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Contents

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Editorial

ONCE again we are able to offer you another *Eureka*, this time with production increased by 18·2 per cent. (Readers can make a "guestimate" of production from this.) Thanks are due to all the contributors and to those who wrote to us during the year.

The purpose of this journal is to provide an outlet for the ideas of undergraduates, research students and others and to enable people outside the University to keep in touch with the trends of Cambridge mathematics. *Eureka* has exchange arrangements with many European journals, most of which seem to be of a more serious nature, particularly those from behind the Iron Curtain. If you have anything you would like to contribute to next year's issue, then please send it to us as soon as possible and remember, articles are paid for at the rate of 10s. per printed page.

Readers may be surprised to hear that Cambridge has a "ridge" (or depression?) as well as London. In the Trinity Fellows' Garden where a unique form of bowls is played, woods have for years behaved peculiarly near one end of the lawn. So this term you may see out there a group of "visitors" from Lord's, carefully taking readings on their theodolites. And no doubt, they will pass on their readings to the Trinity mathematicians!

The Archimedeans

THIS year's programme is again very full and varied, with more social activities now than there have ever been before.

There will be the usual evening meetings, when distinguished mathematicians speak on a wide variety of mathematical topics. One talk of a slightly different nature will be given by Professor Pitt of Nottingham University on the problem of mathematical education in this country. Because of their popularity, there will be more tea meetings than last year, at which research students will discuss some of the less serious branches of mathematics. Places to be visited so far include the National Physical Laboratory at Teddington and the Mathematical Laboratory in Cambridge.

On the social side there will be the usual Punt Party and Ramble, and visits to a Gilbert and Sullivan opera in Cambridge, and a theatre in London.

The Music Group will continue to meet weekly, the Bridge Group will cater for all standards of bridge players, and the Play Reading Group will continue to read plays of all kinds. In addition, a Computer Group and a Mathematical Models Group are being started this year. The bookshop will still give better terms to buyers and sellers than the town bookshops.

The Committee has tried to arrange a very interesting programme, and hopes that members will continue to give their support. If any member has any suggestions to make, please do not hesitate to inform the Secretary, either direct or through the suggestions book in the Arts School.

M. W. THOMAS, *Secretary.*

A Schoolmaster's Mathematics

BY F. M. HALL

IT is a truism that a good teacher must be interested both in his pupils and in his subject. Much has been written about the former, so I intend in this article to deal with the second interest, and to say something about the development of my own mathematical thought during my first year's teaching in a secondary school.

Of course a schoolmaster can hardly hope to be a mathematician at the research level. He hasn't the time, and if this is where his abilities lie he must seek other work, most probably as a university lecturer. There is plenty of scope for mathematical thought apart from this, however, and I have myself felt a renewal of my enjoyment in the subject now that I am freed from the pressure of having to learn a large quantity of new work, increasing exponentially in complexity.

I suppose that the chief awakening, or rather re-awakening, of my interests has been in the byways of mathematics. Puzzles, historical details, conjuring tricks (e.g. Möbius strips) and other sidelines are always interesting to boys and their teachers, and there is plenty of scope for their introduction into a lesson. I am having to extend my meagre library of books dealing with this kind of topic, and I am taking an interest, in my case for the first time, in that happy hunting ground of mathematicians, the construction of polyhedra. Historical information is welcomed by boys of all ages and is easily supplied, even to juniors. For example, when beginning algebra I talked about the origin of the word "Algebra," the Arabs and Omar Khayyam, and intend some day to learn how to write the latter's name in Arabic, thus following in the footsteps of a recent Cambridge don.

In most mathematical teaching it is easy to convince the pupils that you know much more than you do, but I have found difficulty in dealing with the philosophical questions beloved of schoolboys, ranging from "Why do we learn geometry?" to "How can mathematics be an exact science when it sometimes gives us two (or more) possible answers to a question?" I am beginning to realise two things: firstly that I must think much more about the philosophical basis of my subject and the teaching of it, and secondly that I must be careful not to say too much about it. It took me a month to live down a careless remark to the first form about the impossibility of drawing parallel lines.

As far as the technical work is concerned, I am taken back to my own schooldays as I struggle once again with the familiar types of problem. Going back to the same kind of work can be dangerous, in that it is easy merely to pass on the teaching received oneself, without enquiring into its merits and demerits. With our university training we are capable of seeing what is really behind the techniques and theories taught, and with care the work can be presented in a way that takes account of the background structure and of modern methods. What I understood imperfectly at school now takes its place in the total system, and coming back to it from the advanced point of view is fascinating to me and I can make it fascinating to my pupils provided I have the background in my mind, even though I cannot communicate all that I know to them at this stage.

Although they do not understand fully, indeed probably *because* they do not understand fully, boys have an uncanny knack of pointing out, and asking questions about, the places of difficulty in an argument or technique. It is essential to understand the work thoroughly oneself, and very true that a person never really understands something until he has taught it. I have had to think quite deeply about points that I had always taken for granted, and have to be careful not to assume "obvious" steps in my arguments. A good example of this is in integration by substitution. Having emphasised to the form that the symbols dy and dx have no meaning by themselves (at their stage, at any rate), I then proceeded to substitute $x = f(t)$ and replace dx by $f'(t)dt$. Not unnaturally this was pointed out forcefully to me and I had some difficulty in convincing them on the spot that all was in order. I have in this way filled in many of the gaps that were left in my knowledge as first acquired, when I had to pass on to further work before consolidating what I had just learnt.

In common with all teachers I dislike some parts of my work. I hate marking, I find it difficult to teach boys who aren't really interested in the subject, I prefer talking myself to watching the pupils do their own very necessary work, and I therefore dislike "revision," which at the time of writing is an all pervading activity. Despite this I have shown, I hope, that there is much to interest me within mathematics, but the real joy of teaching lies always in the interrelationship between my knowledge of the subject and between the pupils to whom I am trying to pass it on.

Follow That Car!

BY E. O. TUCK

TRAFFIC ENGINEERING, the study of large scale movement of vehicles on highways and roads, is a subject of immense and growing importance. Unfortunately, until recently this subject has been in a highly empirical, and to a mathematician, unsatisfactory state. Traffic theory can be usefully compared with the kinetic theory of gases, in that macroscopic traffic flow properties are determined by individual car interactions just as macroscopic gas properties are determined by molecular interactions. The mathematical theory of traffic flow has been held up firstly because there has been no theory of "traffic dynamics" (cf. the dynamics of particles for gases), nor secondly any way of relating large scale statistical properties of traffic to these small scale effects (cf. statistical mechanics for gases). Both of these difficulties have been partially remedied within the last ten years or so, and I propose to give a simple example from the first of these fields, the dynamics of following, or traffic dynamics.

Suppose you are driving the n-th car in a long line of traffic on a straight highway with no passing allowed. The car's acceleration is thus the only variable at your immediate disposal. How do you, in fact, regulate this acceleration? The obvious answer is that you attempt to keep a constant distance between your car and the one in front of you. This could be done in many ways, but the following control law has been found to agree fairly well with experiment. If the position of your car at time t is $x_n(t)$, we suggest

$$\ddot{x}_n(t + T) = k[\dot{x}_{n-1}(t) - \dot{x}_n(t)]$$

where k is a positive sensitivity constant, and T is your reaction time. Notice that, apart from the effect of the reaction time, you maintain constant speed as long as the separation between you and the $(n-1)$th car is not changing, while if this separation is increasing you apply a positive acceleration in order to catch up—all very reasonable. The presence of a reaction time T means that the correction appropriate to the time t is instead applied at T seconds later, which you may expect will mess up things a little. This equation is still a gross over-simplification, ignoring as it does the effects of passing, emergency braking, and back seat drivers, but is nevertheless a reasonable starting point.

Let us consider the motion of the n-th car, subsequent to a sudden and complete stop by the $(n-1)$th car. Then we have

$$\ddot{x}_n(t + T) + k\dot{x}_n(t) = 0.$$

Try a solution of the form e^{at}, which requires that

$$a\, e^{aT} + k = 0.$$

This equation to determine a has an infinite number of roots, but we can concentrate attention on the root, say a_0, with largest real part, since this will dominate for large t. The investigation of the roots of this transcendental equation is an interesting elementary problem (see Herman, p. 89), with the following conclusions:

(1) If $kT < 1/e$, a_0 is real and negative. Thus the solution is damped and non-oscillatory.

(2) If $1/e < kT < \pi/2$, a_0 is complex and has negative real part. Thus the solution is damped but oscillatory.

(3) If $kT > \pi/2$, a_0 is complex and has positive real part. Thus the solution is undamped and oscillatory.

In both cases (1) and (2) you are safe, and you will probably get out of your car and swear at the driver in front. However in case (2), do not be surprised to hear an almighty crash about six cars behind you as your erratic braking is amplified down the line until it causes a collision. In case (3), pray.

Notice that, as we should expect, safe drivers are drivers with small reaction times T. However, the additional conclusion that safe drivers have low sensitivities k, is not on the face of it quite so obvious. This conclusion implies that the taxi driver (mini-cab probably) breezing along with one hand on the wheel, flicking his cigarette ash out of the window while he argues with his passenger, is a safer driver than the matron with both hands glued on the wheel, eyes similarly glued on the car in front, and foot ready poised over the brake. This is probably perfectly correct, though I doubt if it will console you if you fail your driving test.

Experiments have shown that human drivers have values of kT clustered just below the $1/e$ level, which is just as well. However, as we have seen, the presence of just one slow-witted or over-sensitive driver in a long line of cars (e.g. a driver in class (2)) can and does cause chain collisions further down the line in which the guilty driver is not even involved. He (perhaps more commonly she) will then glance behind and mutter some comment about other people driving dangerously.

I recommend this field as an example of the power of even quite elementary mathematics to enliven an empirical and apparently stagnant subject. The original paper on traffic dynamics was by L. A. Pipes, *Journal of Applied Physics*, **24**, 274 (1953), while the example in this article is taken from Herman *et al.*, *Operations Research*, **7**, 86 (1959). A recent paper, with a bibliography, is G. F. Newell, *Operations Research*, **9**, 209 (1961).

Relations

BY J. F. HARPER

EVEN the most applied of mathematicians will have heard of reflexive, symmetric and transitive dyadic relationships, but there are many others commonly met with but not so often studied in detail.

To refresh the memories of readers, a relationship **R** between two (whence "dyadic") members x, y of a set $\{X\}$ is

reflexive, if $x\,\mathbf{R}\,x$ (e.g. $x = x$)

symmetric, if $x\,\mathbf{R}\,y$ implies $y\,\mathbf{R}\,x$

transitive, if $x\,\mathbf{R}\,y$ and $y\,\mathbf{R}\,z$ together imply $x\,\mathbf{R}\,z$

Since for a given relationship any of these three properties may hold always (i.e. for all x, y in $\{X\}$), sometimes (i.e. for some x, y but not others), or never, there would appear at first sight to be 27 different kinds of dyadic relationship, one for each combination of these properties. In fact 12 of these are logically impossible if there exist any x, y such that $x\,\mathbf{R}\,y$, and it seems not without interest to give an example of each of the other 15.

The set $\{X\}$ will for this purpose be taken as the set of all law-abiding citizens of Great Britain who know their own names.

The abbreviations in the table are as follows:

R = reflexive.	*a* = always.
S = symmetric.	*s* = sometimes.
T = transitive.	*n* = never.

R	S	T	
a	*a*	*a*	x is related to y.
a	*a*	*s*	x and y know each other's names.
a	*s*	*a*	x is not liked more than y.
a	*s*	*s*	x is a descendant of y's grandfather.
s	*a*	*a*	x and y are both male.
s	*a*	*s*	x and y have exactly one friend in common.
s	*s*	*a*	x is a son of y's father.
s	*s*	*s*	x admires y.
n	*a*	*s*	x is a cousin of y.
n	*a*	*n*	x is married to y.
n	*s*	*s*	x is a brother of y.
n	*s*	*n*	x is the only brother of y.
n	*n*	*a*	x is an ancestor of y.
n	*n*	*s*	x knows y's name, but y doesn't know x's.
n	*n*	*n*	x is the father of y.

It will be noticed that there is in the table a counter example to the commonly held belief that a symmetric transitive relationship must also be reflexive, viz "x and y are both male." This is possible because this relationship divides the set $\{X\}$ into two mutually exclusive parts such that in one (law-abiding male citizens of Great Britain who know their own names) the relationship is reflexive, symmetric and transitive, that in the other (law-abiding female citizens of Great Britain who know their own names) it is never true, and that if x belongs to one of these subsets and y to the other it is also never true.

One can without difficulty produce mathematical examples of most of the 15 types of relationship given above (e.g. if $\{X\}$ is the set of all non-zero real numbers, $x/y < 1$ is $R_nS_nT_s$), and the construction of a complete set is left as an exercise for the reader.

Love Under the Integral Sign

By A. H. Faulkner

This story takes place in 1984 when we join our hero Lemma Jordan as he enters Great Gate for the first time. Above him Henry VIII no longer holds a chair leg but a red and green flashing sign—"JOIN T.M.S." Lemma goes to his room in New Court and finds it rather cold and damp. Some things have not changed with the years. While Lemma is settling in we note a few facts about Trinity and T.M.S. Trinity is now a mixed College all the week. Also T.M.S. has just outbid the Quintics for control of the Archimedeans with a final bid of $\frac{1}{2}x/$—, $\sqrt{2}$ apples and $\frac{1}{2}\pi$ sticks of chalk per member as well as free use of the worst blackboard in Europe (Oxford now has a new one). Further to break the market monopoly, T.M.S. has gone into apple-growing and is now the largest apple producing College Mathematical Society outside the United States.

With Lemma now settled in, we rejoin him as he listens to his radio provided by local electronics interests. He is thinking of his national service on Mars when he is interrupted by "Join T.M.S.—for bright sparkling meetings. Nine out of ten mathematicians join T.M.S. See your College Rep. NOW. . . . Do you find your Maths problems won't come out? Then drink Bournvillicks every night. Remember Newton drank Bournvillicks and scientists have proved that more people get Firsts on Bournvillicks. Be fresh at 9 a.m. Buy some now."

His first lecture the next morning is a broadcast one, so at 9.5 a.m. he switches on his bedside television. "The Quintics bring you the first instalment of 'The Conic and You.'" The lecturer is seen writing on his bedside blackboard, and eventually ends with

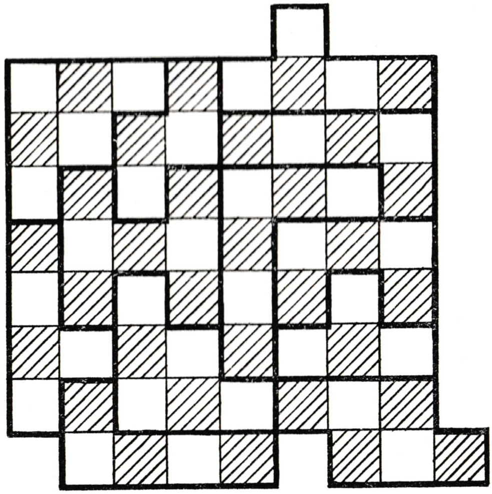
"Tune in next week for the proof of the exciting theorem which Pascal discovered over tea."

Then Lemma has to go to the Arts School for a lecture and there *she* is. So it isn't long before they are seen walking through the Abelian fields and round the semi-circle at infinity in the upper half-plane. (Which is alright as they are convergent). But she won't marry him unless he is president of T.M.S. Unfortunately Lemma is not sponsored by the Committee, so a long and bitter campaign follows, culminating in a T.V. meeting where they outline their policies on such matters as Fermat's last theorem and future apple buying. Suffice it to say that all goes well and they get married and live happily till the Tripos.

EDSAC to the Rescue

By B. D. Josephson

Some years ago I was given a puzzle which consisted of twelve pieces of cardboard, which were shaped as shown in the diagram and had to be fitted together to form a chessboard. I had spent quite a time on it without finding a solution, and so on reading the articles by Dr. Miller and by Drs. C. B. and J. Haselgrove in last year's *Eureka*, about solving such problems on an electronic computer, I decided to try it out on the Mathematical Laboratory computer EDSAC II.



While I was writing the programme various people attempted the puzzle without success, the longest attempt being a systematic search for six hours. The basic ideas of the programme are essentially the same as those described by the Haselgroves and the following description of the modifications made should if possible be read in conjunction with their article.

In the first place advantage was taken of the logical orders which can be used with EDSAC. For example, the method used to find whether it was possible to put a piece on the board, with its leading square in a given position, is as follows. The state of the board is represented by a number whose n-th digit is 0 if square n is occupied and 1 otherwise. In this way the whole of the board can be stored in two registers. The shape of the piece one is trying to put in is represented similarly by supposing it to be placed on the board with its leading square on square 1, and using 1's for full squares and 0's for empty ones. The number representing the state of the board is put into the accumulator and shifted to the left, discarding the zeros at the beginning, till the 1 representing the square one is trying to fill becomes the first digit. Let the resulting number be denoted by a, and the number specifying the orientation of the piece by b. Then the piece can be inserted if and only if to every 1 in b there corresponds a 1 in a, or symbolically $a_n b_n = b_n$. But the logical product c of a and b is just the number whose n-th digit is $a_n b_n$, so one only needs to test whether $b = c$, and this checks every square simultaneously. If the piece can be inserted, the subsequent state of the board is represented simply by $a - b$.

The following method was used to reduce the number of orientations to which the above test had to be applied. First the machine looked at the colour of the square which was to be filled, and removed from the list all orientations of the pieces which had the wrong colour for their leading square. It then looked at the neighbouring squares, and if they were already occupied it removed from the list all the orientations which would cover that square. The process of removing from the list was done by multiplying logically by a number which had 0's for the digits to be removed and 1's for the others.

Another modification to the programme was to make the machine print out its results directly in the form of a diagram consisting of 64 letters in an 8×8 array, with A's representing one piece, B's another and so on.

When the programme was run first, it was arranged that the machine should print out diagrams every time it got eleven pieces in, and to stop after getting all twelve in. It churned out tape almost continuously for nearly three minutes before finding a correct solution. The tape took over half an hour to print out and had 151 diagrams with eleven pieces in, and so it is clearly somewhat difficult to fit in the last piece. This conclusion was confirmed when the programme was modified so as to find all the solutions. To reduce the amount of computation necessary, the machine was made to look only at possibilities with the centre of the H-shaped piece in the top half of the board. This method will find all the essentially different solutions, since the others can be obtained from these by rotation through 180°. Two solutions were found, in two minutes machine time.

A discussion arose as to what would happen if one ignored the colouring of the pieces, and the programme was accordingly modified and run for three minutes during which time less than a fiftieth of the possibilities had been worked through and three solutions had been found.

Finally I should like to thank the staff of the Mathematical Laboratory for permission to use EDSAC, and various colleagues who have offered useful advice on the project.

(The solutions to the original problem can be found on page 32.)

The Gulf Stream

BY KATHLEEN JOHNSON

THE Gulf Stream was well known to sailors in the early 1500's, for it is recorded that Spanish ships bound for America went by way of the Equatorial Current, but on their journey home they followed the Gulf Stream as far as Cape Hatteras and then sailed eastward to Spain. In this way they have favourable winds and currents over the whole voyage. However by the 1770's these facts appeared to be unknown to H.M. mail packets, who used to take two weeks longer to make the journey from England to New England than did the merchant ships. The Postmaster General discussed the problem with an American sea captain, and was told that the mail packets continued to sail against the current of three miles an hour since "they were too wise to be counselled by simple American fishermen."

More recently several Mathematical theories have been advanced to explain the circulation of the oceans and more particularly why the oceans have strong currents, such as the Gulf Stream, on their western edges in both hemispheres.

In setting up the equations of motion applicable to the oceans, account has to be taken of the Earth's rotation. This effect on the horizontal equations of motion can be represented by postulating an apparent deviating force, called the Coriolis force, which is proportional to the velocity of the fluid element, the angular velocity of the earth, and the sine of the latitude. It acts in a direction perpendicular to the velocity. Over most of the ocean the velocities are so small that the Coriolis force is large compared with the inertia and viscous forces. A good approximation to the flow therefore, is obtained by retaining only the Coriolis and pressure terms in the horizontal equations of motion. This is known as the Geostrophic Relationship. As the vertical motions are very small, the vertical equation of motion is approximated to by the hydrostatic equation and using these equations, one can calculate the geostrophic currents from the density field.

The early theories suggested that winds are the driving mechanism of the oceans. If a chart of the winds over an ocean is compared with the currents, it is seen that their directions and strength roughly correspond. But there is no intensification of winds over the western edges of the oceans to explain the strong currents there. In 1950 W. H. Munk succeeded in deducing many features of the ocean circulation by retaining the frictional terms in the equations of motion. In his model he used a mean annual wind stress over a rectangular ocean with a suitable value of the eddy viscosity. His results show a strong current at the western edge of the ocean and also a counter current east of this. The theoretical counter current is very nearly equal to that observed. Over the rest of the ocean he finds that the frictional terms are negligible and that his solution is the same as that obtained by neglecting the frictional forces. The cause of the asymmetry in the ocean's circulation can be found in the variation of the Coriolis force with latitude. If the ocean were flat and rotating with a constant angular velocity, there would be no such asymmetry.

Another theory which also explains the existence of strong currents on the western edges of the oceans was put forward by Morgan in 1956. This theory retains the inertia terms in the equations of motion, but neglects the viscous terms. Although it does not predict a counter current, it is superior to Munk's in that it does not involve an unknown parameter, the eddy viscosity. The inertial theory is also interesting as it is the only known example of an inertial boundary layer, whereas viscous boundary layers are a well known phenomena.

This article has failed to mention other factors influencing the circulation of the oceans, such as the motion caused by the non uniformity of heat salinity. I hope, however, that it has indicated how such a variable and complex phenomenon as the Gulf Stream can be tackled by Mathematical techniques.

A Method for Calculating Partitions

By M. Rowan-Robinson

A *partition* of an integer n is a representation of n as the sum of any number of positive integral parts;
e.g. $5 = 4 + 1 = 3 + 2 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 1 + 1 + 1 = 1 + 1 + 1 + 1 + 1$, are the 7 partitions of 5. The number of different partitions of n is denoted by $p(n)$, i.e. $p(5) = 7$. The order of the parts is arbitrary, so they will be arranged in decreasing order of magnitude, as in the example above, i.e. a partition of n is given by $n = b_1 + b_2 + \ldots + b_s$ where $b_1 \geqslant b_2 \geqslant \ldots \geqslant b_s \geqslant 1$ and $1 \leqq s \leqq n$. Denote by $p_r(n)$ the number of partitions of n which have $b_1 = r$.

$$\text{Then } p(n) = \sum_{r=1}^{n} p_r(n) \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

The $p_r(n)$ are then arranged in a triangular array:—

$n =$	1	2	3	4	5	6	7	8	9	10	11	....n....
$p(n) =$	1	2	3	5	7	11	15	22	30	42	56	....$p(n)$....
$i = 1$	1	1	1	1	1	1	1	1	1	1	1	....$p_n(n)$....
2		1	1	1	1	1	1	1	1	1	1	....$p_{n-1}(n)$....
3			1	2	2	2	2	2	2	2	2	..
4				1	2	3	3	3	3	3	3	..
5					1	3	4	5	5	5	5	..
6						1	3	5	6	7	7	..
7							1	4	7	9	10	..
8								1	4	8	11	..
9									1	5	10	..
10										1	5	..
11											1	..
..												..
..												..
$n - r + 1$												..$p_r(n)$....
..												..
..												..
n												$p_1(n)$....

There are no elements below the diagonal since $p_r(n)$ is undefined if $r < 1$. Clearly $p_n(n) = p_{n-1}(n) = p_1(n) = 1$. $p(n)$ is obtained by adding all the elements in the n-th column [by equation (1)]. The interesting feature of the array is that it may be arrived at without calculating any of the $p_r(n)$ directly.

The differences between adjacent elements along any particular *row*, say the i-th, are given by the $(i - 1)$th *column*, read from the bottom:

$$p_r(n) - p_{r-1}(n - 1) = p_r(n - r) \qquad \ldots \qquad \ldots \qquad (2)$$

The two numbers on the left hand side of this equation are adjacent elements in the $(n - r + 1)$th row, and the right-hand number is the r-th element up the $(n - r)$th column.

E.g. the 6th row is 1 3 5 6 7 7 7....
Differences are: 1 2 2 1 1 0 0....
And the 5th column is: 1 2 2 1 1 (read from the bottom).

At a certain point along each row, the differences become zero, corresponding to when the top of the associated column is reached. [$p_r(n) = 0$ if $r > n$]. If $2r \geqslant n$,

$$p_r(n) = p(n - r) \qquad \ldots \qquad (3)$$

The triangular array can thus be built up by writing down the first row (all elements unity) and then applying the rule that whenever the j-th column is known, the $(j + 1)$th row may be built up using the elements of the j-th column as differences between adjacent elements in the row. This method is based on equation (3), which is easily proved (and is almost obvious) and on equation (2):

$$p_r(n) - p_{r-1}(n - 1) = p_r(n - r)$$

By (3) this is true for $2r \geqslant n$. For $2r < n$, the proof seems to be more complex, and has so far only been achieved for a few cases.

Letter to the Editor

Dear Sir,

I was interested to read Mr. Hall's article on "Group Theory in the Sixth Form" and hope it may stimulate others to repeat this experiment. I have myself made a similar attempt for several years, by introducing Abstract Algebra to the first year university honours class in their first term, before going on to the usual Linear Algebra. My conclusions with this, not dissimilar, audience in general resemble Mr. Hall's. I have one additional suggestion to make; at the start I discuss equivalence in some detail and prove the basic theorem about equivalence classes.

This has several advantages. First, it leads to the investigation of the theory of congruences, an unfamiliar, relatively easy, interesting topic, which also provides welcome illustrations for the later group theory. Next, much of the later work is simplified by using the idea of equivalence; indeed, the course tends to become "reflections on equivalence classes." The integers, rational numbers, complex numbers, are all equivalence classes of suitably chosen number pairs; abstract groups are equivalence classes of groups under isomorphism; cosets of a subgroup H are equivalence classes under

"a related to b means $a = bh$ for some h in H";

conjugate sets in a group G are equivalence classes under

"a related to b means $a = tbt^{-1}$ for some t in G."

Equivalent, similar, and congruent matrices yield further illustrations, and at every stage the exposition is simplified.

Lastly, there is here a fruitful field for the invention of illustrative examples, with numbers, with other mathematical elements, or from "real life," both of equivalence relations and of relations which have only one or two, or even none, of the reflexive, symmetric and transitive properties. I like to leave this as an exercise for the first week-end of term, emphasising the need to be precise not only about the relation but also about the set in which it operates. (The relation "*a* is the brother of *b*" in the set of students at King's is symmetric in Cambridge but not in London.) I find that this investigation is within the grasp of even the most daunted freshman, that as a result they take more kindly to abstract definitions and are more careful to be precise; vagueness about details is followed by disaster more immediately and obviously in abstract algebra than in analysis, where rigour is usually first taught. This year my standard example of a relation which is not transitive, "*a* related to *b*" means that one of *a*'s parents is a parent of *b*, sparked off a discussion on marriage laws, polygamy, divorce. . . .

Subsequent confidence, I suspect, is built up when one has learned to construct examples and counter examples in whatever abstract realm one may enter. This year a Nigerian newcomer produced as the set "confident candidates discussing their answers to an examination question" and the two relations "*a* thinks that *b*'s answer is right" and "*a* thinks *b*'s answer is wrong." I agreed that the first is an equivalence relation; to my unwary comment that the second need not be symmetric came the quiet rejoinder, rebuking my insensitivity to the finer points of English, "I said that they were *confident* candidates."

Yours sincerely,

MARY HARTLEY.

University College,
Legon, Accra,
Ghana.

Another Basic Theorem

BY A DEGENERATE SINGULARITY

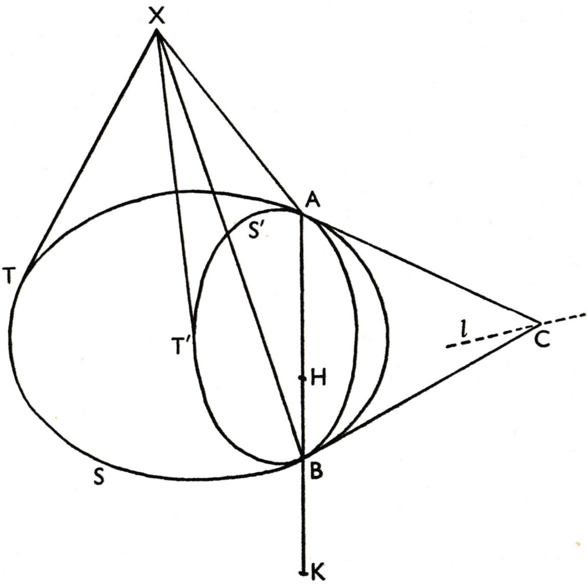
AVID and assiduous readers of this journal will remember the deep and intricate result proved by old friend "A Very Twisted Cubic," in *Eureka* 20, namely, given 3 collinear points in order, A, B, C, then AB = BC. The recipe for the proof was essentially (i) projective geometry, (ii) elementary geometry and (iii) good plain cookery. The methods used below are "the mixture as before" and readers are advised not to imbibe too large quantities immediately before retiring.

With great pleasure, we are at last able to announce a time honoured conjecture, whose truth has long been in doubt.

Theorem: In a plane, there exists at least one fixed conic that passes through all points of the plane.

We consider the following problem:—

S and S′ are two conics having double contact at A and B and C is the common pole of AB. What is the locus L of a point X such that X(A,B;T,T′) is harmonic, where XT, XT′ are tangents to S,S′ respectively?



The required conclusion will be obtained by projecting in two different ways. First, let A,B go into the circular points at infinity I,J. Then S and S′ become concentric circles S_1, S_1' with centre C_1 and the problem is transformed into:—

Find the locus L_1 of points X_1 such that X_1T_1 and X_1T_1', tangents to S_1,S_1' respectively, are at right angles.

Now since the quadrilateral $C_1T_1X_1T_1'$ has 3 right angles, it is a rectangle and so:—

$(C_1X_1)^2 = (C_1T_1)^2 + (C_1T_1')^2 =$ sum of squares of radii of S_1,S_1'.

Hence the locus of X_1 is another complete concentric circle. Moreover, by altering the radii of S_1 and S_1', *any* fixed point can be made to lie upon such a locus L_1. Thus in the original figure we have: the required locus L is a conic which may be made to pass through any point of the plane by altering the conics S and S′ in such a way as to always touch AC, BC at A,B respectively (which we know can be done, there being one degree of freedom).

Again, take any fixed pair of points H,K on AB such that they separate A,B harmonically, and project H,K into the circular points I,J. Then S and S′ are transformed into rectangular hyperbolas S_2,S_2' with common centre C_2 and asymptotes C_2A, C_2B (AB being the line at infinity). The problem becomes:—

Find the locus of points X_2 such that tangents X_2T_2, X_2T_2', one to each of S_2 and S_2', are harmonically separated by X_2A and X_2B, i.e. X_2T_2 and X_2T_2' are equally inclined to the asymptotes which we will take as the x and y axes.

Now any point on the x axis is a point of the locus for the x axis is a tangent (at infinity) to both S_2 and S_2' and these lines (i.e. the x axis taken twice) are equally inclined to the axes (by the usual conventions of projective geometry): similarly, any point on the y axis is a point of the locus and so the required locus (which we know from the first projection to be a conic), is the line-pair $xy = 0$, *whatever pair of hyperbolas of the system are chosen.* Hence in the original figure the locus L is a *fixed* conic, whatever pair of conics S,S′ are taken touching AC, BC at A,B.

Thus we arrive at the climax. By the first projection, all the points of the plane lie on *some* locus L as S,S′ vary, and by the second projection, all points of these loci belong to a *fixed* conic so that our astounding conclusion is unavoidable.

A better known result now follows quickly.

Corollary: $1 = 0$.

Consider one *fixed* pair of conics S,S′ in the above problem. How many points, *other than C itself*, of the locus L that corresponds to S,S′, lie on a general line l through C?

Let l_1 and l_2 be general lines through C_1 and C_2 respectively. There are 2 points of L_1 on l_1 and *no* points of L_2 on l_2, so that, as these results are projectively related, we have:

$$2 = 0$$

Halving both sides, the desired result is obtained.

wordgame

ADDDITION
SUBRACTION

HOM O MOR P HIS M
B RACKET S
INequality

better? . . .

$e^{xponential}$
log_{arithm}

(*c*,*d*,*t*,*s*)

d/differentiate

co ∽ ine

more? . . .

Union
∩ntersection
INTER*POL*ATE
EXTRAPOLATE**

EXT E N D
PERTUMATION
ar l
ra dom v iab e
n

Ph dhesis

more? send them in to ROBIN

Problems Drive 1961

Set by B. D. Josephson and J. M. Boardman

A. When A was three times as old as B was the year before A was a half of B's present age, B was 3 years younger than A was when B was two thirds of A's present age. A's and B's ages now total 73. How old are A and B?

B. Find the next term in each of the following series:—

(*a*)	0	1	5	19	65	..		
(*b*)	1	5	14	30	55	..		
(*c*)	2	3	8	30	144	..		
(*d*)	2	5	11	17	29	37	53	..

C. Show by diagrams all the possible configurations of five soap bubbles in space—e.g. each bubble enclosed by the next larger, or all the bubbles free from each other.

D. Arrange 15 balls, 3 of each of 5 colours, in the triangular array below, so that no two of the same colour lie in any line parallel to any side of the triangle.

```
        O
       O O
      O O O
     O O O O
    O O O O O
```

E. Find the missing digits, denoted by asterisks, in this long division, which has no remainder and is in the scale of ten.

```
        *****
     ___________
    ***)*******
        ***
        ----

        ****
         ***
        -----

          ****
          **8*
          ----
          ....
```

F. In how many ways can the following sums of money be made up, using only sixpences, shillings, florins, and half-crowns?

(*a*) 2s. 6d., (*b*) 4s. 6d., (*c*) 6s.

G. Three towns, A, B, and C, form a triangle. Each pair is joined by a number of telephone lines, and a call between two towns may go either direct or via the third town. If there are 155 routes between B and C (including those through A), and 90 routes between A and C (including those through B), how many routes are there between A and B (including those through C)?

H. In how many distinct ways can the faces of a cube be coloured using every colour at least once, if the number of colours available is:—

(*a*) 2, (*b*) 5?

I. Rearrange the order of the following so as to make true statements.

(*a*) 1 2 3 4 6 + + + =

(*b*) a a b b c c + − − − = ( )

(*c*) angles are has less more no not than than triangle which 70° 2

(*d*) is not parallelogram rectangle a every

J. Solve the following cross-number problem (in the scale of ten), where a, b, c, d, are positive integers.

ACROSS: 1. $b - 2$

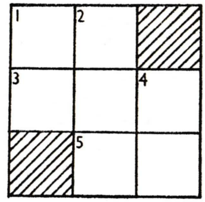
3. $a^2 - c$

5. $3(a + d)$

DOWN: 1. $c^2 - 1$

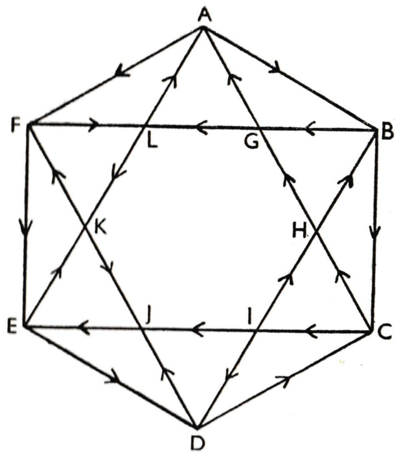
2. $b^2 + d^2$

4. $4a + 2c$



K. Express each integer from 1 to 11 inclusive using exactly three 7's, and the usual arithmetic signs. The symbols for integer part, and functions using letters are not allowed.

L. The map below represents one-way street system of a certain university city, the direction in which travel is allowed being indicated by arrows. An undergraduate living at A wishes to cycle round the city, visiting each intersection just once, and returning to A. What route must he take?



A Dimensional Howler

In this year's G.C.E. "O" Level Examination (Joint Matriculation Board) one candidate, a girl, wrote:

1 sq. yd. costs 18d.

$\therefore$ 1 yd. costs $\sqrt{18}$d. $= 4{\cdot}243$d.

$\therefore$ 1 ft. costs $1{\cdot}414$d.

$\therefore$ 1 sq. ft. costs $(1{\cdot}414)^2$d. $= 2$d.

The Bishop Squared

In our last issue (No. 23), readers were invited to try to "square the bishop" and below are three of the solutions received:

B I S H O P
I L L U M E
S L I D E S
H U D D L E
O M E L E T
P E S E T A

independently from A. L. Cooil of Derby, England, and J. M. Dagnese of Cambridge, Massachusetts, U.S.A., and:

B I S H O P
I N H E R E
S H A R P S
H E R M I T
O R P I N E
P E S T E R

B I S H O P
I M P A L E
S P I N E T
H A N G A R
O L E A T E
P E T R E L

from respectively A. R. B. Thomas of Tiverton, Devon, and independently, R. W. Payne of Dartford, Kent, J. D. E. Konhauser of Pennsylvania, U.S.A., and M. Rumney of London, E.5.

In reference to the punctuation puzzle which appeared in *Eureka* No. 18, L. Mercer of London, N.6, suggests increasing the number of *had's* to twelve. Of two compositors setting similar work in type it could be said that although Smith, where his mate had had "HAD HAD," had had "*had had,*" "HAD HAD" had had the foreman's approval.

Mathematical Association

President: J. T. Combridge, Esq., M.A., M.Sc. (King's College London)

The Mathematical Association, which was founded in 1871 as the *Association for the Improvement of Geometrical Teaching*, aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

The subscription to the association is 21s. per annum: to encourage students, and those who have recently completed their training, the rules of the Association provide for junior membership for two years at an annual subscription of 10s. 6d. Full particulars can be had from The Mathematical Association, Gordon House, Gordon Square, London, W.C.1.

The *Mathematical Gazette* is the journal of the Association. It is published four times a year and deals with mathematical topics of general interest.

Problems Drive Solutions

The average marks (out of 10) obtained by the 24 pairs competing are given in brackets. The highest total was 82.

A. (1·3) A is 39, B is 34.

B. (1·8) (*a*) 211 ($3^{n-1} - 2^{n-1}$), (*b*) 91 (sum of first n squares), (*c*) 840 ($n! + (n-1)!$), (*d*) 67 (next prime after n^2).

C. (3·3) There are 20, best found by induction on the number of bubbles.

D. (6·7) There are two essentially different solutions:—

A		C
D E		E A
C A B		B C D
E B C D		D E A B
B D A E C	and	A B C D E

E. (2·0) The division is 498)5980482, quotient 12009.

F. (5·6) (*a*) 5, (*b*) 13, (*c*) 24.

G. (2·1) 270. There are 11 lines between B and C, 24 between C and A, and 6 between A and B.

H. (1·6) (*a*) 8 by counting cases.

(*b*) 75. One colour must be repeated: if on adjacent faces gives $\frac{1}{2} \times 4!$ ways, if on opposite faces gives $\frac{1}{2} \times 3!$ ways.

I. (6·3) (*a*) $1 + 3 + 4 = 2 + 6$.

(*b*) Several ways, e.g. $a + b - b - (c - c) = a$.

(*c*) No triangle has more than 2 angles which are not less than 70°.

(*d*) Not every parallelogram is a rectangle.

J. (0·9) Across: 1.29; 3.479; 5.78. Down: 1.24; 2.977; 4.98. In fact, $a = 22$, $b = 31$, $c = 5$, $d = 4$.

K. (6·0) $1 = \{\sqrt{(7 \times 7)}\}/7$, $2 = (7 + 7)/7$, $3 = 7/{\cdot}7 - 7$, $4 = 7 - \sqrt{(7/{\cdot}\dot{7})}$, $5 = 7/({\cdot}7 + {\cdot}7)$, $6 = 7 - 7/7$, $7 = 7 \times 7/7$, $8 = 7 + 7/7$, $9 = \{\sqrt{(7 \times 7)}\}/{\cdot}\dot{7}$, $10 = \{\sqrt{(7 \times 7)}\}/{\cdot}7$, $11 = 77/7$.

L. (5·6) AFLKJEDCIHBGA, unique.

Book Reviews

Integral Quadratic Forms. By G. L. WATSON. (Cambridge University Press.) 30s.

The Theory of Numbers is a subject whose results are easy to state and comprehensible but in which the proofs are often complicated. Its spirit is typified by Dr. Watson's book which, in spite of its slim, retiring appearance, includes a complete and detailed account of the fundamentals of quadratic forms. It caters for the specialist in this field and although the author assumes no previous knowledge of the subject on the part of his reader, owing to its intricacy and the hard work necessary to obtain results, the book will not recommend itself to the more general reader.

The book deals with such questions as which integers can be represented by certain types of quadratic forms, what forms can represent zero, and the division of quadratic forms into various types of equivalence classes by means of transformations. If possible, canonical forms for these equivalence classes are found, and, mainly through the use of the Hilbert symbol, $(a, b)_p$, invariants by means of which the classes may be distinguished.

The last chapter is on rational automorphs and in particular their factorisation (an automorph of a form being a transformation which leaves the form unaltered).

It is surprising that more use is not made of results and methods of Linear Algebra and also of p-adic numbers as such; the author seems to favour an "elementary" approach throughout resulting in a certain loss of elegance. Several very recent results are included and the presentation is made tidier by bringing all the footnotes together at the end of the book. P. PLEASANTS.

Homology Theory. By P. J. HILTON and S. WYLIE. (Cambridge University Press.) 75s.

For some years there has been a need in Algebraic Topology for a book that could provide the beginner with an introduction as good as that given 35 years ago by H. Seifert and W. Threlfall in their "Lehrbuch der Topologie." P. J. Hilton and S. Wylie have now done this, and have produced an excellent introduction to those techniques that are now so basic to the subject.

Their book is divided into two parts; in the first they discuss the Homology Theory of Polyhedra and in doing so, bring in quite a lot of Homological Algebra. The second part starts with a fairly rapid account of the Homotopy Theory required later in the book followed by chapters on Obstruction Theory, Singular Homology and Cohomology.

Finally there is an admirable description of spectral sequences and their applications that will help to take away much of the mystery associated with this part of the subject.

Throughout the book there are helpful examples, counter-examples and exercises. It is certain to be a necessary purchase for all who wish to learn the fundamental ideas of Homology Theory. M. MOSS.

A Second Course in Statistics. By R. LOVEDAY. (Cambridge University Press.) 10s.

Essentially a school text book, this is a clear readable explanation of the application of some of the elementary ideas of probability, and of the standard statistical tests. For an undergraduate mathematician it is too elementary, being content with bald statements of conclusions untrammelled by any questioning why. On the other hand, for a natural scientist who has accumulated much data with no clear idea about its reduction, χ^2, Poisson distributions and regression lines need remain mysteries no longer. The wealth of amusing illustrative examples and exercises does much to bring the book to life.

F. P. BRETHERTON.

Special Functions of Mathematical Physics and Chemistry. By I. N. SNEDDON. (Oliver and Boyd.) Second Edition. 10s. 6d.

This new edition is identical with that reviewed in *Eureka* No. 21, except that the type has been reset and the printing is larger. It is a pity that all the errors pointed out in the previous review have not been corrected in the new edition, as this limits its use by research workers, who would otherwise find it a very useful book. K. JOHNSON.

Figures for Fun and *More Figures for Fun.* By J. A. H. HUNTER (Phoenix House, London.) 10s. 6d. and 11s. 6d. respectively.

The author is a retired Commander in the Royal Navy who hated mathematics at school yet when he graduated from the Royal Naval College at Dartmouth, was awarded the prize for mathematics. He has done much through his newspaper columns to spread the popularity of the lighter side of the subject throughout the Commonwealth.

Each book contains one hundred and fifty problems each in the form of a small story. These stories, though ingenious, are often forced and it would perhaps have been better if they had been more natural.

Most problems require elementary algebra, usually simultaneous equations or simple indeterminate equations with integral solutions. The remainder are exercises in simple logic and these are probably the most interesting. All of them require clear thinking and careful reading of the story. Both books contain sets of model solutions which will be of great benefit to the reader.

The general standard of mathematics required is not high but these books will be of use to someone who has just started algebra and an interesting relaxation to the non-mathematician. J. C. ALEXANDER.
P. C. CHATWIN.

Introduction to Fourier Analysis and Generalised Functions. By M. J. LIGHTHILL. (Cambridge University Press.) 10s. 6d.

This book was reviewed in *Eureka* No. 21 but is now available in paperback form, the text being unaltered. To those who have said, on meeting the Dirac delta function for the first time, "What is this all about?," Lighthill's book can be thoroughly recommended. Functional analysts may feel that the treatment does not ascend (?) to their level of contemplation, but all others should be enthralled by the exciting results obtained with simple tools from such homely material as "good functions," "fairly good functions," and the irresistable "smudge function." J. FAULKNER.

Mathematical Puzzles and Diversions from the "Scientific American." By MARTIN GARDNER. (G. Bell & Sons Ltd.) 17s. 6d.

If you find it difficult to open a book which sets out to be chatty about "Magic with a Matrix," or "Tick tack toc" (Noughts and Crosses)—try this one. For in spite of the banalities, there are some very good problems here. From among a very wide range indeed (sixteen chapters—and references for further reading too!) of "mathematical puzzles and diversions," your reviewer would single out particularly the extraordinary Sam Lloyd puzzles, and some of the probability paradoxes. I can well believe that "Hexaflexagons" are absorbingly interesting, provided you actually get down to making the paper models.

And, there are two chapters—"Nine Problems," "Nine More Problems"—for those who like actually solving things: it would be a change, though, if we could read about somebody else here, and not the inevitable Jones and Professor Smith!

R. E. HARTE.

The Real Projective Plane. By H. S. M. COXETER. (Cambridge University Press.) 18s. 6d.

In this book, now available in a paper-back edition, the geometry of the real projective plane is developed as a logical system. The reader is carefully led from the primitive concepts of point and line, and the axioms of incidence, through the principle of duality, projectivities in two dimensions, and conics, to the beginnings of affine geometry. The last two chapters are devoted to the introduction of co-ordinates and their use, showing the similarities between the geometry of real homogeneous co-ordinates and the abstract system developed in the previous chapters.

Perhaps some of the proofs suffer from wordiness. Otherwise this book is written in a style which is both imaginative and attractive. At the beginning of many chapters are short introductions which outline the historical background of the ensuing work—a pleasing feature, all too rarely remembered in modern textbooks.

A well written book, which should prove useful to many.

J. J. MCCUTCHEON.

Special Relativity. By W. RINDLER. (Oliver and Boyd.) 10s. 6d.

This latest addition to the University Mathematical Texts will be welcomed as filling an important gap in the series.

It begins with a brief discussion of the Michelson-Morley experiment, and an introduction to the basic concepts of Special Relativity. The Lorentz Transformation has been derived by the end of the first chapter, after which the theory is successively applied to various branches of physics.

Tensor analysis is used in the second half of the book, and is dealt with in an appendix. The notation here is such as will facilitate the transition to tensors of General Relativity; for instance, the device peculiar to the Special Theory of taking a pure imaginary time co-ordinate is not employed at all.

The work can be used with profit both by Tripos students requiring an introduction to Relativity Theory and its applications, and by others interested in the subject. The low price is a further recommendation.

D. B. PEARSON.

Fourier Transforms. By R. R. GOLDBERG. (Cambridge University Press.) 21s.

This book provides the necessary background of the classical Fourier transform theory for use in abstract harmonic analysis. Half the book is devoted to the Fourier transform on L^1 with the Riemann-Lebesgue theorem, inversion, the "approx. identity" function, a proof that an analytic function of a Fourier transform is locally a Fourier transform, and Wiener's famous result on the closure of translates in L^1. Next the Fourier transform on L^2 is discussed with a proof of Plancherel's theorem, then the generalisations of Wiener's theorem and Bochner's characterisation of Fourier-Stieltjes transforms of non-decreasing bounded functions.

A feature of the book is the introductory chapter which states all the theorems on integration used in the subsequent chapters and provides a very ready reference. The book should be useful to applied mathematicians as it is concisely and well written so that the price is far more reasonable than it first appears to be. A. M. J. DAVIS.

A Treatise on the Analytical Dynamics of Particles and Rigid Bodies. By E. T. Whittaker. (Cambridge University Press.) 30s.

This is a paperback edition of the book first published in 1904 and subsequently revised several times. A quote on the front cover describes it as "The classical treatise on the subject of analytical dynamics—a work at once eminently readable, rigorously exact and almost encyclopaedically comprehensive" and this edition may well replace Ramsey's two volumes as the popular text book on the subject.

Lagrange's equations are introduced at an early stage and used whenever possible in the solution of the problems discussed. These include particle orbits and motion on a surface, Poinsot's representation, tops, vibrations about equilibrium and about steady motion, non-holonomic systems, the principles of least action and least curvature, the three body problem and the general theory of orbits. But, like Ramsey, the author does not use vector methods so that topics involving their use, e.g. Euler's equations and moving axes problems, are not covered. A. M. J. DAVIS.

Pure Mathematics. By F. GERRISH. (Cambridge University Press.) Vol. I (Calculus), 25s. Vol. II (Algebra, Trigonometry and Co-ordinate Geometry), 35s.

For many years the sixth former studying for the Entrance Scholarship Examinations has had to delve through ungainly text books for snippets of information on algebra, trigonometry and calculus, and has in practice, had to rely heavily on his schoolmasters for knowledge of these subjects. These two volumes by F. Gerrish fill the gap admirably. Co-ordinate geometry in two and three dimensions is included and natural scientists at school and university will like the chapters on differential equations, convergence tests and the summation of series. "Pictures" are used extensively to explain the calculus and for some reason, the word "steadily" appears in place of "monotonically." Nevertheless, the treatment is much more thorough than is needed by school mathematicians and so should be of value to Part I students. A vast number of problems is provided and answers are given at the end of each volume. A. M. J. DAVIS.

An Introduction to Homological Algebra. By D. G. NORTHCOTT. (Cambridge University Press.) 42s. 6d.

Professor Northcott showed in his tract "Ideal Theory" that he has the gift—why is it so extraordinarily rare?—of the ability to write a lucid, compact and entertaining treatise: a book on which the research student can confidently base his future reading.

The present book is written to satisfy a similar need. For since the extraordinary expansion of the subject following the appearance of Cartan and Eilenberg's famous (though unreadable) work there is practically no branch of pure mathematics that is not saturated with homological methods.

To my mind Professor Northcott's new book does not reach the height of perfection of his earlier work. Perhaps the nature of the subject does not allow it. On the one hand one has to introduce a highly abstract and complicated technique: while on the other hand one wishes as soon as possible to produce concrete and reassuring applications of this technique. Perhaps this book is as good a compromise as can be made. Certainly for the majority of those who want a pleasant introduction to the subject, it is, as was said in another context, the best we have.

Particularly useful are the explanatory notes at the back, which might, in my view, have been greatly expanded. The difficulty in this subject is never to follow *what* is being done; it is to appreciate *why* anyone should want to do it. T. G. MURPHY.

Real Variable. By J. M. HYSLOP. (Oliver and Boyd.) 8s. 6d.

Professor Hyslop's *Real Variable* makes a welcome addition to the Oliver and Boyd series of University Mathematical Texts. As the preface says, "it includes material which is taken for granted in such Texts as *Integration* and *Infinite Series* and may therefore be regarded as the foundation on which each of these rests." Consistent with this, the integral approach to exponential, logarithmic and the circular functions is avoided and their properties are derived from their expansions in infinite series.

It is necessary, therefore, to develop the theory of Taylor Series and the various forms of the remainder to prove rigorously, for example, that *exp* x is equal to $\sum_{n=0}^{\infty} \frac{x^n}{n!}$, a point which is sidestepped by many authors. The convergence to zero of the remainder after n terms (R_n) causes most trouble in the cases of *log* $(1 + x)$ and $(1 + x)^k$. Professor Hyslop goes through the full details of the proof in both cases, though they might have been simplified if the expressions for R_n/R_{n+1} corresponding to the Lagrange and Cauchy forms of the remainder had been used. In the notation of the book, these are

$$\frac{R_n}{R_{n+1}} = \frac{h\rho}{n + 1 + \theta h\rho} \quad \text{and} \quad \frac{R_n}{R_{n+1}} = \frac{h\rho(1 - \theta)}{n}$$

where $\rho = \dfrac{f^{(n+1)}}{f^{(n)}}(a + \theta h)$, $0 < \theta < 1$ and

$f^{(n)}$ denotes the n-th derivative of the function f. (In general, the θ's in the two cases will be distinct.) T. T. WEST

Field Computations in Engineering and Physics. By A. THOM and C. J. APELT. (Van Nostrand.) 30s.

This is an exposition of the numerical solution of elliptic partial differential equations in two-dimensional regions, or fields. In the first three chapters the method of finite differences is explained, and in subsequent chapters it is shown how it can be used to solve a variety of types of problems, including Dirichlet, Laplace free-boundary, and Navier-Stokes.

The techniques used differ somewhat from those of Southwell: considerable use is made of conformal mapping to eliminate curved boundaries; and the linear algebraic equations resulting from the finite difference approximation are solved by Thom's method of squares, which is the same as Liebmann's method.

The work of the senior author stretches over the past 30 years, and consequently the methods described have been developed for hand computation. With the advent of digital computers methods are being revolutionized, and, in particular, the sections of the book dealing with the solution of linear equations are out of date. Despite this, this is a valuable book. The authors have had considerable experience and have succeeded in presenting a great deal of practical know-how, the chapter on the nonlinear Navier Stokes equations being especially interesting. C. W. CRYER.

Postal Subscriptions and Back Numbers

For the benefit of persons not resident in Cambridge we have a postal subscription service. You may enrol as a personal subscriber and either pay for each issue on receipt or, by advancing 10s. or more, receive future issues as published at approximately 25 per cent. discount, with notification when credit has expired. The rates this year are:—

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Solution to Chessboard Dissection Puzzle

In each diagram squares with a common letter belong to a single piece.

A	B	B	C	C	C	D	E
A	A	B	B	C	D	D	E
A	A	A	C	C	C	D	E
F	F	F	F	G	E	E	E
H	H	I	F	G	G	G	G
H	I	I	I	I	J	J	J
H	H	K	L	L	L	J	J
K	K	K	K	L	L	L	J

A	B	B	B	C	C	C	D
A	A	B	E	E	C	C	D
A	B	B	B	E	E	C	D
F	F	F	F	G	D	D	D
H	H	I	F	G	G	G	G
H	I	I	I	I	J	J	J
H	H	K	L	L	L	J	J
K	K	K	K	L	L	L	J