CASSIOPEIA'S ToE

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SCIENCE INTRODUCTION

Observations of nature quickly reveal motions that seem to follow an order, and the study of those motions led (Newton) to the equations that describe that motion e.g.:

$$\mathbf{F = ma} \qquad \mathbf{p = mv} \qquad \mathbf{E = \tfrac{1}{2}\, mv^2 + V}$$

But there are other equivalent approaches to obtaining equations of motion that generalize to modern physics more easily than Newton's approach. The Hamiltonian approach led to Schrodinger's Equation, and the Lagrangian approach capitalizes on Hamilton's ***principle of least action,*** where the basic Lagrangian function is L = (Kinetic Energy – Potential Energy), and the Action, S, is given by

$$S = \int_{t_1}^{t_2} \mathcal{L}(x_i, v_i)\, dt$$

This approach generalizes to relativity, and to quantum field theory. All you have to do is figure out the Lagrangian for the situation at hand, and solve the equations. The Lagrangian itself will vary its precise form depending on the existence of any of the various fields and forms of energy that we know about.

As a relativistic quantum field theory, the standard model is described by a Lagrangian which can be schematically written as

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \,. \tag{1.2}$$

The various terms are:

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4} G^A_{\mu\nu} G^{A\,\mu\nu} - \tfrac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} \,, \tag{1.3}$$

$$\mathcal{L}_{\text{kinetic}} = \bar{Q}^i_L \, i\not{D}\, Q^i_L + \bar{u}^i_R \, i\not{D}\, u^i_R + \bar{d}^i_R \, i\not{D}\, d^i_R + \bar{l}^i_L \, i\not{D}\, l^i_L + \bar{e}^i_R \, i\not{D}\, e^i_R \,, \tag{1.4}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \tfrac{v^2}{2} \right)^2 \,, \tag{1.5}$$

$$\mathcal{L}_{\text{Yukawa}} = -g^{ij}_u \, \bar{Q}^i_L \, \epsilon H^* u^j_R - g^{ij}_d \, \bar{Q}^i_L \, H d^j_R - g^{ij}_e \, \bar{L}^i_L \, H e^j_R + \text{h.c.} \,. \tag{1.6}$$

(Lagrangians are taken from *Baryon Number Violation beyond the Standard Model* a Thesis by Bartosz Fornal at California Institute of Technology (2014))

Relativity introduces the idea that the motion we want to describe can look very different to observers in different frames of reference. So we require that our theories and equations be the same for all these observers – that they be invariant under certain changes in reference frame. These transformation constraints fall into three main groups.

Translations – changes in the location coordinates between frames
Rotations – changes in orientation between frames
Boosts – changes in (constant) velocity between frames

The group of these transformations that preserve the Lagrangian and thus the laws of physics is called the Poincare Group, and the last two types together form the familiar Lorentz Transformations known best from Einstein's Theory of Special Relativity.

Quantum Mechanics (a precursor to Quantum Field Theory) was the recognition within the last 100 years that objects have both a particle and a wave nature, that recognized that nature at the smallest scales restricts objects to discrete values, and that realized that there are limits on the precision with which some physical quantities can be known. However, it has the limitation of being unable to address the creation and annihilation of particles… a problem that Field Theory addresses.

Also worth mentioning here are our various assumptions about space and time. Starting from equations in a Euclidian, 3-dimensional, flat space, Special Relativity took us immediately to a 4-dimensional, space-time (Minkowski) manifold. But in a Minkowski manifold, space-time is still assumed to be constant, flat, continuous, and differentiable. These are also the spaces that form the backdrop for the Standard Model and Quantum Field Theory (sometimes generalized to more than 4 dimensions), but General Relativity, on the other hand, introduced the idea of a curved and deformable space-time – a Riemannian Manifold.

With all this in mind then, physics has succeeded in building two very successful descriptions of the universe on different scales. Quantum Field Theory and the Standard Model describe the very small while General Relativity describes the very large.

Quantum Field Theory describes all forces and all particles as excitations of several (many) underlying, all-pervasive (gauge) fields. These fields are the fundamental objects. The fields are quantized, and they have symmetries that allow us to put various categories and "flavors" of particles into groups that transform according the rules of Group Theory. Their external and dynamic symmetries give rise to conservation laws of momentum, energy, and angular momentum. And their internal symmetries give rise to conservation laws of charge and spin. Note that ***charge*** here refers to more than the electric charge.

It refers to the generators of the various forces which are best described within symmetry groups: Electric charge for QED, U(1), Color charge for QCD, SU(3), and Weak-Isospin for Weak Interactions, SU(2).

General Relativity describes space as being deformed by the presence of (matter and) energy. If we view space in this manner, then there are no forces necessary to describe the motion of objects in a gravitational field. It is space itself that is curved, and moving objects are simply following the geodesic world lines of the curved space.

Now, let’s proceed to search for a visual model that finds the common ground between these two successful theories.