EVALUATING FORMULA BIAS IN VARIOUS INDEXES USING SIMULATIONS

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INTRODUCTION

Current research on the consumer price index (CPI) has focused upon several sources of potential bias. Reducing this bias is important because many federal tax and expenditure programs are indexed to changes in the CPI. This means that a change of 0.1 percent per year changes federal expenditures and revenues by $600 million dollars in the first year. An upward bias of 1.0 percent per year then represents a loss to the federal government of approximately $6,000 million dollars.

In response to one of the problems described in Reinsdorf (1994), the Bureau of Labor Statistics (BLS) recently implemented a method to reduce 'formula bias.' This bias, also known as functional form bias and elementary index bias, occurs when the BLS estimates a Laspeyres price index in which the quantity weights are measured in a past 'base period.' Because the BLS lacks information about these quantity weights, it estimates them by dividing total expenditures on a good by its unit price. But because these base period prices are themselves missing, they are imputed, causing a problem known as 'proxy' bias. Imputing with the prices when they first appear in the index leads to the special case known as formula bias, and its correction involves using prices from some other time period.

In spite of the large sums of money involved, the size of this bias is not well understood because its current estimates rely upon either educated guesswork or strong parametric assumptions about the distribution of prices. This reflects the fact that researchers face the same problem confronting the

BLS: without the base period prices, the Laspeyres index cannot be constructed, and so we cannot know the size of the bias.

In this paper I evaluate the magnitude of the bias of several different price indexes and provide measures of their closeness to the Laspeyres index without making the strong parametric assumptions in previous work. This is done by treating the price quotes collected by the BLS as a representative population from which price quotes can be drawn and calculating the population Laspeyres index. By sampling quotes from this population in the same manner as the BLS samples from the actual population, I then simulate price indexes and their distributions and compare their closeness to the population Laspeyres index, including an estimate of the size of the bias of the current BLS method. Because both the current method and the method recently implemented by the BLS are simulated, the reduction in bias from the new implementation is also estimated.

My results are: First, the estimate of bias size using the old imputation method is lower than previous estimates, being about 0.20 point per annum for the commodities and services component of the CPI.[1] Second, the recently implemented 'seasoning' method reduces the estimated bias to about -0.02. Finally, estimates of other distance measures, such as root mean square error (RMSE) and mean absolute bias, are also lower with the seasoning procedure than with the previous index.

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[1] Because this component excludes shelter, it represents about three-quarters of all expenditures in the CPI's market basket.

However, because the choice of a base period partially determines the level of the population Laspeyres index, ceteris paribus, the distance of any index to the Laspeyres varies with the base period. This implies that other indexes may better approximate a Laspeyres index for some base period. Nevertheless, the simulations suggest that an index using seasoning is closer to the Laspeyres index than others considered here for reasonable definitions of the base period.

A SIMPLE DESCRIPTION OF THE PROBLEM

To clarify the nature of the problem, Erickson (1995) considers several simplified examples. Here one of those cases is generalized.[2] The index to be estimated is $E(P_{j,T}/P_{j.B})/E(P_{j,S}/P_{j.B})$, where $P_{j,T}$ is the price of good j in time T, $P_T>0$ and $0 < S < T$.

This index does not resemble a standard Laspeyres index in two respects. First, a Laspeyres index weights the prices in times T and S with quantities, which can be calculated by dividing expenditures by prices. For this example, we suppress the expenditures and weight prices with the inverse of prices in period B, the 'base period.' In addition, in a standard Laspeyres index, the base period would be period S, while here we assume it is earlier. This follows BLS methodology, where period S is no earlier than the "link" period L which itself is later than period B.

[2] I would like to thank Tim Erickson for several useful conversations regarding this example. Other descriptions of the problem can be found in Reinsdorf (1994) and Erickson (1995).

The index above can be consistently estimated with $(n^{-1}\Sigma_j P_{j,T}/P_{j,B})/(n^{-1}\Sigma_j P_{j,S}/P_{j,B})$, where n is the number of prices in the index. Because we do not actually observe prices in the base period, we impute them using a proxy $\hat{P}_{i,B}$ and form the index $(n^{-1}\Sigma_j P_{j,T}/\hat{P}_{i,B})/(n^{-1}\Sigma_j P_{j,S}/\hat{P}_{i,B})$. Assuming that $\{P_{j,S}, P_{j,T}, P_{j,B}, \hat{P}_{i,B}\}$ is independently and identically distributed with a nondegenerate distribution, the 'proxy' bias can be written as the difference between the expected value of the calculated index and the target:

$$(1)\ \beta = \frac{E(P_T / \hat{P}_B)}{E(P_S / \hat{P}_B)} - \frac{E(P_T / P_B)}{E(P_S / P_B)} = \frac{E(P_T / \hat{P}_B)E(P_S / P_B) - E(P_T / P_B)E(P_S / \hat{P}_B)}{E(P_S / \hat{P}_B)E(P_S / P_B)},$$

where the subscripts denoting goods are suppressed. By noting that

$$E\left(\frac{P_T}{\hat{P}_B}\right)E\left(\frac{P_S}{P_B}\right) = E\left(\frac{P_T}{\hat{P}_B}\frac{P_S}{P_B}\right) - \mathrm{cov}\left(\frac{P_T}{\hat{P}_B}, \frac{P_S}{P_B}\right)$$

and that a similar equality holds for the second component of the numerator in Equation (1), we may write the bias as

$$\beta = \frac{\mathrm{cov}\left(P_T/P_B, P_S/\hat{P}_B\right) - \mathrm{cov}\left(P_T/\hat{P}_B, P_S/P_B\right)}{E(P_S / \hat{P}_B)E(P_S / P_B)}.$$

If these two covariances are equal, then the bias is zero. Selection of $\hat{P}_B$ is therefore a balancing act between one covariance and the other. Ceteris paribus, as the correlation between the proxy and, say P_S, increases, the first covariance goes to zero while the second moves towards negative infinity.

BLS imputation procedure prior to January 1995 can be represented by $\hat{P}_B = \rho P_L$, where ρ is a positive constant defined in the next section. The bias is then

$$\beta = \frac{\mathrm{cov}\left(P_T/P_B, P_S/P_L\right) - \mathrm{cov}\left(P_T/P_L, P_S/P_B\right)}{E(P_S / P_L)E(P_S / P_B)}.$$

Because period L will always be closer to S than to T, one might expect the first covariance to be smaller in absolute magnitude than the second and the second to be negative. The overall bias would then be positive.

Erickson considers the special case of formula bias, when S equals L. Then,

$$\beta = \frac{-\operatorname{cov}\left(P_T / P_L , P_L / P_B \right)}{E(P_L / P_B)} .$$

Erickson notes: "My intuition is that [the bias] will often be positive, because for given P_T and P_B an increase in P_L decreases P_T/P_L while increasing P_L/P_B." Moulton (1996) also points out that in this case larger variances in P_L cause larger biases by increasing the covariance between $1/P_L$ and P_L. If we simplify even further by assuming that $1/P_B$ is uncorrelated with P_T and P_L, β reduces to $E(P_T/P_L) - E(P_T)/E(P_L)$. Here the bias clearly exists and, by Jensen's inequality, must be positive.

THE PROBLEM AS IT EXISTS IN THE CPI

To relate the example in the previous section to the CPI, it is first necessary to explain some aspects of the CPI's construction. The BLS divides services and commodities into expenditure classes (such as expenditure class 07: fish and seafood), which are composed of several item strata (such as item stratum 0702: fresh or frozen fish and seafood). Item strata are themselves composed of one or more entry level items (ELI), such as ELI 07021: Shellfish, excluding canned. Once a specific good to be priced -- such as one pound of crab meat -- has been selected, a price quote for the good is collected either monthly or bimonthly.

With these price quotes, indexes for each of the 207 item strata are calculated in 44 geographic areas. For each item stratum, these 44 geographic indexes are aggregated into a single national index. This national item stratum index is then aggregated into the all-items CPI using expenditure weights from the Consumer Expenditure Survey (CES). Every year, about 20 percent of the price quotes in each population statistical unit (PSU). Each of the 44 geographic areas consists of one or more PSU, which are phased out and replaced with price quotes on new items. In the 'link period,' the old sample is used to compare prices with those of the previous month, while price quotes in the new sample are collected so they can be compared with prices in the following month. Thereafter, only price quotes in the new sample are used.

To examine proxy bias, consider the calculation of the relative change in the (modified) Laspeyres index of a specific item strata in a specific area. This is the target that the BLS estimates when it calculates indexes for each item stratum and area. Given a sample of n prices, the relative price change between times T and T-1 is calculated as:

$$(2)\ R_{T,T-1} = \frac{\sum_j^n Q_{j,B} P_{j,T}}{\sum_j^n Q_{j,B} P_{j,T-1}},$$

where

$$Q_{j,B} = E_{j,B} / P_{j,B},$$

and E_{jB} is the total expenditures on good j in the base period B, the time period during which the expenditures are made. Rather than occurring in a single month, this period varies from one week to five years, depending upon the item strata. In addition, the expenditures need to be multiplied by several adjustment factors, such as a correction for the difference in the definition of goods between the survey which collects expenditures and the survey which collects the prices.

There are two separate problems complicating the calculation of Equation (2). First, for each item stratum, the BLS collects only a small proportion of all possible price quotes in a given area. To represent this, replace $E_{j,B}$ with $\overline{E}_{j,B} \equiv W_{j,B} * N_{j,B}$, with $N_{j,B}$ being a random variable equal to unity if a particular outlet and item is chosen and zero otherwise. The BLS sets the probability of choosing a given outlet proportional to the outlet's expenditures. Within each outlet, the price of a specific good is 'initiated' by being randomly selected in a similar manner. This ensures that the expected value of $\overline{E}_{j,B}$ equals E_{jB}. The variable $W_{j,B}$ is the product of the adjustment factors mentioned above a sample weight and total expenditures of the sampled outlets $\Sigma E_{j,B} * N_{j,B}$. The modified Laspeyres of such a sample is then:

$$(3)\ \hat{R}_{T,T-1} = \frac{\sum_j^n \left[\overline{E}_{j,B} / P_{j,B}\right] P_{j,T}}{\sum_j^n \left[\overline{E}_{j,B} / P_{j,B}\right] P_{j,T-1}} .$$

Although the index in Equation (3) is a consistent estimator of the index in Equation (2), it suffers from small sample bias β_s; i.e., the expectation of the index calculated on a small sample does not necessarily equal the index calculated on the entire population. In spite of the fact that the numerator and denominator in Equation (3) are each unbiased estimators of their population counterparts, the ratio is not. Therefore, the right-hand side of Equation (3) is not an unbiased estimator of its population counterpart. While this bias is often assumed to be positive, this need not be the case.

The source of the second problem, proxy bias, occurs because we do not observe any base period prices $P_{j,B}$. Prior to January 1995, the BLS imputed these base period prices by deflating the link period prices by the index, defined as ρ in the previous section. The left-hand side of Equation (2) would then be estimated with the relative $\hat{R}^L_{T,T-1}$:

$$(4)\quad \hat{R}^{L}{}_{T,T-1} = \frac{\sum_j^n \left[\overline{E}_{j,B} \Big/ \left(P_{j,L} \Big/ \hat{R}^{L}_{L,B}\right)\right] P_{j,T}}{\sum_j^n \left[\overline{E}_{j,B} \Big/ \left(P_{j,L} \Big/ \hat{R}^{L}_{L,B}\right)\right] P_{j,T-1}}.$$

For simplicity, assume from here on that all quotes in the area rotate simultaneously (which occurs where a geographic area contains only one PSU) so that $\hat{R}^{L}_{L,B}$ cancels out of Equation (4). In this case, Equation (4) becomes

$$(5)\quad \hat{R}^{L}{}_{T,T-1} = \frac{\sum_j^n \left[\overline{E}_{j,B} \big/ P_{j,L}\right] P_{j,T}}{\sum_j^n \left[\overline{E}_{j,B} \big/ P_{j,L}\right] P_{j,T-1}}.$$

Further, taking expectations and subtracting the right-hand side of Equation (3) from the right-hand side of Equation (5) yields an expression analogous to Equation (1). As in the simple example above, if $P_{j,L}$ is more correlated with $P_{j,T-1}$ than to $P_{i,T}$, then one might expect the bias to be positive.

The analog to the formula bias case considered in Erickson (1995) occurs when T-1 is equal to L. Then:

$$(6)\quad \hat{R}^{L}{}_{T,L} = \frac{\sum_j^n (\overline{E}_{j,B} \big/ P_{j,L}) P_{j,T}}{\sum_j^n (\overline{E}_{j,B} \big/ P_{j,L}) P_{j,L}} = \frac{\sum_j^n \overline{E}_{j,B}\, P_{j,T} \big/ P_{j,L}}{\sum_j^n \overline{E}_{j,B}}.$$

Again, intuition suggests that the bias will be positive. Higher variance in $P_{j,L}$ is also expected to cause a greater level of formula bias.

ALTERNATIVE SOLUTIONS

It should be noted that, due to small sample bias, none of the proposed methods are unbiased. Indexes that impute base period prices will also generally suffer from the proxy bias defined in Equation (1). In addition, because we can only observe expenditures in the base period, desirable indexes, such as superlative indexes, cannot be estimated. However, it may be possible to select an index that is closer to the modified Laspeyres than an index that imputes with the link month.

One alternate imputation method involves using the period in which price quotes are first 'initiated' and the prices collected. This initiation period occurs approximately four months prior to the link month. Because the price quotes are collected at this time, the initiation period price can be used for imputation.[3]

This approach will be most effective in reducing bias if prices become increasingly uncorrelated as the time period between them increases. It also renders the worst case scenario in Erickson (1995) impossible. Using the initiation period prices $P_{j,I}$ instead of the link month period prices results in the index:

$$(7)\ \ \hat{R}^{I}_{T,T-1} = \frac{\sum_j^n (\overline{E}_{j,B}/P_{j,I})P_{j,T}}{\sum_j^n (\overline{E}_{j,B}/P_{j,I})P_{j,T-1}}.$$

A virtually identical procedure to using the initiation period is to collect the price quotes at the link month, but wait several months before actually linking in the new quotes. This procedure is termed seasoning and the BLS has used it on food at home item strata since January 1995.[4] The BLS has recently extended this method to non-food items.

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[3] Armknecht, Moulton and Stewart (1995) describe this method and several others, such as the geometric mean, as alternative methods to alleviate the formula bias for the food at home component of the CPI. See also Moulton and Smedley (1995) for simulations of the current method, the Dutot and the Geometric means indexes.

[4] Food at home items are those numbered below 1900. See Table A1 for a listing of all items included in this study. In addition, quotes in several apparel ELI's are seasoned for either

It is also possible to use some other procedure to impute the base period prices. For example, imputing the base period with any non-zero constant yields the weighted ratio of expectations (Dutot) relative:

$$(8)\ \hat{R}^{r}_{T,T-1} = \frac{\sum_j^n \overline{E}_{j,B} P_{j,T}}{\sum_j^n \overline{E}_{j,B} P_{j,T-1}}.$$

Because the expenditure weights are not divided by prices, this index will give too large a weight to high-priced items. This will be an especially severe problem for groupings of widely disparate goods such as item stratum number 6001, 'sports vehicles, including bicycles' which can include boats with prices exceeding $300,000 as well as bicycles with prices of $50.

A final imputation procedure would be to estimate the base period prices using a hedonic equation:

$$(9)\ \hat{R}^{H}_{T,T-1} = \frac{\sum_j^n (\overline{E}_{j,B} / \hat{P}_{j,B}) P_{j,T}}{\sum_j^n (\overline{E}_{j,B} / \hat{P}_{j,B}) P_{j,T-1}},$$

where $\hat{P}_{j,B}$ is the estimated price from a hedonic regression. The major problem with this method is the difficulty in correctly specifying hedonic equations for the hundreds of item strata that are used to make up the CPI.[5] It also requires that the item possesses useful and identifiable characteristics with which prices can be estimated.[6]

An alternative to imputing the base period price is to use the geometric means (geomeans or Jevons) relative

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eight or four months. In the simulations below, all of the quotes in any strata with such an ELI are seasoned for eight months.

[5] For example, the single market model of Kahn and Lang (1988) requires correct specification of the functional form for model identification.

[6] For these reasons hedonic regressions are currently used only for quality adjustment on a small set of items in the CPI.

$$(10) \qquad \hat{R}^{G}_{T,T-1} = \prod_j^n \left[P_{j,T} / P_{j,T-1} \right]^{\hat{E}_{j,B} / \sum \hat{E}_{j,B}} .$$

It has often been noted that the geometric means relative is lower than the Laspeyres relative because the Laspeyres is a true cost-of-living index when the elasticity of substitution among goods is zero, while the geometric means relative is a true cost-of-living index when the elasticity of substitution among goods is unity.

This statement only applies when the base month is equal to the link month. As the base month prices become increasingly uncorrelated with the prices in times T and T-1, this condition no longer holds. When the base month prices are entirely uncorrelated with the prices in times T and T-1, the modified Laspeyres index equals the ratio of the arithmetic average prices. In this case, the modified Laspeyres index and the geometric means index are simply different measures of the central tendency of the distribution of prices. The modified Laspeyres and the geometric means will then only differ to the extent that the variance of prices changes over time.[7]

SIMULATING THE INDEXES

The primary problem with comparing the performance of the above alternatives is that we cannot observe the base month prices. If we could, we wouldn't need to impute. This problem is approached here by using price quotes collected by the BLS nationally and arbitrarily designate specific months as the base month, initiation month and link month. After assuming that these quotes are the population for a representative area I constructed

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[7] I would like to thank Marshall Reinsdorf and Brent Moulton for several useful conversations regarding this point.

the Laspeyres index, the alternative methods described in Equations (4), (7), (8) and (10), and an index that seasons the food components but uses link month imputation elsewhere.[8] Further, the distribution of the indexes are simulated by sampling these quotes randomly 10,000 times with the probability of selection proportional to expenditures. Several measures of the closeness of the alternative methods to the Laspeyres were then applied and rough estimates of the magnitude of the small-sample bias were made. These Monte Carlo simulations were performed using 162 of the item strata in the commodities and services section of the CPI's market basket, which represents approximately three-quarters of expenditures.

The link period is January 1994, and the initiation period is set to be September 1993.[9] As mentioned above, use of the initiation period method is also equivalent to seasoning the quotes for four months. The sample Laspeyres index uses a base period of January 1993 for monthly item strata and bimonthly strata price quotes collected in odd months. To form the population Laspeyres index, the base period price is defined as the average price in one of two time periods. Base period price 1 is the average price from July 1992 to January 1993.[10] This is as similar to the base period for expenditures as the data limitation to months after June 1992 will allow. In this sense, it is the base period definition most consistent with forming the

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[8] The mixture of link and seasoning methods represents the seasoning of food items that started in January 1995 and the eight month seasoning of some apparel ELI's started in 1991. Because of the complexity in performing hedonic regressions, this method is not included.

[9] As described in the Appendix, quotes from even-numbered months of bimonthly item stratum were assumed to be odd-numbered.

[10] The months prior to 1993 became available too late for use in the Monte Carlo simulations. Therefore, all Monte Carlo simulations of the Laspeyres index use a base period of January 1993. As described in the Appendix, by construction, any quote not available in July 1992 was available no later than January 1993.

quantity weights in Equation (2). Base period price 2 is the average price for the year prior to the initiation period September 1993, which is the recall period used when initiating prices.

If a quote was missing in the initiation period or link period, it was imputed using the quote in the first available month. Missing quotes in other months were imputed using the last available quote and inflating by the index formed without the quote. All of the indexes calculated compare short-run price changes from one time period to the next. The long-run relatives from January 1994 to September 1994 were calculated by estimating the monthly relatives and then multiplying them together. Because actual expenditures on an outlet were unavailable, the outlet expenditure used in the weights and sampling was taken to be the average expenditure on all outlets in the area. For more details on the construction of the data set, see the Appendix. Table A1 reports the means of the sample indexes for the 162 separate item strata.

RESULTS

Figure 1 depicts the proxy bias for the various short-run population indexes from January to September 1994. Because this is the formula bias in Equation (6), there is a large bias with the link month method in the link month. The bias of the link month method then tapers off.

As mentioned above, this decline can occur under conditions that favor the seasoning method. In the January/February period, the link/seasoning index, which seasons food items and uses link month imputation on others, reduces the bias by more than 85 percent. Seasoning the non-food items reduces the bias in this month an additional 12 percent. In the following months, however, there is relatively little bias reduction to seasoning of food items and a much greater proportionate reduction to seasoning the non-food items. While the link month method always has a positive bias, the seasoning month method starts with a positive bias but falls below zero in the latter months.

Figure 2 describes the cumulative effects of this bias. This shows that most of the benefit from seasoning food comes in the first comparison. After that, the link/season index tracks the link month imputation method. The seasoning method, in contrast, starts with a small positive bias and appears to have a shallow negative trend. Given sufficiently strong seasonality, this bias would continue to be negative until one year after the link, when it turn positive. In contrast, the Dutot method has a negative bias that trends lower over time. The geometric means index also appears to underestimate the population Laspeyres, with the difference growing over time.

Although not designed specifically to measure small sample bias, this study can provide some evidence. These results should be interpreted with caution because of the small number of months examined. In theory, the link month imputation method should have no small sample bias when the quotes first link. For subsequent short run comparisons, bias is possible, although the cumulative bias should remain quite close to zero.[11] In contrast, the seasoning method, the Dutot method and the geomeans method should all have some degree of small sample bias.

[11] The existence of missing quotes can cause the bias to vary from zero.

Figure 3 reports the small sample biases for the various short-run indexes. In the link month, the link month method has an extremely small bias: 0.008. The remaining methods have some degree of bias, but only of about 0.05. For subsequent months, the bias of the link month method is larger, but always below that of other methods. In contrast, the geomeans and the Dutot always have the greatest level of small sample bias.

The cumulative bias is shown in Figure 4. The link month bias starts near zero and rises to approximately 0.04 before declining below zero. Because it should equal zero asympotically if no quotes are missing, the bias should hover around zero for indexes beyond September. The bias in the seasoning method, while not equal to zero, peaks below 0.15 and appears to be declining towards zero at the end of the sample, where it equals about 0.07.

To estimate the size of proxy bias in the commodities and services section of the CPI, I compare the population Laspeyres index with the various population alternatives. The raw bias estimates are divided by five because about twenty percent of the quotes rotate every year, and there is probably little if any sample bias after the first year.

It is clear that the two base periods result in fundamentally the same estimate.[12] Table 1 lists both the estimate of the bias using the link month method of about 0.20 per year, as compared with some previous estimates of 0.50 per year. The intermediate link/seasoning method appears to cut the bias about in half, while the pure seasoning method seems to have eliminated most of the additional bias.

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[12] Moulton (1996) multiplies the figures in table one by 0.75 to represent the fact that housing, which represents about twenty five percent of the index, is omitted.

Table 2 lists the results from applying several criteria for evaluating the ability of sample indexes to estimate a population Laspeyres index that uses base period price 1. In order to measure how precisely the current methodology estimates the Laspeyres index, I use the root mean square error (RMSE) criteria and the related mean absolute error criteria. Because the mean bias has received the most attention, this criteria is also selected. Because this number includes small sample bias and is not adjusted for the percent of quotes rotating per year, it cannot be interpreted as an estimate of the actual bias. Finally, the very conservative maximum absolute error criteria and the frequency with which an index is within 0.5 percent of the target are included.

As the table shows, the RMSE using the new imputation method is below both the link/seasoning methodology and the link month method. The mean absolute error is also lower for the seasoning method than for other methods. In addition, the proportion of simulations with which the index is within 0.5 of the Laspeyres index is greater using the new method.

On the other hand, the maximum error of the seasoning method exceeds that of the Dutot and the geomeans, implying that imputing the base period generates occasional large outliers. In Table 3, the results from using the second base period are listed. These results are qualitatively similar to those of Table 2.

However, the above results depend upon the definition of the base period. To show this, the RMSE of the indexes as a function of the base period of the population Laspeyres index were recalculated. These base periods ran from

July 1992 to August 1993.[13] The results are shown in Figure 3, where the most noticeable feature is the effect of seasonality.

For all methods not using link-month imputation, the RMSE peaks with a base period of February 1992 and falls away after that. For the link month imputation method, RMSE is minimized at that period and rises elsewhere. Because the link month is either January or February 1993, this suggests that seasonality is the source of this pattern. Comparing the RMSE's of July 1992 with July 1993 also suggests seasonality. For these two months, the RMSE of the link month imputation method is equal to or lower than that for the seasoning method. For any other month, the seasoning method is preferred.

To examine the effects of base period selection upon the mean bias, a similar experiment were performed. The results are described in Figure 4. The bias for each method is minimized when the base period is set to January or February 1992. For the link-month imputation method, the bias is very nearly zero for February 1992. In addition, bias for the seasoning/imputation method is near zero when the base period is September or October of 1992, one year prior to the initiation month. Again, seasonality appears to be the cause.

CONCLUSION

In this paper I provide estimates of the proxy bias of several price indexes and provide measures of their closeness to the Laspeyres index. By using

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[13] Odd bimonthly quotes use either the same month as the monthly quotes, if it is an odd month, or one month earlier, if it is even. Even bimonthly quotes are one month later than the odd bi-monthly quotes.

Monte Carlo simulations with BLS data, these estimates are made without the strong parametric assumptions of previous work. The results show that the recently implemented seasoning procedure more closely estimates the Laspeyres index than previous BLS methods, using two definitions of the base period. I also estimate the proxy bias using the old imputation method at about 0.20 point. The recently implemented "seasoning procedure" reduces the estimated bias to about -0.02, although it is difficult to attach a sign to this number because of fluctuations in the bias over time.

Because these results are sensitive to the definition of the base period, some care must be used in interpreting the results. In particular, the effects of seasonality need to be considered. Nevertheless, for reasonable definitions of the base period, the results suggest that the seasoning procedure is much closer to the Laspeyres index than the older imputation procedure.

Table 1 Net Bias Estimates With Two Laspeyres Indexes				
Imputation	Base Period Price One		Base Period Price Two	
Method	Per Rotation	Per Year	Per Rotation	Per Year
Link Month	1.048	0.210	0.989	0.198
Link/Season	0.622	0.124	0.563	0.113
Seasoning	-0.074	-0.015	-0.133	-0.027
Geomeans	-0.265	-0.053	-0.324	-0.065
Dutot	-0.455	-0.091	-0.514	-0.103

Base Period Price One is the average price from July 1992 to January 1993. Base Period Price Two is the average price from September 1992 to August 1993. Approximately 20 percent of the quotes rotate per year.

Table 2

Comparison of Estimation Methods of Long Run Relatives, Using the Population Laspeyres (With Base Period Price One) as the Target

Sample Index	Root Mean Square Error	Mean Absolute Error	Mean Error	Maximum Absolute Error	Proportion within .5 of Laspeyres
Link Month	1.181	1.041	1.032	5.378	0.170
Link/Season	0.854	0.693	0.636	5.160	0.409
Seasoning	0.514	0.404	-0.001	3.010	0.683
Geomeans	0.536	0.426	-0.114	2.966	0.651
Dutot	0.622	0.499	-0.240	2.713	0.572

The Link/Season index seasons for all food items and imputes with the link month for all non-food items.

Table 3

Comparison of Estimation Methods of Long Run Relatives, Using the Population Laspeyres (With Base Period Price Two) as the Target

Sample Index	Root Mean Square Error	Mean Absolute Error	Mean Error	Maximum Absolute Error	Proportion within .5 of Laspeyres
Link Month	1.130	0.985	0.973	5.319	0.199
Link/Season	0.811	0.650	0.577	5.101	0.448
Seasoning	0.517	0.409	-0.060	2.951	0.672
Geomeans	0.552	0.441	-0.173	2.907	0.630
Dutot	0.647	0.521	-0.299	2.654	0.550

The Link/Season index seasons for all food items and imputes with the link month for all non-food items.

Figure 1
Proxy Bias of Short-Run Population Indexes When Estimating A Population Laspeyres Index (With Base Period Price One)

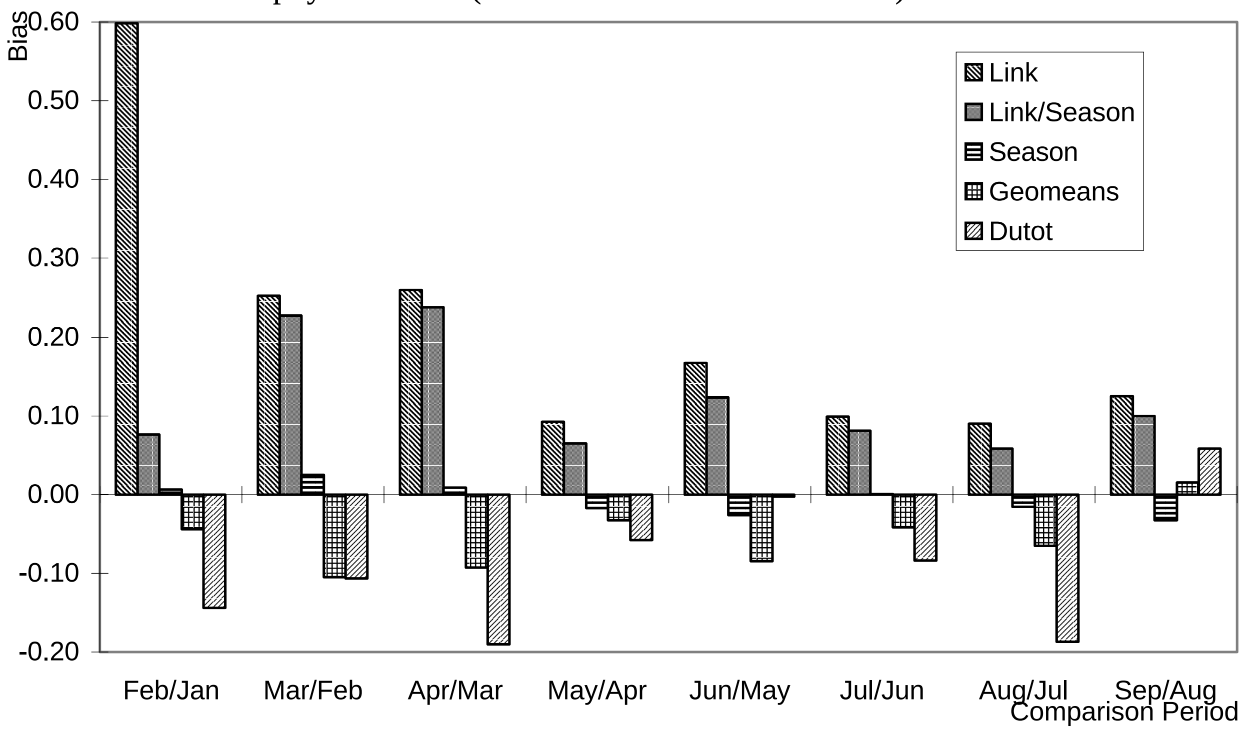


Figure 2
Proxy Bias of Cumulative Population Indexes When Estimating A Population Laspeyres Index (With Base Period Price One)

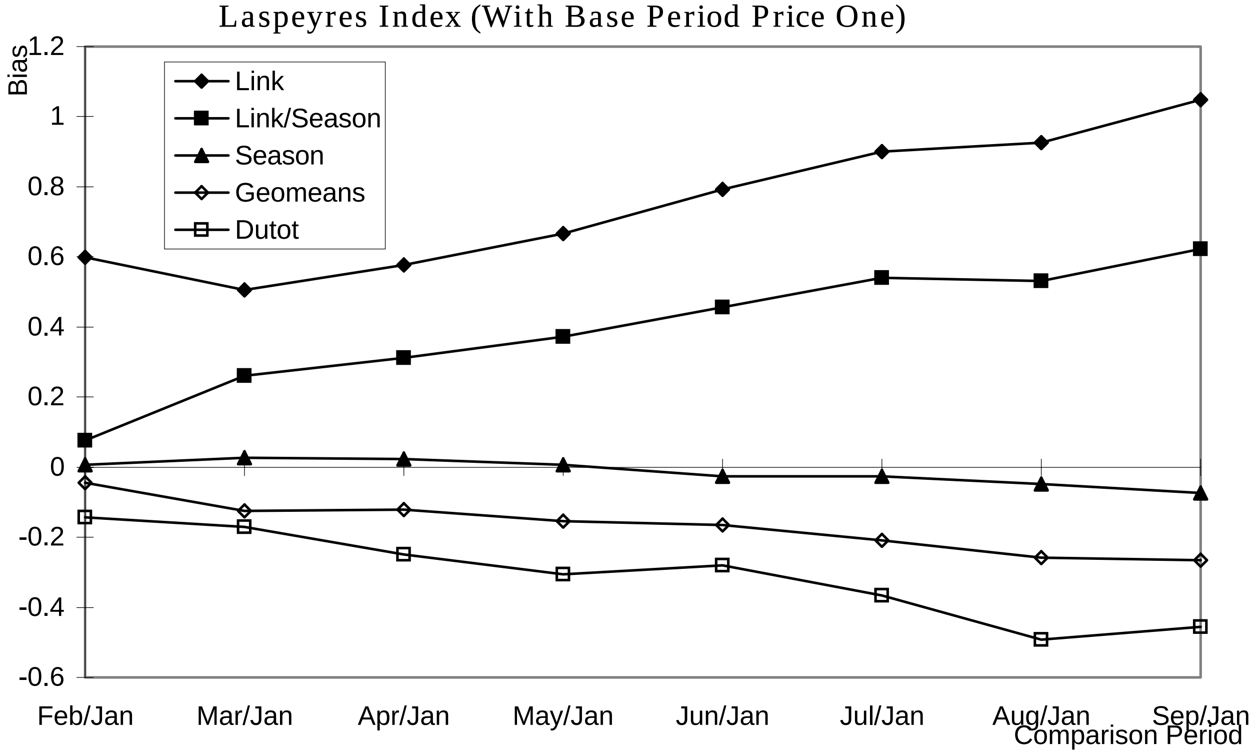


Figure 3
Small Sample Bias of Short-Run Indexes

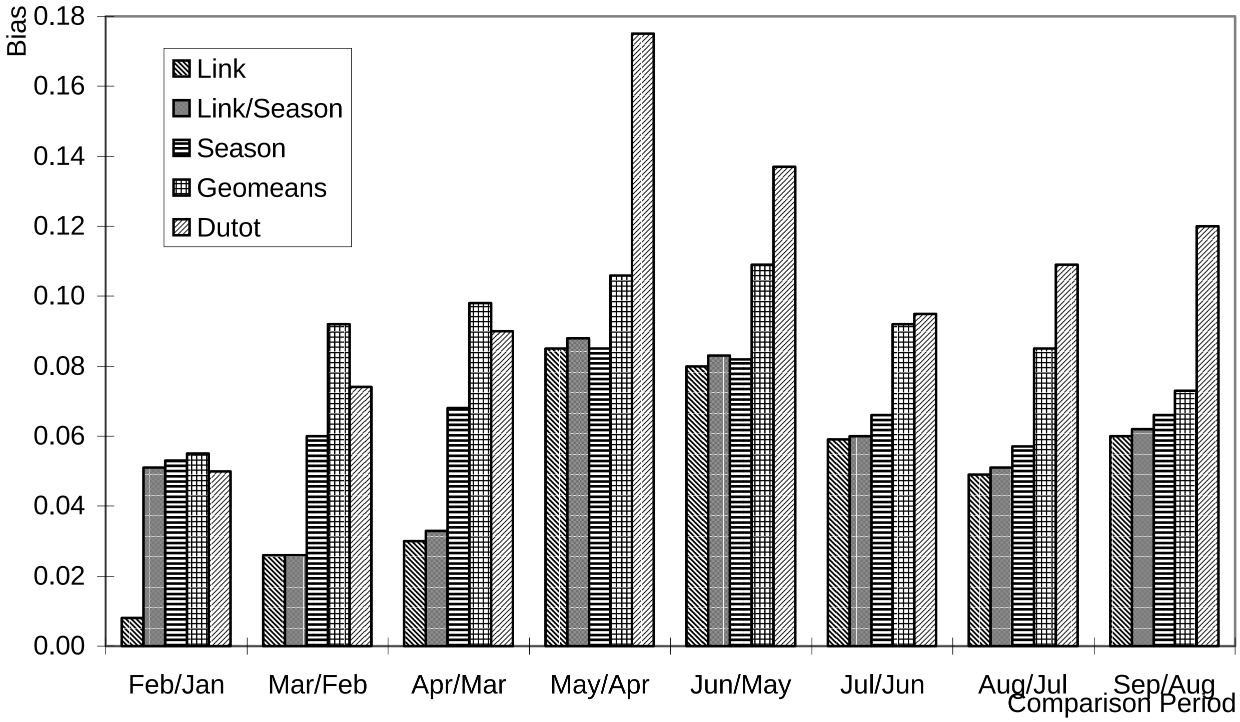


Figure 4
Small Sample Bias of Cumulative Indexes

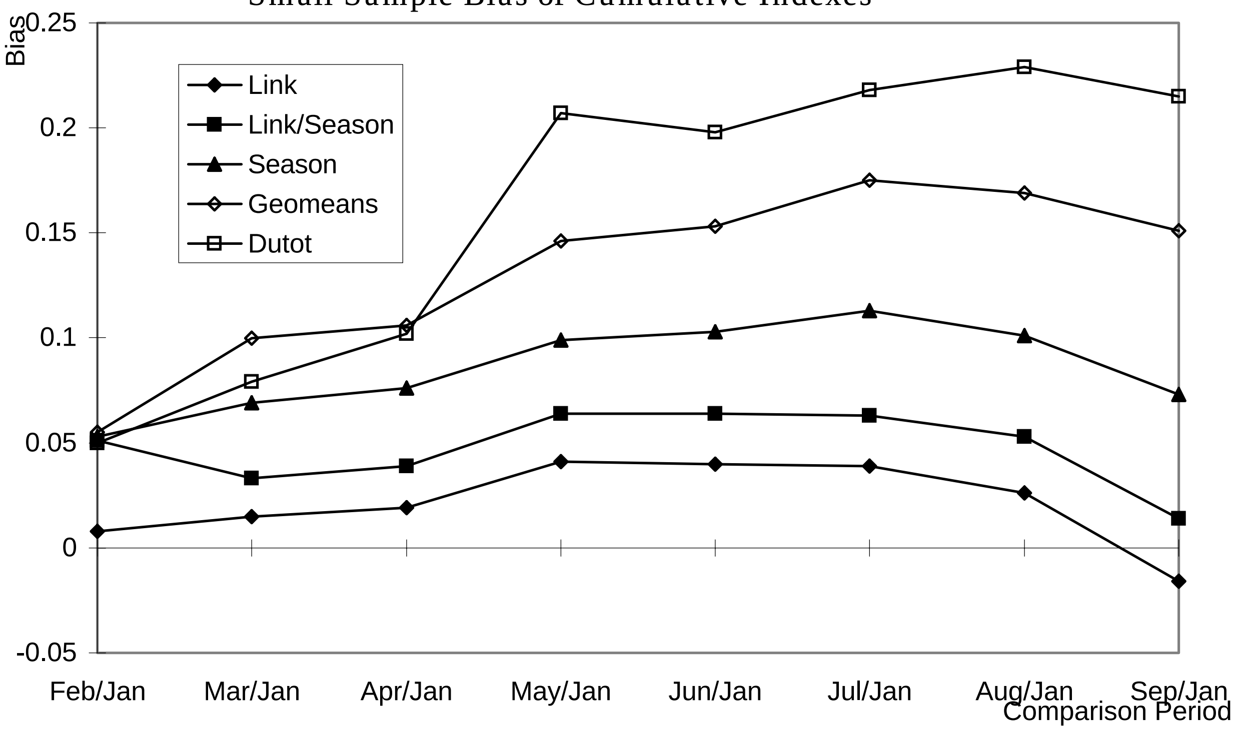


Figure 5
RMSE of Indexes as a Function of the Base Period of the Population Laspeyres Index

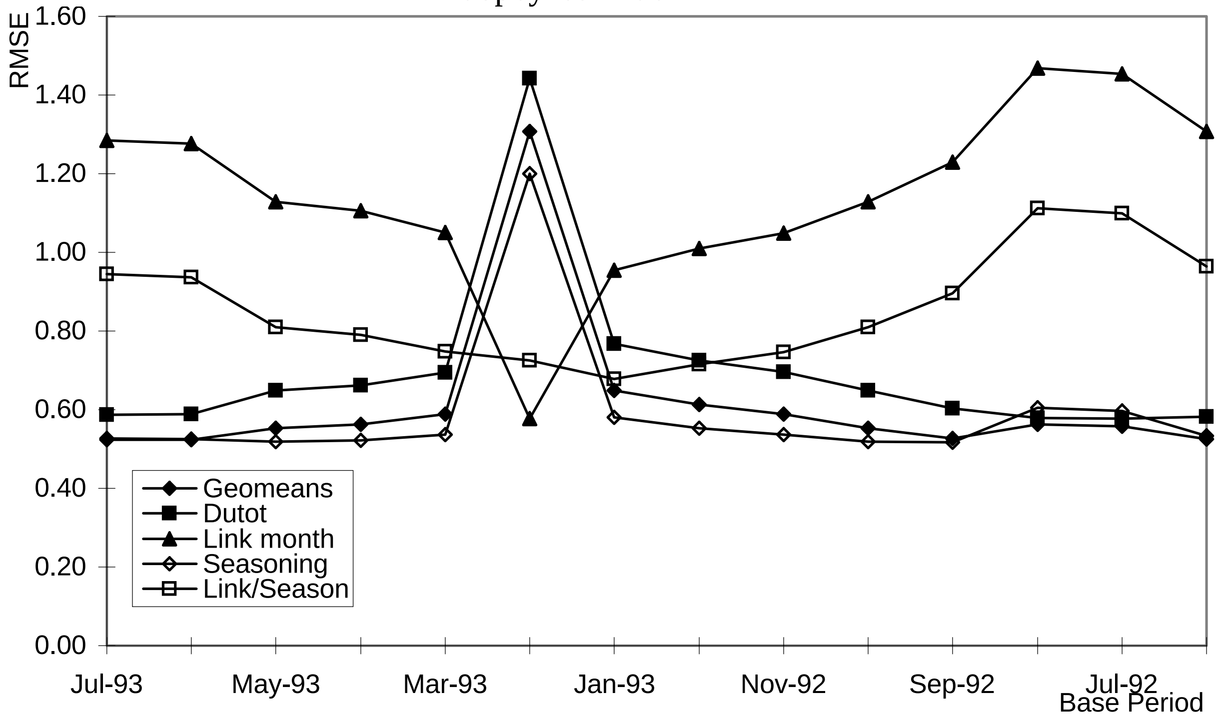
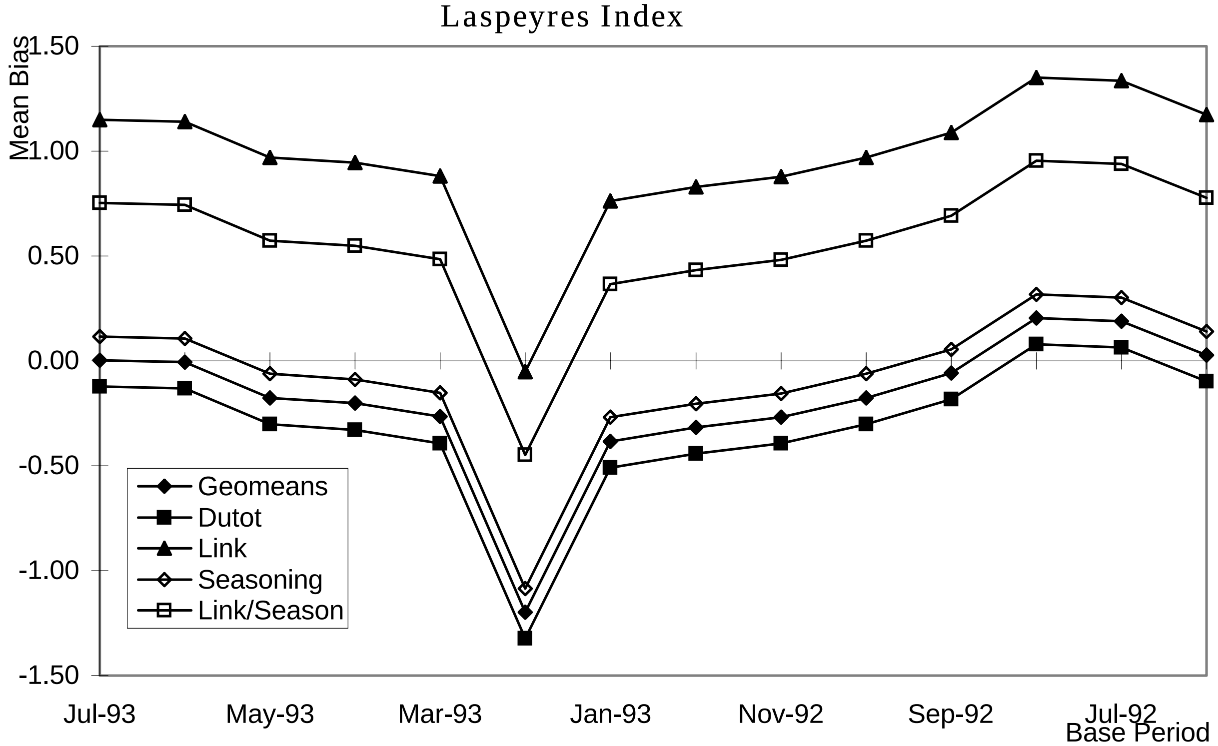


Figure 4
Mean Bias of Indexes as a Function of the Base Period of the Population Laspeyres Index



APPENDIX

DATA SET CONSTRUCTION

The prices used in this analysis are the prices of items in the commodities and services section that are used in calculating the CPI. Item classes in which the average number of quotes was less than two were deleted -- calculated by dividing the total number of quotes by 44, the total number of geographic areas for which indexes are created-- as were strata with an unusually large number of missing prices or with anomalous price behavior. The total weight of these three classes in the CPI is less than 0.5 percent. The number of remaining item strata is 162.

Any quote that failed to exist continuously from January 1993 to November 1994 or for which the number of quotes in the area was unknown was deleted. Because approximately 20 percent of the sample rotates every year this left about 60 percent of the original sample. If the price for a given quote was missing for every month from January to November of 1994, then the quote was deleted. Quotes with missing prices were not otherwise deleted. The total number of price quotes remaining was 48,390. This forms the population from which quotes were drawn, either all at once to form the population indexes, or several at time in the Monte Carlo simulations.

For the 82 item groups where prices are collected monthly, the ‘short run’ index is calculated from one month to the next, starting in January and ending in September 1994. For the 83 groups where prices were collected bimonthly, even-month indexes run from February through August and odd-month indexes run from January through September. The even-numbered months were treated as odd-month quotes (i.e., quotes from February were combined with the January quotes.) There were also several large metropolitan areas that collect the bimonthly items on a monthly basis. Odd-month quotes of this type were used and even-month quotes were ignored.

The outlet expenditures used for the expenditure weights were taken to be the average expenditures in the area from which a price quote was taken. All the adjustment factors in the weights, such as the percent of POPS correction, were used intact. This procedure inevitably resulted in the expenditure information being older than would be true in the actual Bureau procedure. By construction, the simulated link month of January 1994 is after the month the price was actually linked, which must have been prior to January 1993 to avoid being deleted. This problem exists to the extent that relative outlet expenditures varied from the month in which the survey was actually conducted to the month in which it should have been conducted in the simulations.

To reflect Bureau procedure, 'long run' indexes between January and August 1994 were formed by multiplying 'short run' indexes together. If there were no missing prices, this process yields exactly the same value as calculating the long-run index directly. As with BLS policy, if at least one quote existed in both periods of the index, then missing values were imputed using the index formed by the existing quotes. This procedure creates an index slightly different from the directly calculated index.

The long run quotes were aggregated using the relative importance weights for December 1994. These weights are the product of the index in a given period and the item's expenditure share estimated from the 1982-4 Consumer Expenditure Survey. While this method is slightly inaccurate, the inaccuracy is only a function of how the relative inflation rates across goods has changed from January to December 1994.

For the Monte Carlo simulations, the average number of quotes were drawn from the population 10,000 times, using the definition of average described above. The total number of simulations computed for each index formula was then 10,000*(82*8 + 80*4)= 9,760,000. The probability of an outlet being selected was proportional to its expenditures, as defined above.

Official Bureau procedure is to select randomly one entry level item (ELI) to represent the item strata of goods and then randomly select outlets from that ELI proportional to their expenditures. Here, I do not separate the quotes by ELI, but simply sample outlet/quote combinations from the entire item stratum.

If a short run index was missing because all of the values were missing, a randomly selected index for the month from another simulation was substituted. This is reflective of Bureau procedure, where missing indexes are imputed using the index for a different area or the index for a different class of goods in the same area. To check for the sensitivity of the results to this procedure, all the index formulas were calculated on the item classes in which there were no missing indexes. The resulting aggregate indexes were essentially identical to those that substituted for missing indexes.

Table A1

Mean Long Run Relative For Simulations Using Mean Number of Quotes

Item Strata	Name	Laspeyres	Imputation Link Month	Imputation Season	Dutot	Geomeans
101	Flour and prepared flour mixes	102.52	103.20	102.35	102.18	101.51
102	Cereal	101.06	102.05	101.18	100.93	100.97
103	Rice, pasta, and cornmeal	102.95	103.68	102.64	102.23	101.69
201	White bread	103.82	104.87	103.69	103.75	103.70
202	Fresh other bread, biscuits rolls	101.78	102.05	101.65	101.42	102.86
204	Cookies, fresh cakes, and cupcakes	99.79	100.16	99.70	99.75	100.91
206	Other bakery products	107.77	109.25	106.46	107.37	105.33
301	Ground beef other than canned	96.97	97.59	96.76	96.64	97.31
302	Chuck roast	99.43	102.19	98.92	98.90	99.22
303	Round roast	98.03	100.28	97.42	97.30	97.44
304	Other beef and veal	99.12	100.34	99.21	98.90	99.05
305	Round steak	98.12	100.56	98.39	98.03	98.22
306	Sirloin steak	100.46	102.74	100.62	100.42	100.23
401	Bacon	98.41	99.80	98.74	98.32	98.64
402	Pork chops	101.29	103.19	101.74	101.44	101.34
403	Ham	106.65	109.33	106.00	106.65	105.38
404	Other pork, including sausage	100.23	101.26	99.98	99.58	99.66
501	Other meats	100.57	101.82	100.39	100.43	100.18
601	Fresh whole chicken	100.03	101.76	99.64	99.68	99.51
602	Fresh and frozen chicken parts	100.56	102.37	101.07	100.28	99.10
603	Other poultry	105.82	106.47	105.96	105.14	104.77
701	Canned fish and seafood	100.78	101.33	100.56	100.60	100.29
702	Fresh and frozen fish and seafood	100.26	101.27	99.07	99.16	99.80
801	Eggs	95.68	96.63	95.46	95.36	95.63
901	Fresh whole milk	98.31	98.52	98.32	98.22	98.34
902	Other fresh milk and cream	99.23	99.73	98.99	98.97	98.97
1001	Other dairy products incl. butter	100.01	100.93	100.22	100.10	100.20
1002	Cheese	100.41	100.95	100.39	100.29	100.61
1004	Ice cream and related products	102.78	104.61	102.42	102.80	101.60
1101	Apples	110.17	111.83	107.81	107.45	107.48
1102	Bananas	104.95	111.43	103.88	105.64	106.02
1103	Oranges, including tangerines	157.10	171.58	160.79	153.27	154.95
1104	Other fresh fruits	102.35	111.16	90.02	92.74	90.24
1201	Potatoes	100.94	103.76	99.93	98.45	100.15
1202	Lettuce	119.71	132.73	119.59	120.65	119.21
1203	Tomatoes	65.04	69.31	65.49	64.64	66.39
1204	Other fresh vegetables	90.38	96.15	88.37	88.76	90.06
1301	Fruit juices and frozen fruit	96.91	98.29	97.21	97.15	96.98

Table A1
Mean Long Run Relative For Simulations Using Mean Number of Quotes

Item Strata	Name	Laspeyres	Imputation Link Month	Season	Dutot	Geomeans
1303	Canned and dried fruits	98.88	99.42	98.80	98.69	99.10
1401	Frozen vegetables	101.68	101.95	101.38	101.33	100.92
1402	Other processed vegetables	104.35	105.75	104.21	104.31	103.36
1501	Sweets, including candy	100.38	100.60	100.45	100.22	100.55
1502	Sugar and artificial sweeteners	100.49	100.80	100.40	100.33	101.29
1601	Fats and oils	103.18	104.08	103.39	103.12	102.23
1701	Carbonated drinks	99.05	100.46	99.09	98.97	99.03
1703	Coffee	158.72	159.36	158.47	155.99	146.38
1705	Other noncarbonated drinks	98.37	98.63	98.37	97.97	97.11
1801	Canned and packaged soup	103.69	103.93	103.62	103.38	103.07
1802	Frozen prepared food	101.80	102.94	101.72	101.86	101.57
1803	Snacks	103.09	104.77	102.57	102.93	103.20
1804	Seasoning condiments sauces, spices	103.80	104.61	103.16	103.46	104.90
1806	Miscellaneous prepared food, including baby food	102.21	102.73	102.16	102.07	101.93
1901	Lunch Away from Home	101.09	101.13	101.08	101.01	100.95
1902	Dinner Away from Home	101.12	101.15	101.14	101.06	101.51
1903	Other meals and snacks Away from Home	101.09	101.29	101.10	100.97	100.93
2001	Beer and ale at home	99.95	100.12	99.85	99.88	99.79
2002	Distilled spirits at home	100.41	100.60	100.39	100.42	100.39
2003	Wine at home	97.56	98.18	97.65	97.66	97.33
2005	Alcoholic beverages away from home	100.54	100.70	100.66	100.42	101.07
2102	Lodging while out of town	104.96	106.13	100.13	102.02	97.48
2103	Lodging while at school	103.65	103.76	103.72	103.66	103.34
2401	Material supplies and equipment for home	98.15	98.62	97.99	98.17	98.27
2501	Fuel oil	97.82	97.86	97.72	97.75	97.94
2502	Other household fuel commodities	97.20	97.22	97.09	97.10	96.01
2601	Electricity	106.86	107.22	105.74	106.46	105.76
2602	Utility (piped) gas	96.90	97.09	96.51	96.49	97.14
2701	Telephone local charges	99.82	99.81	99.81	99.76	99.84
2702	Water and sewerage maintenance	102.10	102.08	102.08	101.95	102.76
2703	Cable television	97.08	97.62	97.55	97.15	97.07
2704	Refuse collection	102.77	102.78	102.79	102.52	102.01
2705	Telephone interstate toll calls	105.89	105.96	105.93	105.80	103.28
2706	Telephone intrastate toll calls	99.84	99.85	99.85	99.79	99.52
2801	Textile House furnishings	98.84	100.07	98.68	98.41	100.33
2901	Bedroom furniture	100.41	100.69	100.34	99.84	102.19

Table A1
Mean Long Run Relative For Simulations Using Mean Number of Quotes

Item Strata	Name	Laspeyres	Imputation Link Month	Imputation Season	Imputation Dutot	Imputation Geomeans
2902	Sofas	101.14	101.33	101.33	100.92	101.42
2903	Living room chairs and tables	99.99	100.11	99.93	99.09	99.78
2904	Other furniture	101.70	101.88	101.58	101.18	103.39
3001	Refrigerators and home freezers	99.89	99.81	99.77	99.76	99.71
3002	Laundry equipment	102.91	102.93	102.92	102.80	102.84
3003	Stoves, ovens, dishwashers and air conditioners	99.12	100.40	100.35	100.06	100.33
3101	Televisions	98.10	98.28	98.05	98.00	98.05
3103	Audio products	99.21	99.04	99.04	98.62	95.88
3201	Floor and window covering infants' laundry,	98.88	98.53	98.75	98.08	99.26
3202	Clocks, lamps, and decor items	97.95	98.55	97.58	97.81	98.04
3203	Tableware serving pieces, and nonelectric kitchenware	99.50	99.92	99.76	99.26	100.29
3204	Lawn equipment power tools, etc.	100.15	100.05	99.95	99.75	99.90
3205	Sewing, floor cleaning small kitchen...	96.63	95.72	96.38	94.26	94.70
3206	Indoor plants and fresh cut flowers	100.31	101.40	101.06	100.78	101.50
3301	Laundry and cleaning products including soap	100.72	101.15	100.78	100.69	100.59
3303	Household paper products and stationary supplies	99.96	100.47	99.77	99.74	99.63
3305	Other household lawn, and garden supplies	101.53	102.48	102.05	101.68	101.88
3401	Postage	100.00	100.00	100.00	100.00	100.00
3404	Gardening, other household services	101.06	101.08	101.05	101.00	101.65
3601	Men's suits, sport coats, coats, and	104.31	106.26	103.60	103.99	106.14
3603	Men's furnishing and special clothing	103.30	106.04	103.25	103.92	104.87
3604	Men's shirts	96.04	97.17	95.40	94.43	93.91
3605	Men's dungaree jeans, and trousers	97.59	99.90	98.04	97.61	97.61
3701	Boys apparel	98.27	100.55	97.68	97.37	96.82
3801	Women's coats and jackets	96.89	101.45	96.50	95.70	97.48
3802	Women's dresses	104.50	116.03	106.84	105.97	101.72
3803	Women's separated and sportswear	99.53	106.27	98.83	99.43	100.65
3804	Women's underwear nightwear hosiery, and accessories	99.26	100.75	98.28	97.92	101.44
3805	Women's suits	99.70	110.38	100.77	99.03	99.21
3901	Girls apparel	93.66	98.50	94.50	93.56	93.44
4001	Men's footwear	99.18	100.46	98.91	98.79	98.71

Table A1
Mean Long Run Relative For Simulations Using Mean Number of Quotes

Item Strata	Name	Laspeyres	Imputation Link Month	Imputation Season	Dutot	Geomeans
4002	Boys' and girls' footwear	97.05	99.55	97.78	97.83	97.78
4003	Women's footwear	99.75	102.38	99.34	98.66	99.69
4101	Infants and toddlers apparel	103.51	103.35	102.98	101.94	101.14
4201	Sewing materials, notions and luggage	102.89	103.23	101.58	101.03	101.88
4302	Jewelry	103.16	106.75	104.31	103.09	101.52
4401	Other apparel services	101.31	101.32	101.31	101.21	101.06
4402	Laundry and dry cleaning	102.05	102.03	102.04	101.88	102.09
4501	New cars	100.45	100.51	100.44	100.38	99.74
4502	New trucks	102.70	102.75	102.73	102.65	102.70
4503	New motorcycles	106.01	106.03	105.96	105.90	105.90
4701	Motor fuel	112.65	112.86	112.56	112.58	112.06
4702	Motor oil, coolant, and other products	103.04	103.22	103.11	102.90	103.22
4801	Tires	100.02	100.19	100.10	99.96	99.70
4802	Other automobile parts and equipment	99.46	99.55	99.50	99.15	99.55
4901	Automobile body work	101.57	101.55	101.60	101.42	101.40
4902	Automobile drive train, brake, and miscellaneous	101.89	102.05	101.98	101.81	102.11
4903	Automobile maintenance and servicing	101.15	101.29	101.25	101.02	102.42
4904	Automobile power plant repair	102.02	102.11	102.04	101.97	102.06
5201	Automobile registration licensing and inspection fees	100.15	100.14	100.14	100.14	100.18
5205	Other automobile fees	102.56	102.75	102.70	102.35	103.31
5301	Airline fares	97.32	97.87	96.51	96.05	97.57
5302	Other intercity public transportation	100.41	100.48	100.25	99.77	98.79
5303	Intracity public transportation	100.39	100.40	100.40	100.39	100.77
5401	Prescription Drugs	101.97	102.03	102.00	101.87	102.20
5502	Internal and respiratory over-the-counter drugs	101.50	101.86	101.55	99.48	99.38
5503	Nonprescription medical equipment and supplies	100.32	100.81	100.76	100.13	99.47
5601	Physicians' services	102.45	102.57	102.49	102.14	102.17
5602	Dental services	103.41	103.49	103.44	103.13	103.04
5603	Eye care	101.01	100.99	100.92	100.56	99.76
5604	Services by other medical professionals	101.74	101.96	101.92	101.69	101.27
5701	Hospital room	103.13	103.20	103.12	102.95	102.41

Table A1 Mean Long Run Relative For Simulations Using Mean Number of Quotes						
		Imputation				
Item Strata	Name	Laspeyres	Link Month	Season	Dutot	Geomeans
5702	Other inpatient hospital services	104.16	104.17	104.02	103.78	103.68
5703	Outpatient services	105.84	105.86	105.75	105.31	105.24
5901	Newspapers	102.61	102.61	102.63	102.33	102.69
5902	Magazines periodicals, and books	102.10	102.20	102.15	101.90	102.68
6001	Sport vehicles including bicycles	98.94	98.99	99.00	98.79	100.03
6002	Other sporting goods	100.87	100.98	100.65	100.04	100.87
6101	Toys, hobbies, and music equipment	101.31	101.69	101.51	101.22	102.39
6102	Photography supplies and equipment	100.99	101.35	100.76	100.95	99.79
6103	Pet supplies and expense	100.83	101.08	100.84	100.44	100.21
6201	Club memberships	99.78	100.21	99.69	96.52	99.36
6202	Fees for participant sports, exclusive club memberships	102.48	102.83	102.78	102.01	101.21
6203	Admissions	101.15	101.31	101.24	99.54	102.12
6205	Other entertainment services	101.14	101.23	101.13	101.08	101.18
6301	Tobacco products	101.82	101.98	101.96	101.62	100.32
6401	Other toilet goods and small personal care	104.09	104.29	103.81	103.60	102.10
6403	Cosmetic bath, nail preparations manicure.	98.04	98.06	98.00	97.83	98.76
6501	Beauty parlor services for females	101.76	101.87	101.81	101.46	101.88
6502	Haircuts and other barber shop services for...	100.95	101.02	101.01	100.92	100.84
6601	School books and supplies, college	102.26	102.45	102.33	102.25	102.26
6701	College tuition	106.20	106.12	106.10	106.05	105.90
6702	Elementary and high school tuition	104.87	104.83	104.83	104.77	104.69
6703	Daycare and nursery school	102.93	102.95	102.94	102.91	103.15
6704	Tuition for technical/business/ and other schools	104.45	104.45	104.44	104.42	104.13
6801	Legal service fees	100.94	100.88	100.91	100.83	100.42
6802	Personal financial services	100.70	100.71	100.71	100.70	100.83
6803	Funeral expenses	102.97	102.98	102.96	102.92	102.86

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