge occurring over a period of time Δt.

In the garden hose example, your body picks up equal amounts of positive and negative charge, resulting in no change in your total charge, so the electrical current flowing into you is zero.

Interpretation of $\Delta q/\Delta t$ — *example 1*

▷ How should the expression $\Delta q/\Delta t$ be interpreted when the current isn't constant?

▷ You've seen lots of equations of this form before: $v = \Delta x/\Delta t$, $F = \Delta p/\Delta t$, etc. These are all descriptions of rates of change, and they all require that the rate of change be constant. If the rate of change isn't constant, you instead have to use the slope of the tangent line on a graph. The slope of a tangent line is equivalent to a derivative in calculus; applications of calculus are discussed in section 21.7.

Ions moving across a cell membrane — *example 2*

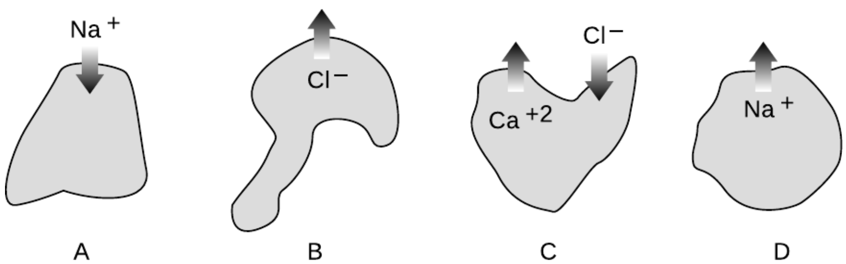
▷ Figure g shows ions, labeled with their charges, moving in or out through the membranes of four cells. If the ions all cross the membranes during the same interval of time, how would the currents into the cells compare with each other?

▷ Cell A has positive current going into it because its charge is increased, i.e., has a positive value of Δq.

Cell B has the same current as cell A, because by losing one unit of negative charge it also ends up increasing its own total charge by one unit.

Cell C's total charge is reduced by three units, so it has a large negative current going into it.

Cell D loses one unit of charge, so it has a small negative current into it.



g / Example 2

It may seem strange to say that a negatively charged particle going one way creates a current going the other way, but this is quite ordinary. As we will see, currents flow through metal wires via the motion of electrons, which are negatively charged, so the direction of motion of the electrons in a circuit is always opposite to the direction of the current. Of course it would have been convenient of Benjamin Franklin had defined the positive and negative signs of charge the opposite way, since so many electrical devices are based on metal wires.

Number of electrons flowing through a lightbulb *example 3*

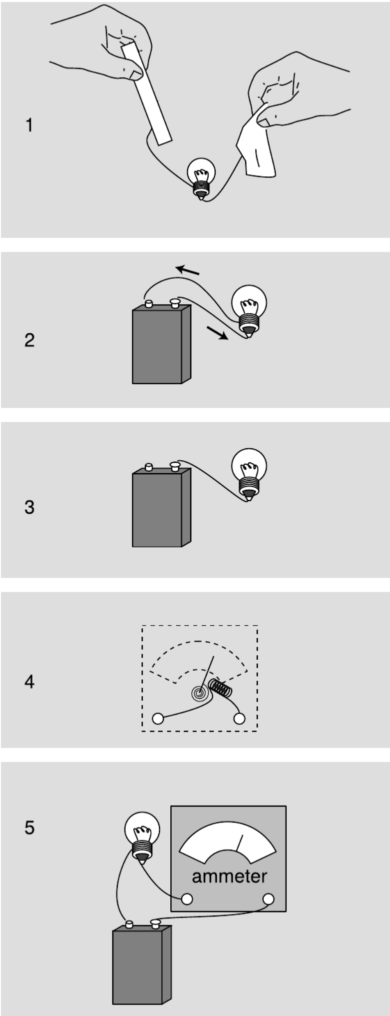
▷ If a lightbulb has 1.0 A flowing through it, how many electrons will pass through the filament in 1.0 s?

▷ We are only calculating the number of electrons that flow, so we can ignore the positive and negative signs. Solving for $\Delta q = I\Delta t$ gives a charge of 1.0 C flowing in this time interval. The number of electrons is

$$\begin{aligned}\text{number of electrons} &= \text{coulombs} \times \frac{\text{electrons}}{\text{coulomb}} \\ &= \text{coulombs}/\frac{\text{coulombs}}{\text{electron}} \\ &= 1.0\ \text{C}/e \\ &= 6.2 \times 10^{18}\end{aligned}$$

21.4 Circuits

How can we put electric currents to work? The only method of controlling electric charge we have studied so far is to charge different substances, e.g., rubber and fur, by rubbing them against each other. Figure h/1 shows an attempt to use this technique to light a lightbulb. This method is unsatisfactory. True, current will flow through the bulb, since electrons can move through metal wires, and



h / 1. Static electricity runs out quickly. 2. A practical circuit. 3. An open circuit. 4. How an ammeter works. 5. Measuring the current with an ammeter.

the excess electrons on the rubber rod will therefore come through the wires and bulb due to the attraction of the positively charged fur and the repulsion of the other electrons. The problem is that after a zillionth of a second of current, the rod and fur will both have run out of charge. No more current will flow, and the lightbulb will go out.

Figure h/2 shows a setup that works. The battery pushes charge through the circuit, and recycles it over and over again. (We will have more to say later in this chapter about how batteries work.) This is called a *complete circuit*. Today, the electrical use of the word "circuit" is the only one that springs to mind for most people, but the original meaning was to travel around and make a round trip, as when a circuit court judge would ride around the boondocks, dispensing justice in each town on a certain date.

Note that an example like h/3 does not work. The wire will quickly begin acquiring a net charge, because it has no way to get rid of the charge flowing into it. The repulsion of this charge will make it more and more difficult to send any more charge in, and soon the electrical forces exerted by the battery will be canceled out completely. The whole process would be over so quickly that the filament would not even have enough time to get hot and glow. This is known as an *open circuit*. Exactly the same thing would happen if the complete circuit of figure h/2 was cut somewhere with a pair of scissors, and in fact that is essentially how an ordinary light switch works: by opening up a gap in the circuit.

The definition of electric current we have developed has the great virtue that it is easy to measure. In practical electrical work, one almost always measures current, not charge. The instrument used to measure current is called an *ammeter*. A simplified ammeter, h/4, simply consists of a coiled-wire magnet whose force twists an iron needle against the resistance of a spring. The greater the current, the greater the force. Although the construction of ammeters may differ, their use is always the same. We break into the path of the electric current and interpose the meter like a tollbooth on a road, h/5. There is still a complete circuit, and as far as the battery and bulb are concerned, the ammeter is just another segment of wire.

Does it matter where in the circuit we place the ammeter? Could we, for instance, have put it in the left side of the circuit instead of the right? Conservation of charge tells us that this can make no difference. Charge is not destroyed or "used up" by the lightbulb, so we will get the same current reading on either side of it. What is "used up" is energy stored in the battery, which is being converted into heat and light energy.

21.5 Voltage

The volt unit

Electrical circuits can be used for sending signals, storing information, or doing calculations, but their most common purpose by far is to manipulate energy, as in the battery-and-bulb example of the previous section. We know that lightbulbs are rated in units of watts, i.e., how many joules per second of energy they can convert into heat and light, but how would this relate to the flow of charge as measured in amperes? By way of analogy, suppose your friend, who didn't take physics, can't find any job better than pitching bales of hay. The number of calories he burns per hour will certainly depend on how many bales he pitches per minute, but it will also be proportional to how much mechanical work he has to do on each bale. If his job is to toss them up into a hayloft, he will get tired a lot more quickly than someone who merely tips bales off a loading dock into trucks. In metric units,

$$\frac{\text{joules}}{\text{second}} = \frac{\text{haybales}}{\text{second}} \times \frac{\text{joules}}{\text{haybale}}.$$

Similarly, the rate of energy transformation by a battery will not just depend on how many coulombs per second it pushes through a circuit but also on how much mechanical work it has to do on each coulomb of charge:

$$\frac{\text{joules}}{\text{second}} = \frac{\text{coulombs}}{\text{second}} \times \frac{\text{joules}}{\text{coulomb}}$$

or

$$\text{power} = \text{current} \times \text{work per unit charge}.$$

Units of joules per coulomb are abbreviated as *volts*, 1 V=1 J/C, named after the Italian physicist Alessandro Volta. Everyone knows that batteries are rated in units of volts, but the voltage concept is more general than that; it turns out that voltage is a property of every point in space. To gain more insight, let's think more carefully about what goes on in the battery and bulb circuit.

i / Alessandro Volta (1745-1827).

The concept of voltage (electrical potential) in general

To do work on a charged particle, the battery apparently must be exerting forces on it. How does it do this? Well, the only thing that can exert an electrical force on a charged particle is another charged particle. It's as though the haybales were pushing and pulling each other into the hayloft! This is potentially a horribly complicated situation. Even if we knew how much excess positive or negative charge there was at every point in the circuit (which realistically we don't) we would have to calculate zillions of forces using Coulomb's law, perform all the vector additions, and finally calculate how much work was being done on the charges as they moved along. To make things even more scary, there is more than one type of charged particle that moves: electrons are what move in the wires and the bulb's filament, but ions are the moving charge carriers inside the battery. Luckily, there are two ways in which we can simplify things:

The situation is unchanging. Unlike the imaginary setup in which we attempted to light a bulb using a rubber rod and a piece of fur, this circuit maintains itself in a steady state (after perhaps a microsecond-long period of settling down after the circuit is first assembled). The current is steady, and as charge flows out of any area of the circuit it is replaced by the same amount of charge flowing in. The amount of excess positive or negative charge in any part of the circuit therefore stays constant. Similarly, when we watch a river flowing, the water goes by but the river doesn't disappear.

Force depends only on position. Since the charge distribution is not changing, the total electrical force on a charged particle depends only on its own charge and on its location. If another charged particle of the same type visits the same location later on, it will feel exactly the same force.

The second observation tells us that there is nothing all that different about the experience of one charged particle as compared to another's. If we single out one particle to pay attention to, and figure out the amount of work done on it by electrical forces as it goes from point A to point B along a certain path, then this is the same amount of work that will be done on any other charged particles of the same type as it follows the same path. For the sake of visualization, let's think about the path that starts at one terminal of the battery, goes through the light bulb's filament, and ends at the other terminal. When an object experiences a force that depends only on its position (and when certain other, technical conditions are satisfied), we can define an electrical energy associated with the position of that object. The amount of work done on the particle by electrical forces as it moves from A to B equals the drop in electrical energy between A and B. This electrical energy is what is being converted into other forms of energy such as heat and light. We therefore define ΔV in general as electrical energy per unit charge:

definition of potential difference
The ΔV between two points in space is defined as

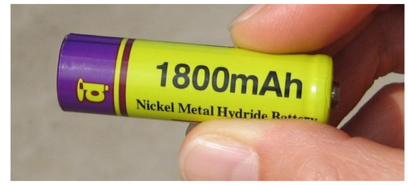
$$\Delta V = \Delta PE_{elec}/q,$$

where ΔPE_{elec} is the change in the electrical energy of a particle with charge q as it moves from the initial point to the final point.

In this context, where we think of the voltage as being a scalar function that is defined everywhere in space, it is more common in formal writing to refer to it as the electrical *potential*.

The amount of power dissipated (i.e., rate at which energy is transformed by the flow of electricity) is then given by the equation

$$P = I\Delta V.$$



j / Example 4.

Energy stored in a battery *example 4*

▷ The 1.2 V rechargeable battery in figure j is labeled 1800 milliamp-hours. What is the maximum amount of energy the battery can store?

▷ An ampere-hour is a unit of current multiplied by a unit of time. Current is charge per unit time, so an ampere-hour is in fact a funny unit of *charge*:

$$\begin{aligned}(1\text{ A})(1\text{ hour}) &= (1\text{ C/s})(3600\text{ s})\\ &= 3600\text{ C}\end{aligned}$$

1800 milliamp-hours is therefore $1800 \times 10^{-3} \times 3600$ C = 6.5×10^3 C. That's a huge number of charged particles, but the total loss of electrical energy will just be their total charge multiplied by the voltage difference across which they move:

$$\begin{aligned}\Delta PE_{elec} &= q\Delta V\\ &= (6.5 \times 10^3\text{ C})(1.2\text{ V})\\ &= 7.8\text{ kJ}\end{aligned}$$

Units of volt-amps *example 5*

▷ Doorbells are often rated in volt-amps. What does this combination of units mean?

▷ Current times voltage gives units of power, $P = I\Delta V$, so volt-amps are really just a nonstandard way of writing watts. They are telling you how much power the doorbell requires.

Power dissipated by a battery and bulb *example 6*

▷ If a 9.0-volt battery causes 1.0 A to flow through a lightbulb, how much power is dissipated?

▷ The voltage rating of a battery tells us what voltage difference ΔV it is designed to maintain between its terminals.

$$\begin{aligned}P &= I\,\Delta\text{V}\\ &= 9.0\text{ A}\cdot\text{V}\\ &= 9.0\,\frac{\text{C}}{\text{s}}\cdot\frac{\text{J}}{\text{C}}\\ &= 9.0\text{ J/s}\\ &= 9.0\text{ W}\end{aligned}$$

The only nontrivial thing in this problem was dealing with the units. One quickly gets used to translating common combinations like A · V into simpler terms.

Here are a few questions and answers about the voltage concept.

Question: OK, so what *is* voltage, really?
Answer: A device like a battery has positive and negative charges inside it that push other charges around the outside circuit. A higher-voltage battery has denser charges in it, which will do more work on each charged particle that moves through the outside circuit.

To use a gravitational analogy, we can put a paddlewheel at the bottom of either a tall waterfall or a short one, but a kg of water that falls through the greater gravitational energy difference will have more energy to give up to the paddlewheel at the bottom.

Question: Why do we define voltage as electrical energy divided by charge, instead of just defining it as electrical energy?
Answer: One answer is that it's the only definition that makes the equation $P = I\Delta V$ work. A more general answer is that we want to be able to define a voltage difference between any two points in space without having to know in advance how much charge the particles moving between them will have. If you put a nine-volt battery on your tongue, then the charged particles that move across your tongue and give you that tingly sensation are not electrons but ions, which may have charges of $+e$, $-2e$, or practically anything. The manufacturer probably expected the battery to be used mostly in circuits with metal wires, where the charged particles that flowed would be electrons with charges of $-e$. If the ones flowing across your tongue happen to have charges of $-2e$, the electrical energy difference for them will be twice as much, but dividing by their charge of $-2e$ in the definition of voltage will still give a result of 9 V.

Question: Are there two separate roles for the charged particles in the circuit, a type that sits still and exerts the forces, and another that moves under the influence of those forces?
Answer: No. Every charged particle simultaneously plays both roles. Newton's third law says that any particle that has an electrical force acting on it must also be exerting an electrical force back on the other particle. There are no "designated movers" or "designated force-makers."

Question: Why does the definition of voltage only refer to voltage *differences*?
Answer: It's perfectly OK to define voltage as $V = PE_{elec}/q$. But recall that it is only *differences* in interaction energy, U, that have direct physical meaning in physics. Similarly, voltage differences are really more useful than absolute voltages. A voltmeter measures voltage differences, not absolute voltages.

Discussion questions

A A roller coaster is sort of like an electric circuit, but it uses gravitational forces on the cars instead of electric ones. What would a high-voltage roller coaster be like? What would a high-current roller coaster be like?

B Criticize the following statements:

"He touched the wire, and 10000 volts went through him."

"That battery has a charge of 9 volts."

"You used up the charge of the battery."

C When you touch a 9-volt battery to your tongue, both positive and negative ions move through your saliva. Which ions go which way?

D I once touched a piece of physics apparatus that had been wired incorrectly, and got a several-thousand-volt voltage difference across my hand. I was not injured. For what possible reason would the shock have had insufficient power to hurt me?

21.6 Resistance

Resistance

So far we have simply presented it as an observed fact that a battery-and-bulb circuit quickly settles down to a steady flow, but why should it? Newton's second law, $a = F/m$, would seem to predict that the steady forces on the charged particles should make them whip around the circuit faster and faster. The answer is that as charged particles move through matter, there are always forces, analogous to frictional forces, that resist the motion. These forces need to be included in Newton's second law, which is really $a = F_{total}/m$, not $a = F/m$. If, by analogy, you push a crate across the floor at constant speed, i.e., with zero acceleration, the total force on it must be zero. After you get the crate going, the floor's frictional force is exactly canceling out your force. The chemical energy stored in your body is being transformed into heat in the crate and the floor, and no longer into an increase in the crate's kinetic energy. Similarly, the battery's internal chemical energy is converted into heat, not into perpetually increasing the charged particles' kinetic energy. Changing energy into heat may be a nuisance in some circuits, such as a computer chip, but it is vital in an incandescent lightbulb, which must get hot enough to glow. Whether we like it or not, this kind of heating effect is going to occur any time charged particles move through matter.

What determines the amount of heating? One flashlight bulb designed to work with a 9-volt battery might be labeled 1.0 watts, another 5.0. How does this work? Even without knowing the details of this type of friction at the atomic level, we can relate the heat dissipation to the amount of current that flows via the equation $P = I\Delta\text{V}$. If the two flashlight bulbs can have two different values of P when used with a battery that maintains the same ΔV, it must be that the 5.0-watt bulb allows five times more current to flow through it.


k / Georg Simon Ohm (1787-1854).

For many substances, including the tungsten from which lightbulb filaments are made, experiments show that the amount of current that will flow through it is directly proportional to the voltage difference placed across it. For an object made of such a substance, we define its electrical *resistance* as follows:

> **definition of resistance**
> If an object inserted in a circuit displays a current flow proportional to the voltage difference across it, then we define its resistance as the constant ratio
>
> $$R = \Delta V/I$$

The units of resistance are volts/ampere, usually abbreviated as ohms, symbolized with the capital Greek letter omega, Ω.

Resistance of a lightbulb *example 7*

▷ A flashlight bulb powered by a 9-volt battery has a resistance of 10 Ω. How much current will it draw?

▷ Solving the definition of resistance for I, we find

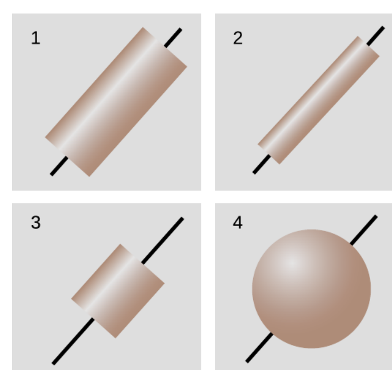
$$\begin{aligned} I &= \Delta V/R \\ &= 0.9\ \text{V}/\Omega \\ &= 0.9\ \text{V}/(\text{V}/\text{A}) \\ &= 0.9\ \text{A} \end{aligned}$$

Ohm's law states that many substances, including many solids and some liquids, display this kind of behavior, at least for voltages that are not too large. The fact that Ohm's law is called a "law" should not be taken to mean that all materials obey it, or that it has the same fundamental importance as Newton's laws, for example. Materials are called *ohmic* or *nonohmic*, depending on whether they obey Ohm's law. Although we will concentrate on ohmic materials in this book, it's important to keep in mind that a great many materials are nonohmic, and devices made from them are often very important. For instance, a transistor is a nonohmic device that can be used to amplify a signal (as in a guitar amplifier) or to store and manipulate the ones and zeroes in a computer chip.

If objects of the same size and shape made from two different ohmic materials have different resistances, we can say that one material is more resistive than the other, or equivalently that it is less conductive. Materials, such as metals, that are very conductive are said to be good *conductors*. Those that are extremely poor conductors, for example wood or rubber, are classified as *insulators*. There is no sharp distinction between the two classes of materials. Some, such as silicon, lie midway between the two extremes, and are called semiconductors.

On an intuitive level, we can understand the idea of resistance by making the sounds "hhhhhh" and "ffffff." To make air flow out of your mouth, you use your diaphragm to compress the air in your chest. The pressure difference between your chest and the air outside your mouth is analogous to a voltage difference. When you make the "h" sound, you form your mouth and throat in a way that allows air to flow easily. The large flow of air is like a large current. Dividing by a large current in the definition of resistance means that we get a small resistance. We say that the small resistance of your mouth and throat allows a large current to flow. When you make the "f" sound, you increase the resistance and cause a smaller current to flow.

Note that although the resistance of an object depends on the substance it is made of, we cannot speak simply of the "resistance of gold" or the "resistance of wood." Figure l shows four examples of



l / Four objects made of the same substance have different resistances.

objects that have had wires attached at the ends as electrical connections. If they were made of the same substance, they would all nevertheless have different resistances because of their different sizes and shapes. A more detailed discussion will be more natural in the context of the following chapter, but it should not be too surprising that the resistance of l/2 will be greater than that of l/1 — the image of water flowing through a pipe, however incorrect, gives us the right intuition. Object l/3 will have a smaller resistance than l/1 because the charged particles have less of it to get through.

Superconductors


m / A medical MRI scanner, which uses superconductors.

All materials display some variation in resistance according to temperature (a fact that is used in thermostats to make a thermometer that can be easily interfaced to an electric circuit). More spectacularly, most metals have been found to exhibit a sudden change to *zero* resistance when cooled to a certain critical temperature. They are then said to be superconductors. Currently, the most important practical application of superconductivity is in medical MRI (magnetic resonance imaging) scanners. The mechanism of MRI is explained on p. 483, but the important point for now is that when your body is inserted into one of these devices, you are being immersed in an extremely strong magnetic field produced by electric currents flowing through the coiled wires of an electromagnet. If these wires were not superconducting, they would instantly burn up because of the heat generated by their resistance.

There are many other potential applications for superconductors, but most of these, such as power transmission, are not currently economically feasible because of the extremely low temperatures required for superconductivity to occur.

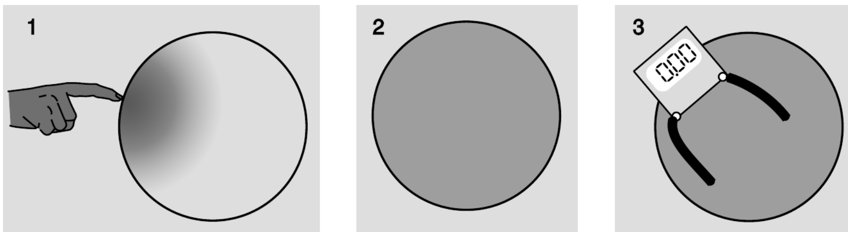
However, it was discovered in 1986 that certain ceramics are superconductors at less extreme temperatures. The technological barrier is now in finding practical methods for making wire out of these brittle materials. Wall Street is currently investing billions of dollars in developing superconducting devices for cellular phone relay stations based on these materials.

There is currently no satisfactory theory of superconductivity in general, although superconductivity in metals is understood fairly well. Unfortunately I have yet to find a fundamental explanation of superconductivity in metals that works at the introductory level.

Constant voltage throughout a conductor

The idea of a superconductor leads us to the question of how we should expect an object to behave if it is made of a very good conductor. Superconductors are an extreme case, but often a metal wire can be thought of as a perfect conductor, for example if the parts of the circuit other than the wire are made of much less conductive materials. What happens if R equals zero in the equation

$R = \Delta V/I$? The result of dividing two numbers can only be zero if the number on top equals zero. This tells us that if we pick any two points in a perfect conductor, the voltage difference between them must be zero. In other words, the entire conductor must be at the same voltage.

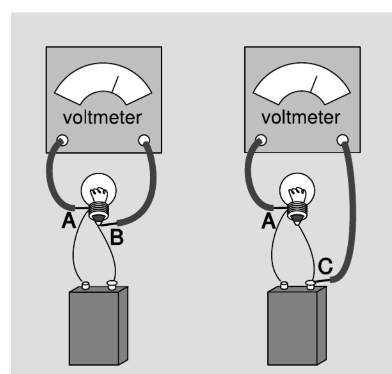


n / 1. The finger deposits charges on the solid, spherical, metal doorknob and is then withdrawn. 2. Almost instantaneously, the charges' mutual repulsion makes them redistribute themselves uniformly on the surface of the sphere. The only *excess* charge is on the surface; charges do exist in the atoms that form the interior of the sphere, but they are balanced. Charges on the interior feel zero total electrical force from the ones at the surface. Charges at the surface experience a net outward repulsion, but this is canceled out by the force that keeps them from escaping into the air. 3. A voltmeter shows zero difference in voltage between any two points on the interior or surface of the sphere. If the voltage difference wasn't zero, then energy could be released by the flow of charge from one point to the other; this only happens before equilibrium is reached.

Constant voltage means that no work would be done on a charge as it moved from one point in the conductor to another. If zero work was done only along a certain path between two specific points, it might mean that positive work was done along part of the path and negative work along the rest, resulting in a cancellation. But there is no way that the work could come out to be zero for all possible paths unless the electrical force on a charge was in fact zero at every point. Suppose, for example, that you build up a static charge by scuffing your feet on a carpet, and then you deposit some of that charge onto a doorknob, which is a good conductor. How can all that charge be in the doorknob without creating any electrical force at any point inside it? The only possible answer is that the charge moves around until it has spread itself into just the right configuration so that the forces exerted by all the little bits of excess surface charge on any charged particle within the doorknob exactly cancel out.

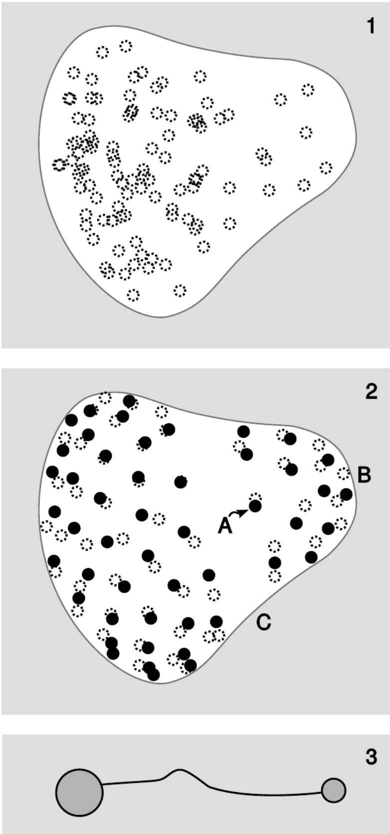
We can explain this behavior if we assume that the charge placed on the doorknob eventually settles down into a stable equilibrium. Since the doorknob is a conductor, the charge is free to move through it. If it was free to move and any part of it did experience a nonzero total force from the rest of the charge, then it would move, and we would not have an equilibrium.

Excess charge placed on a conductor, once it reaches its equilibrium configuration, is entirely on the surface, not on the interior. This should be intuitively reasonable in figure n, for example, since the charges are all repelling each other. A proof is given in example 16 on p. 657.



o / The voltmeter doesn't care which of these setups you use.

Since wires are good conductors, constancy of voltage throughout a conductor provides a convenient freedom in hooking up a voltmeter to a circuit. In figure o, points B and C are on the same piece of conducting wire, so $V_B = V_C$. Measuring $V_B - V_A$ gives the same result as measuring $V_C - V_A$.



p / Example 8. In 1 and 2, charges that are visible on the front surface of the conductor are shown as solid dots; the others would have to be seen through the conductor, which we imagine is semi-transparent.

The lightning rod *example 8*

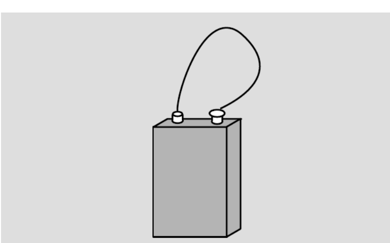
Suppose you have a pear-shaped conductor like the one in figure p/1. Since the pear is a conductor, there are free charges everywhere inside it. Panels 1 and 2 of the figure show a computer simulation with 100 identical electric charges. In 1, the charges are released at random positions inside the pear. Repulsion causes them all to fly outward onto the surface and then settle down into an orderly but nonuniform pattern.

We might not have been able to guess the pattern in advance, but we can verify that some of its features make sense. For example, charge A has more neighbors on the right than on the left, which would tend to make it accelerate off to the left. But when we look at the picture as a whole, it appears reasonable that this is prevented by the larger number of more distant charges on its left than on its right.

There also seems to be a pattern to the nonuniformity: the charges collect more densely in areas like B, where the surface is strongly curved, and less densely in flatter areas like C.

To understand the reason for this pattern, consider p/3. Two conducting spheres are connected by a conducting wire. Since the whole apparatus is conducting, it must all be at one voltage. As shown in problem 43 on p. 624, the density of charge is greater on the smaller sphere. This is an example of a more general fact observed in p/2, which is that the charge on a conductor packs itself more densely in areas that are more sharply curved.

Similar reasoning shows why Benjamin Franklin used a sharp tip when he invented the lightning rod. The charged stormclouds induce positive and negative charges to move to opposite ends of the rod. At the pointed upper end of the rod, the charge tends to concentrate at the point, and this charge attracts the lightning. The same effect can sometimes be seen when a scrap of aluminum foil is inadvertently put in a microwave oven. Modern experiments (Moore *et al.*, Journal of Applied Meteorology 39 (1999) 593) show that although a sharp tip is best at starting a spark, a more moderate curve, like the right-hand tip of the pear in this example, is better at successfully sustaining the spark for long enough to connect a discharge to the clouds.



q / Short-circuiting a battery. Warning: you can burn yourself this way or start a fire! If you want to try this, try making the connection only very briefly, use a low-voltage battery, and avoid touching the battery or the wire, both of which will get hot.

Short circuits

So far we have been assuming a perfect conductor. What if it is a good conductor, but not a perfect one? Then we can solve for $\Delta V = IR$. An ordinary-sized current will make a very small result when we multiply it by the resistance of a good conductor such as a metal wire. The voltage throughout the wire will then be nearly constant. If, on the other hand, the current is extremely large, we can have a significant voltage difference. This is what happens in a *short-circuit:* a circuit in which a low-resistance pathway connects the two sides of a voltage source. Note that this is much more specific than the popular use of the term to indicate any electrical malfunction at all. If, for example, you short-circuit a 9-volt battery as shown in figure q, you will produce perhaps a thousand amperes of current, leading to a very large value of $P = I\Delta V$. The wire gets hot!

self-check C

What would happen to the battery in this kind of short circuit? ▷ Answer, p. 1043

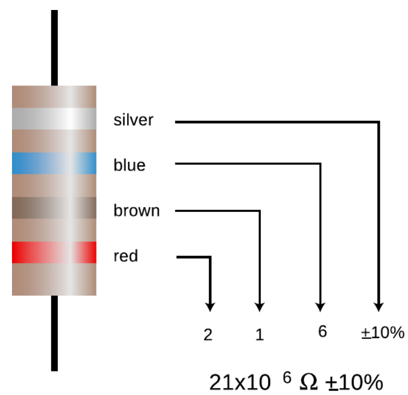
Resistors.

Resistors

Inside any electronic gadget you will see quite a few little circuit elements like the one shown in the photo. These *resistors* are simply a cylinder of ohmic material with wires attached to the end.

r / The symbol used in schematics to represent a resistor.

At this stage, most students have a hard time understanding why resistors would be used inside a radio or a computer. We obviously want a lightbulb or an electric stove to have a circuit element that resists the flow of electricity and heats up, but heating is undesirable in radios and computers. Without going too far afield, let's use a mechanical analogy to get a general idea of why a resistor would be used in a radio.



s / An example of a resistor with a color code.

The main parts of a radio receiver are an antenna, a tuner for selecting the frequency, and an amplifier to strengthen the signal sufficiently to drive a speaker. The tuner resonates at the selected frequency, just as in the examples of mechanical resonance discussed in chapter 18. The behavior of a mechanical resonator depends on three things: its inertia, its stiffness, and the amount of friction or damping. The first two parameters locate the peak of the resonance curve, while the damping determines the width of the resonance. In the radio tuner we have an electrically vibrating system that resonates at a particular frequency. Instead of a physical object moving back and forth, these vibrations consist of electrical currents that flow first in one direction and then in the other. In a mechanical system, damping means taking energy out of the vibration in the form of heat, and exactly the same idea applies to an electrical system: the resistor supplies the damping, and therefore controls the width of the resonance. If we set out to eliminate all resistance in the tuner circuit, by not building in a resistor and by somehow getting rid of all the inherent electrical resistance of the wires, we would have a useless radio. The tuner's resonance would be so narrow that we could never get close enough to the right frequency to bring in the station. The roles of inertia and stiffness are played by other circuit elements we have not discusses (a capacitor and a coil).

black	0
brown	1
red	2
orange	3
yellow	4
green	5
blue	6
violet	7
gray	8
white	9
silver	±10%
gold	±5%

t / Color codes used on resistors.

Many electrical devices are based on electrical resistance and Ohm's law, even if they do not have little components in them that look like the usual resistor. The following are some examples.

Lightbulb

There is nothing special about a lightbulb filament — you can easily make a lightbulb by cutting a narrow waist into a metallic gum wrapper and connecting the wrapper across the terminals of a 9-volt battery. The trouble is that it will instantly burn out. Edison solved this technical challenge by encasing the filament in an evacuated bulb, which prevented burning, since burning requires oxygen.

Polygraph

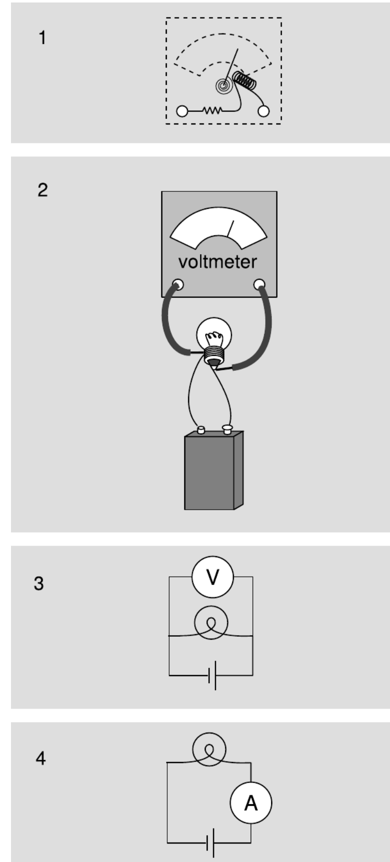
The polygraph, or "lie detector," is really just a set of meters for recording physical measures of the subject's psychological stress, such as sweating and quickened heartbeat. The real-time sweat measurement works on the principle that dry skin is a good insulator, but sweaty skin is a conductor. Of course a truthful subject may become nervous simply because of the situation, and a practiced liar may not even break a sweat. The method's practitioners claim that they can tell the difference, but you should think twice before allowing yourself to be polygraph tested. Most U.S. courts exclude all polygraph evidence, but some employers attempt to screen out dishonest employees by polygraph testing job applicants, an abuse that ranks with such pseudoscience as handwriting analysis.

Fuse

A fuse is a device inserted in a circuit tollbooth-style in the same manner as an ammeter. It is simply a piece of wire made of metals having a relatively low melting point. If too much current passes through the fuse, it melts, opening the circuit. The purpose is to make sure that the building's wires do not carry so much current that they themselves will get hot enough to start a fire. Most modern houses use circuit breakers instead of fuses, although fuses are still common in cars and small devices. A circuit breaker is a switch operated by a coiled-wire magnet, which opens the circuit when enough current flows. The advantage is that once you turn off some of the appliances that were sucking up too much current, you can immediately flip the switch closed. In the days of fuses, one might get caught without a replacement fuse, or even be tempted to stuff aluminum foil in as a replacement, defeating the safety feature.

Voltmeter

A voltmeter is nothing more than an ammeter with an additional high-value resistor through which the current is also forced to flow. Ohm's law states that the current through the resistor is related directly to the voltage difference across it, so the meter can be calibrated in units of volts based on the known value of the resistor. The voltmeter's two probes are touched to the two locations in a circuit between which we wish to measure the voltage difference, u/2. Note how cumbersome this type of drawing is, and how difficult it can be to tell what is connected to what. This is why electrical drawing are usually shown in schematic form. Figure u/3 is a schematic representation of figure u/2.



u / 1. A simplified diagram of how a voltmeter works. 2. Measuring the voltage difference across a lightbulb. 3. The same setup drawn in schematic form. 4. The setup for measuring current is different.

The setups for measuring current and voltage are different. When we are measuring current, we are finding "how much stuff goes through," so we place the ammeter where all the current is forced to go through it. Voltage, however, is not "stuff that goes through," it is a measure of electrical energy. If an ammeter is like the meter that measures your water use, a voltmeter is like a measuring stick that tells you how high a waterfall is, so that you can determine how much energy will be released by each kilogram of falling water. We do not want to force the water to go through the measuring stick! The arrangement in figure u/3 is a *parallel* circuit: one in there are "forks in the road" where some of the current will flow one way and some will flow the other. Figure u/4 is said to be wired in *series:* all the current will visit all the circuit elements one after the other. We will deal with series and parallel circuits in more detail in the following chapter.

If you inserted a voltmeter incorrectly, in series with the bulb and battery, its large internal resistance would cut the current down so low that the bulb would go out. You would have severely disturbed the behavior of the circuit by trying to measure something about it.

Incorrectly placing an ammeter in parallel is likely to be even more disconcerting. The ammeter has nothing but wire inside it to provide resistance, so given the choice, most of the current will flow through it rather than through the bulb. So much current will flow through the ammeter, in fact, that there is a danger of burning out the battery or the meter or both! For this reason, most ammeters have fuses or circuit breakers inside. Some models will trip their circuit breakers and make an audible alarm in this situation, while others will simply blow a fuse and stop working until you replace it.

Discussion questions

A In figure u/1, would it make any difference in the voltage measurement if we touched the voltmeter's probes to different points along the same segments of wire?

B Explain why it would be incorrect to define resistance as the amount of charge the resistor allows to flow.

21.7 ∫ Applications of calculus

As discussed in example 1 on page 582, the definition of current as the rate of change of charge with respect to time must be reexpressed as a derivative in the case where the rate of change is not constant,

$$I = \frac{dq}{dt}.$$

Finding current given charge *example 9*

▷ A charged balloon falls to the ground, and its charge begins leaking off to the Earth. Suppose that the charge on the balloon is given by $q = ae^{-bt}$. Find the current as a function of time, and interpret the answer.

▷ Taking the derivative, we have

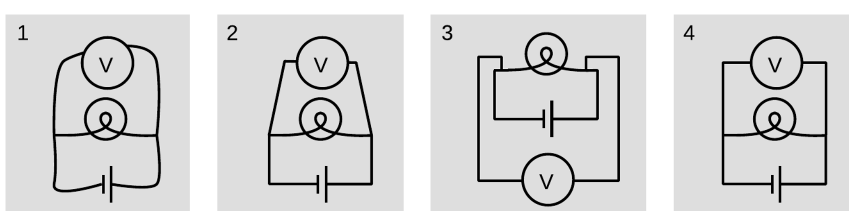
$$\begin{aligned} I &= \frac{dq}{dt} \\ &= -abe^{-bt} \end{aligned}$$

An exponential function approaches zero as the exponent gets more and more negative. This means that both the charge and the current are decreasing in magnitude with time. It makes sense that the charge approaches zero, since the balloon is losing its charge. It also makes sense that the current is decreasing in magnitude, since charge cannot flow at the same rate forever without overshooting zero.

21.8 Series and parallel circuits

Schematics

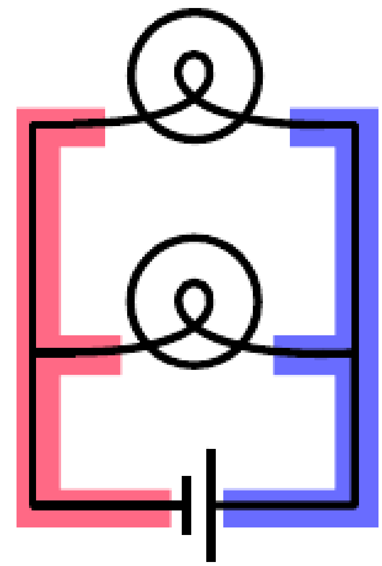
I see a chess position; Kasparov sees an interesting Ruy Lopez variation. To the uninitiated a schematic may look as unintelligible as Mayan hieroglyphs, but even a little bit of eye training can go a long way toward making its meaning leap off the page. A schematic is a stylized and simplified drawing of a circuit. The purpose is to eliminate as many irrelevant features as possible, so that the relevant ones are easier to pick out.



v / 1. Wrong: The shapes of the wires are irrelevant. 2. Wrong: Right angles should be used. 3. Wrong: A simple pattern is made to look unfamiliar and complicated. 4. Right.

An example of an irrelevant feature is the physical shape, length, and diameter of a wire. In nearly all circuits, it is a good approximation to assume that the wires are perfect conductors, so that any piece of wire uninterrupted by other components has constant voltage throughout it. Changing the length of the wire, for instance, does not change this fact. (Of course if we used miles and miles of wire, as in a telephone line, the wire's resistance would start to add up, and its length would start to matter.) The shapes of the wires are likewise irrelevant, so we draw them with standardized, stylized shapes made only of vertical and horizontal lines with right-angle bends in them. This has the effect of making similar circuits look more alike and helping us to recognize familiar patterns, just as words in a newspaper are easier to recognize than handwritten ones. Figure v shows some examples of these concepts.

The most important first step in learning to read schematics is to learn to recognize contiguous pieces of wire which must have constant voltage throughout. In figure w, for example, the two shaded E-shaped pieces of wire must each have constant voltage. This focuses our attention on two of the main unknowns we'd like to be able to predict: the voltage of the left-hand E and the voltage of the one on the right.



w / The two shaded areas shaped like the letter "E" are both regions of constant voltage.

Parallel resistances and the junction rule

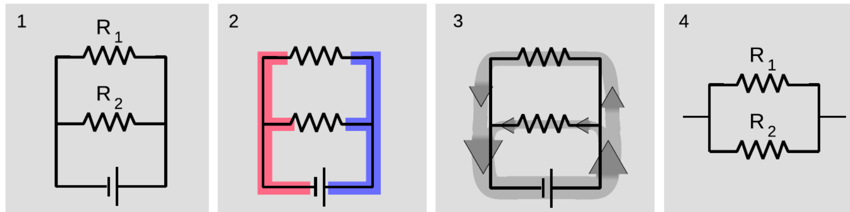
One of the simplest examples to analyze is the parallel resistance circuit, of which figure w was an example. In general we may have unequal resistances R_1 and R_2, as in x/1. Since there are only two constant-voltage areas in the circuit, x/2, all three components have the same voltage difference across them. A battery normally succeeds in maintaining the voltage differences across itself for which it was designed, so the voltage drops ΔV_1 and ΔV_2 across the resistors must both equal the voltage of the battery:

$$\Delta V_1 = \Delta V_2 = \Delta V_{battery}.$$

Each resistance thus feels the same voltage difference as if it was the only one in the circuit, and Ohm's law tells us that the amount of current flowing through each one is also the same as it would have been in a one-resistor circuit. This is why household electrical circuits are wired in parallel. We want every appliance to work the same, regardless of whether other appliances are plugged in or unplugged, turned on or switched off. (The electric company doesn't use batteries of course, but our analysis would be the same for any device that maintains a constant voltage.)

Of course the electric company can tell when we turn on every light in the house. How do they know? The answer is that we draw more current. Each resistance draws a certain amount of current, and the amount that has to be supplied is the sum of the two individual currents. The current is like a river that splits in half, x/3, and then reunites. The total current is

$$I_{total} = I_1 + I_2.$$



x / 1. Two resistors in parallel. 2. There are two constant-voltage areas. 3. The current that comes out of the battery splits between the two resistors, and later reunites. 4. The two resistors in parallel can be treated as a single resistor with a smaller resistance value.

This is an example of a general fact called the junction rule:

> **the junction rule**
> In any circuit that is not storing or releasing charge, conservation of charge implies that the total current flowing out of any junction must be the same as the total flowing in.

Coming back to the analysis of our circuit, we apply Ohm's law to each resistance, resulting in

$$\begin{aligned} I_{total} &= \Delta V/R_1 + \Delta V/R_2 \\ &= \Delta V\left(\frac{1}{R_1}+\frac{1}{R_2}\right). \end{aligned}$$

As far as the electric company is concerned, your whole house is just one resistor with some resistance R, called the *equivalent resistance*. They would write Ohm's law as

$$I_{total} = \Delta V/R,$$

from which we can determine the equivalent resistance by comparison with the previous expression:

$$\begin{aligned} 1/R &= \frac{1}{R_1}+\frac{1}{R_2} \\ R &= \left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1} \end{aligned}$$

[equivalent resistance of two resistors in parallel]

Two resistors in parallel, x/4, are equivalent to a single resistor with a value given by the above equation.

Two lamps on the same household circuit *example 10*

▷ You turn on two lamps that are on the same household circuit. Each one has a resistance of 1 ohm. What is the equivalent resistance, and how does the power dissipation compare with the case of a single lamp?