Practical and Theoretical Considerations of Panel Effects for the Current Population Survey's Composite Estimator November 2017

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Abstract

Labor Force surveys are often designed with rotating panels of households, where households remain in the survey for a set number of months and then rotate out of the sample. Rotation patterns often induce correlations that can be exploited to increase an estimate's efficiency. To further improve precision, the US Current Population Survey (CPS) uses an AK Composite estimator that consists of two primary components: a calibrated estimate, and an estimate of change using overlapping panels between adjacent months.

Panels may exhibit effects in their Labor Force estimates based on the number of months a panel remains in the survey—often called month-in-sample effects. Panel effects affect the AK Composite estimator in a particular fashion, and effects change over time. It's important to consider carefully how these panel effects influence the Composite estimator. This paper looks at the practical aspects of panel effects, and how those practical considerations interact with theoretical considerations. In particular, we look at the following topics: choosing an estimator, estimation of inputs such as panel effects, and optimizing a Composite estimator for multiple estimates of change.

Keywords: Labor Force Survey, Relative Bias, Linear Estimator, Month-in-Sample Bias, Mean Square Error

1. Introduction

Labor Force surveys are important tools to measure the state of an economy. To help ensure a quality, unbiased measure of the Labor Force, statistical agencies use a random sample of households, and one or more individuals are asked specific questions about Labor Force participation and demographic characteristics of each household member. Samples often use complex designs involving multiple stages of sampling and complex rotation patterns. The rotation patterns ensure that a household remains in the sample a specific number of months over the course of a year and between years. The month-in-sample (MIS) is used to refer to the number of months a household has been in the sample.

Estimation methods typically involve nonresponse adjustments and some form of calibration to population totals to help reduce variances and ensure accurate population

[1] The opinions expressed in this paper are those of the author and do not reflect official policy at the Bureau of Labor Statistics.

estimates for demographic categories such as race, sex, age ranges, ethnicity, and their various cross-classifications. The design creates positive correlations across time for various estimates, which makes estimates of changes more efficient. One method used by Labor Force surveys to increase efficiency is Composite estimation. The Composite estimation used by the Current Population Survey is a two-parameter model called the AK Composite estimator (Gurney and Daly, 1965; Ernest and Huang, 1981). While the estimator's efficiency increases, the bias may also increase due to consistently different Labor Force estimates within each month's panel, depending on how long the panel has remained in the sample. The phenomenon is often referred to as "month-in-sample bias." The MIS bias patterns and their changes over time may impact the choice of composite estimate as well as the composite estimator's parameters. If no bias exists in the MIS, then a simpler estimator may be preferred.

This paper looks at Composite estimation in the Current Population Survey, how Labor Force estimates differ between MIS, and the theoretical and practical impacts of those differences. The paper consists of 6 sections including the introduction. Section 2 provides a brief overview of the Current Population Survey. Section 3 describes differences in month-in-sample effects. Section 4 describes different Composite estimators and their attempts to account for the MIS bias. Section 5 describes several practical considerations for Composite estimates as well as the theoretical considerations for each issue. Section 6 provides a summary.

2. Current Population Survey

The CPS is a Labor Force survey conducted by the U.S. Census Bureau for the Bureau of Labor Statistics (BLS). The CPS uses a multi-stage, stratified systematic sample of households. The first stage stratifies groups of similar counties by characteristics related to Labor Force participation and unemployment. A single primary sampling unit (PSU) is selected within each stratum, and a systematic random sample of households is drawn from within each PSU. The systematic sample allows samples of households to be easily divided into panels. Each panel consists of approximately 9,000 households. Panels follow a 4-8-4 rotation design, where panels remain in the sample for 4 consecutive months, fall out of the sample for the next 8 months, and then return to the sample for a final 4 months. This rotation design allows a high level of household overlap between consecutive months and between the same months in consecutive years.

An example may better illustrate the rotation design. In January, 2010, a panel enters the survey for the first month. It remains in the sample for 3 additional months, through April, 2010. It then drops from the sample for 8 months. The panel enters the sample again in January, 2011, and finally exits the CPS after April, 2011. January, 2010 – April, 2010 are referred to MIS 1 – 4, and January, 2011 through April, 2011 are referred to a MIS 5 – 8.

Estimation involves multiple stages of weight adjustments to account for nonresponse and proper accounting of different demographic groups. After the initial weighting steps for nonresponse, there are three main post-stratification weighting steps that align the survey estimates to population totals:

1. A "State Coverage" and "National Coverage" adjustment.
2. A raking step to match specific Age/Race/Ethnicity combinations at the National and State levels. The estimate is referred to as Second Stage (SS) estimator by CPS. It contains three distinct adjustments by Race, Ethnicity, and Age, for both National and State estimates.
3. A Composite estimate combining the Ratio Raking estimate and a measure of the over the month change using the continuing panels. A weighting procedure aligns all estimates to a set of composite estimates for select demographic groups.

Tech paper 66 from the U.S. Census Bureau provides more information on these estimation methods.

The composite estimation step uses the AK Composite estimator of Gurney and Daly (1965) and discussed and studied for CPS by Ernst & Huang (1981).

$$Y_t^{'} = (1-K)\hat{Y}_t + K\left(Y_{t-1}^{'} + \frac{4}{3}\Delta_{t-1,t}\right) + A\beta_t$$

$$\hat{Y}_t = \sum_{i=1}^{8} y_{i,t} \;\; ; \;\; Y_0^{/} = \hat{Y}_0$$

$$\Delta_{t-1,t} = \sum_{\substack{i=2,3,4,\\6,7,8}} y_{i,t} - \sum_{\substack{i=1,2,3,\\5,6,7}} y_{i,t-1} \; ; \; \beta_t = \sum_{i=1,5} y_{i,t} - \frac{1}{3}\sum_{\substack{i=2,3,4,\\6,7,8}} y_{i,t}$$

where $y_{i,t}$ is a sum of Second Stage weights for a given Labor Force status for month-in-sample *i* at time *t*, and $\hat{Y}_t$ is the Second Stage estimate. Provided that the expected value of each panels' Labor Force estimate is unbiased, the Composite estimator is also unbiased (Rao & Graham, 1964). In the absence of any reliable measure of bias, we usually assume that the Second Stage estimates are unbiased. As we discuss in the following section, each MIS provides noticeably different Labor Force estimates.

3. Panel Bias

While the impact of MIS bias was noted in early research on Composite estimates, Bailar (1975) provides one of the more prominent discussions of its impact on the CPS. With consistent additive MIS effects, she shows how these effects impact Labor Force estimates of levels as well as changes. Solon (1986) discussed and challenged the assumption of additive MIS effects, and devised a test to determine the effect type. Erkens (2012) provides an updated overview of additive MIS effects' impact by looking at the time series. In his paper, he provides the following definitions for MIS biases $d_{i,t}$:

$$E\left(\hat{Y}_t\right) = Y_t \quad ; \quad E\left(y_{i,t}\right) = \frac{Y_{i,t}}{8} \quad ; \quad \mathrm{E}\left(\hat{d}_{i,t}\right) = \frac{Y_{i,t} - Y_t}{8}$$

$Y_{i,t}$ are panel i's estimate of the population value Y_t. The $\hat{d}_{i,t}$ are monthly estimates of the MIS bias for each panel.

Provided that the MIS effects are constant, the bias of level estimates for the AK Composite estimator is

$$Bias\left(Y'_{t,AK}\right) = \left(\frac{4}{3}K\left(\sum_{i=4,8} d_i\right) - \frac{4}{3}(K-A)\left(\sum_{i=1,5} d_i\right)\right)\frac{1}{1-K}$$

Note that only four MIS impact the bias. The AK Composite estimator provides a lower Labor Force estimate as the MIS effects for 1 and 5 increase and the effects of MIS 4 and 8 decrease (Erkens, 2012).

Figure 1 shows smoothed additive effects of unemployed for each MIS. A LOESS regression fit (smoothing parameter set to .35) is plotted instead of the actual time series to aid interpretation. The vertical reference lines indicate the 1994 and 2003 CPS redesigns.

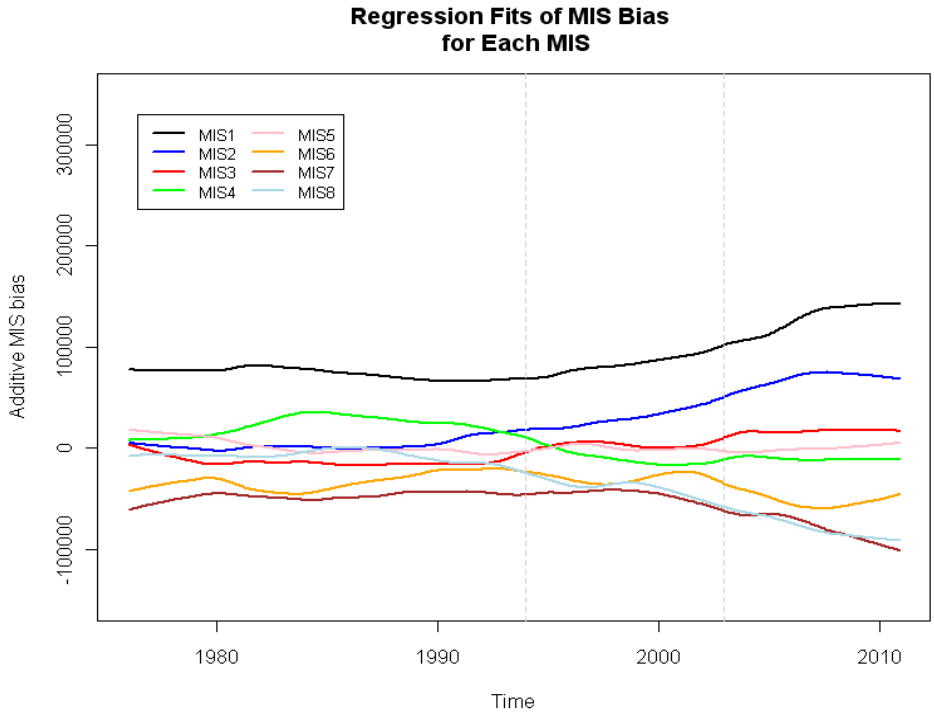

Figure 1: Time series plot of MIS biases. Actual values of Level estimates of Unemployed smoothed with LOESS.

Inspecting MIS 1,4, 5, and 8 show that the necessary pattern mentioned before occurs over the time interval in Figure 1. Judging by this chart, the MIS biases were somewhat stable prior to 1990. After 1990, the bias patterns started to change. Erkens (2012) provides an additional discussion of MIS effects and their changes. The impact of these changing MIS effects is considered in section 5.

4. Composite Estimation

The CPS has used some form of a Composite estimator since 1954 (Tech Paper 66). Huang & Ernst (1981) reviewed the impact of the AK Composite on CPS Labor Force estimates, measured the impact of the variance and the Mean Square Error, and decided on appropriate parameters.

Later research by Breau and Ernst (1983) provided a more general form for the Composite estimator called the Generalized Composite Estimator (GCE):

$$Y_t^{'} = \sum_{i=1}^{8} a_i x_{i,t} - K\sum_{i=1}^{8} b_i x_{i,t-1} + KY_{t-1}^{'}$$

$$\sum_{i=1}^{8} a_i = \sum_{i=1}^{8} b_i = 8$$

The AK Composite is a special form of the GCE in which the ***a*** and ***b*** parameters take the following values:

MIS	***a***	***b***	***MIS***	***a***	***b***
1	1 – K + A	4/3	5	1 – K + A	4/3
2, 3, 4	1 + (K – A)/3	4/3	6, 7, 8	1 + (K – A)/3	4/3

Table 1: *a* and *b* parameter values for the AK Composite estimator using the GCE.

The Composite estimator was extended beyond the CPS to general balanced rotation designs in a paper by Cantwell (1989), who makes the following assumptions:

1. $Var(y_{i,t}) = \sigma^2$
2. $Cov(y_{i,t}, y_{j,t}) = 0$
3. $Cov(y_{i,t}, y_{j,s}) = \rho_{|t-s|}\sigma^2$

The first condition states that the variance is stationary and constant across panels. Shao, Zhou, and Cheng (2014) use a similar set of assumptions to optimize the AK Composite estimator. Those authors present a simulation showing improved MSEs for monthly level estimates, but the work did not extend the results to estimates of change—which are some of the more important estimates for the CPS.

Cantwell's work was extended by Park, Kim, and Hoi (2001). They defined the conditions for a balanced, rotating design, relaxed the Composite estimator's independence and variance assumptions, and assumed the following correlation structure:

1. $Var(y_{i,t}) = \sigma_i^2$
2. $Cov(y_{i,t}, y_{j,t}) = 0$
3. $Cov(y_{i,t}, y_{j,s}) = \rho_{1|t-s|}\sigma_i\sigma_j$
4. $Cov(y_{i,t}, y_{i,t'}) = \rho_{2|t-t'|}\sigma_i\sigma_i$

where $t \neq t'$ and the panels at time periods t and t' are from an adjacent systematic sample. In the CPS if a panel is in MIS 1 at time t, then a panel with similar households will enter the sample at time $t' = t + 17$. The households at the two distinct time periods are different, but the neighborhoods often have similar characteristics. Park et al (2001) calls this a second-order correlation, and notes that accounting for positive correlations has significant impact on the variances of Labor Force estimates (pp. 1491-1492).

They also allowed the panel variances σ_i^2 to differ amongst the MIS. Given the generally categorical nature of the Labor Force responses, the variances for the CPS are described in terms of a binomial response, so a different proportion of Employed or Unemployed in a MIS may lead to a different variance. Park still maintains the stationary variances to develop his formulas and assumes stationary MIS effects, and we discuss and explore those assumptions later.

Similar to Cantwell (1989), Park et al (2001) provides variance formula for different types of GCE estimates. Park also provides formulas for the MSE and Langragian equations to optimize the GCE for these quantities for multiple types of estimates.

Erkens (2012) also worked with the GCE, but reformulated it and provided a simpler form to estimate optimal parameters. He worked with the MSE of a level estimate for Unemployment, but the results extend to other Labor Forces and estimates of change. Under his formulation both the variance and bias are quadratic. To achieve a quadratic form, sums over the *t* months are replaced with exponentially weighted moving averages. The final form is more simplistic, and only the parameter vectors ***a*** and ***b*** are optimized. A value of *K* must be selected beforehand and remain unaltered over time.

We provide some additional exposition of that paper's results here. The GCE's recursion may be expanded to the initial month $t = 0$, which achieves the following form:

$$Y_t^{'} = \sum_{i=1}^{8} a_i \sum_{j=1}^{t} K^{t-j} y_{i,j} - K \sum_{i=1}^{8} b_i \sum_{j=0}^{t-1} K^{t-j-1} y_{i,j} + K^t \hat{Y}_0 \qquad (1)$$

The summations involving K and indexed by j can also be expressed in terms of an exponentially weighted moving average. Consider a t-term exponential moving average:

$$S_t(y) = \sum_{j=1}^{t} \alpha(1-\alpha)^{t-j} y_j + (1-\alpha)^t y_0$$

Substituting $(1-\alpha)$ = K, the leftmost summand is similar to the weighted sums of t terms in the GCE, and those sums may be expressed as:

$$\sum_{j=1}^{t} K^{t-j} \hat{y}_{i,j} = \frac{S_t(\hat{y}_i) - K^t \hat{y}_{i,1}}{(1-K)} \approx \frac{S_t(\hat{y}_i)}{(1-K)} \text{ for large values of } t \text{ since } 0 < K < 1.$$

A derivation is in the appendix. The previous expression is associated with the GCE's ***a*** parameters, and a similar expression is available for the ***b*** parameters. The $K^t \hat{y}_{i,1}$ acts as an intercept, but its influence degrades exponentially with time. When considering the bias and variance, we decided to ignore these intercept terms for the following two reasons:

1. K is fixed, so variances from the intercept terms will exponentially decay. Even for employment with K = .7, variances decay to a marginal level after 12 months ($.7^{24} \approx .02\%$).
2. It is easier computationally to ignore the intercepts.
3. We're more interested in the long-term effects of compositing as MIS effects evolve over months and years.

For these reasons we decided to ignore any product with K^t for large t. After doing so we have the following approximate value of the GCE:

$$Y_t' \approx \sum_{i=1}^{8} \left(a_i S_t(\hat{y}_i) - K b_i S_{t-1}(\hat{y}_i)\right)(1-K)^{-1}$$

The approximate variance of the GCE is therefore

$$Var\left(Y_t'\right) \approx \left[\mathbf{a}\ \mathbf{b}\right]^T \left(COV \odot \left(\mathbf{-1}\quad \mathbf{K}\right)^T \left(\mathbf{-1}\quad \mathbf{K}\right)\right)(1-K)^{-2} \left[\mathbf{a}\ \mathbf{b}\right], \qquad (2)$$

where COV is a variance-covariance matrix of the integrated moving averages of each MIS in months t and t-1, and $\left[\mathbf{a}\ \mathbf{b}\right]$ is the column vector of the ***a*** and ***b*** parameters from the GCE. The [**1** **-K**] is a concatenation of row vectors where **1** is a 1 x 8 vector of 1, and **K** is a 1 x 8 vector of the parameter K. The $\odot$ is a Hadamard product.

We note again that terms with K^t are removed here since we're most interested in optimal composite estimates over multiple years. After a single year, these terms have a negligible impact on the variance in the formula given above. These terms should be accounted for during the initial months following a composite estimates initialization.

Cantwell (1989) and Park et al (2001) each use a correlation matrix in their variance and MSE formulas. Equation (2) may be expressed in a similar form.

$$Var\left(Y_t^{/}\right) \approx \left[\mathbf{a}\ \mathbf{b}\right]^T \left(diag(\sigma)\ COR\ diag(\sigma) \odot \left(\mathbf{-1} \quad \mathbf{K}\right)^T \left(\mathbf{-1} \quad \mathbf{K}\right)\right)\left[\mathbf{a}\ \mathbf{b}\right]\left(1-K\right)^{-2} \quad (3)$$

Where σ is a vector of the 8 MIS standard errors, and COR is a correlation matrix. The $diag(\sigma)$ is a square matrix with σ_i of the σ vector in row *i* column *i* and zeroes elsewhere.

To derive the GCE's bias we may rewrite the GCE in terms of the MIS effects $\hat{d}_{i,t}$. After revising the GCE and ignoring terms with K^t, we have the following formula:

$$Y_t^{'} \approx \hat{Y}_t + \sum_{i=1}^{8}\left(a_i S_t\left(\hat{d}_i\right) - Kb_i S_{t-1}\left(\hat{d}_i\right)\right)\left(1-K\right)^{-1}$$

And the approximate bias is clearly

$$Bias\left(Y_t^{'}\right) \approx \sum_{i=1}^{8}\left(a_i S_t\left(d_i\right) - Kb_i S_{t-1}\left(d_i\right)\right)\left(1-K\right)^{-1}$$

The squared bias expressed in matrix form is

$$Bias^2\left(Y_t^{'}\right) \approx \left[\mathbf{a}\ \mathbf{b}\right]^T \left(B^T B \odot \left(-\mathbf{1} \quad \mathbf{K}\right)^T \left(-\mathbf{1} \quad \mathbf{K}\right)\right)\left[\mathbf{a}\ \mathbf{b}\right]\ \left(1-K\right)^{-2}$$

where *B* is a row vector of the MIS biases. The bias and variance both have a quadratic form, so the final formula for the MSE of the GCE is approximately

$$MSE\left(Y_t^{/}\right) \approx \left[\mathbf{a}\ \mathbf{b}\right]^T \left\lfloor \left(Var\left(Y_t^{/}\right) + Bias^2\left(Y_t^{'}\right)\right)\right\rfloor \left[\mathbf{a}\ \mathbf{b}\right] \quad (4)$$

Note that the variance and the bias are separate components. The *MSE* has the same form for level estimates, estimates of change, and annual averages. As a result, components from different estimators may be easily combined to optimize for different aspects of a Labor Force estimate – such as the variance of a change estimate and the bias of a level estimate. This characteristic is explored in section 5.

5. Practical Considerations

When considering a survey estimator, we must consider both theoretical and practical aspects. For example, the GCE contains more parameters and may likely give a more efficient estimate, but in the absence of MIS effects the efficiency gains are negligible. Previous Compositing research specifies some formulas and the assumptions behind those formulas—such as the variance and correlation assumptions noted in section 4. Estimating those inputs is not necessarily a simple process. The variances and MIS effects contain sampling error, so estimates may be imprecise and hurt the efficiency of Composite estimates. Some of the assumptions may also not hold true, which further effects efficiency.

In this section we consider the following practical topics and their impact on the GCE: looking beyond the MSE, estimating MIS effects, estimating MIS variances, and optimizing for multiple estimates.

5.1 Exploring both Variance and Bias components

The two main versions of Compositing considered for the CPS have been the AK Composite estimator and the GCE. Each version of Compositing comes with benefits and drawbacks. The AK Composite provides a fairly transparent estimator that is easily described to data users, but it only possesses two parameters which may limit efficiency. The GCE contains many more parameters and greater flexibility, but its form is not as transparent for a nontechnical audience. For the loss of transparency, the GCE would need to provide some substantial benefits, which depend upon the impact of the MIS effects.

Most papers discussing Composite estimation focus on the reduced variance and/or MSE of Composite estimates, but it's important to consider the tradeoffs when accounting for the relative bias. Figure 2 and 3 plot respectively the relative biases and standard errors of the over-the-month changes for Unemployed level estimates for the Second Stage, the Current AK Composite, an Optimized AK Composite, and the GCE. We defaulted to the current values of K used in the CPS for Unemployed ($K = .4$). Instead of optimizing for an MSE, both of the optimized composite estimators target the bias of monthly level estimates and the variance of the OTMC. All values are smoothed with LOESS to show the general pattern.

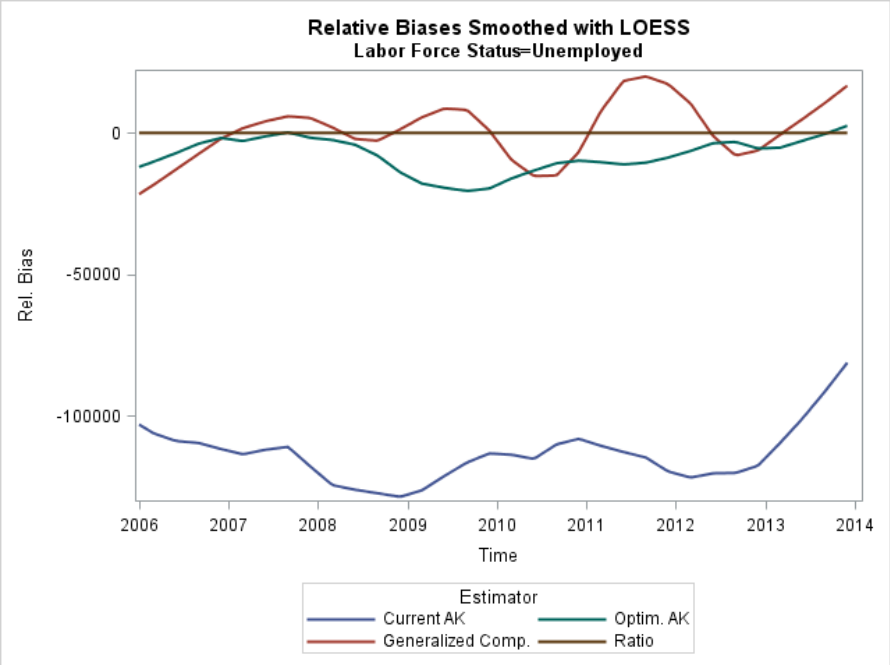

Figure 2: Relative Bias of level estimates optimized

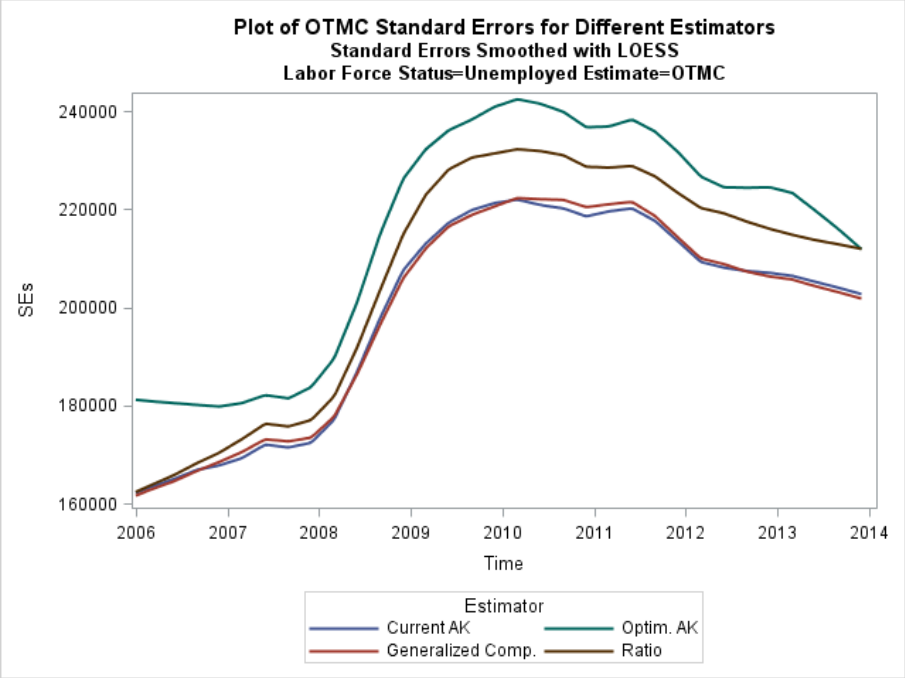

Figure 3: Standard error of over-the-month change estimates for Second Stage (Ratio) estimate and various composite estimates.

While the relative biases of the AK and GCE are comparable, the optimized AK estimator (green) provides a larger standard error than the Second Stage estimator (gold). Some authors (Shao, Zhou, and Cheng, 2014) optimize the MSE alone for the AK. While their theoretical results are interesting, it's important to consider both the variances and the biases of targeted estimates. If eliminating the bias in monthly levels is important, then previous charts indicate that the AK estimator may be insufficient, as its SEs tend to exceed the Second Stage estimate.

5.2 Estimating MIS Effects

Figure 1 showed smoothed MIS effects over 25 years, but those effects contain a significant amount of variation. Figure 4 plots additive MIS effects for Unemployment from 1996 – 2013. The darker lines are LOESS smoothed values for effects from MIS 1 and 8, while the lighter lines show the actual time series for each MIS.

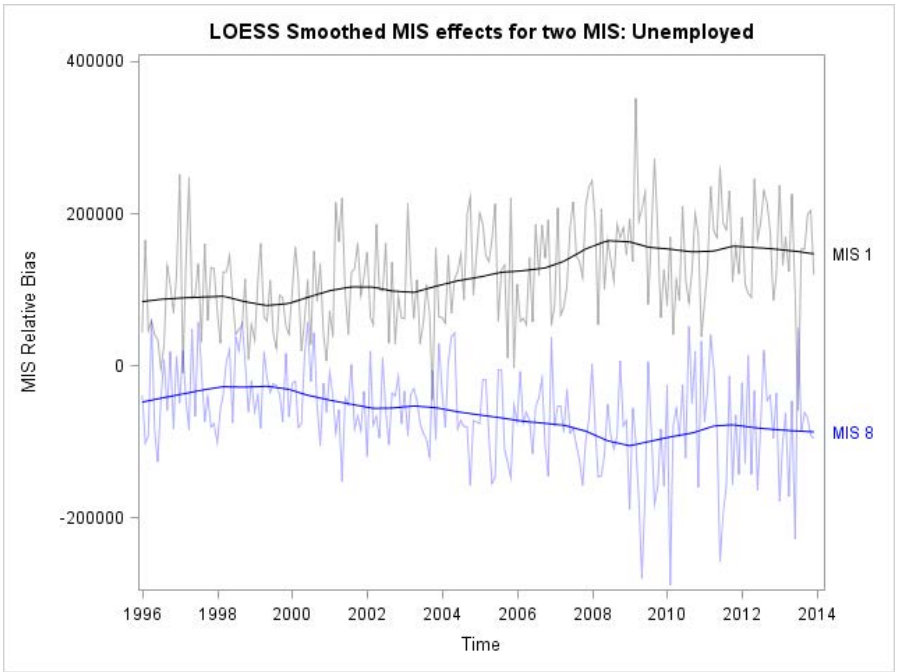

Figure 4: Plot of original and LOESS smoothed series of MIS relative biases.

Note that there's a considerable amount of noise in the series. Some autocorrelation still exists in a first-differenced transformed series, but it is not large enough to construct good forecasts with an ARIMA model. Equation 1 shows that most of the bias comes from the first 4-6 months when K^{t-1} is larger, so averaging MIS effects over a series of months seems problematic given the nonstationary behavior and the lopsided impact of near-term values. To better understand the impact, we ran a simulation with the following two different inputs for the MIS effects B for month t in equation 4:

1. A mean average of MIS effects for months $t - 24$ through t
2. The exponentially weighted moving average of each MIS effect through month t

Figure 5 below shows the impact on the relative bias of 6-month change estimates for employed for the two different estimates of the MIS effects. Two characteristics are immediately noteworthy. First, most authors assume that MIS effects are constant. This assumption implies that estimates of levels may be biased while estimates of change remain unbiased, but for some measures of change this assumption is clearly not true. Second, how the MIS effects are measured affects the relative bias. The left panel in figure 5 uses $S_t\left(\hat{d}_i\right)$ in the vector of bias terms in equation 4. The right panel uses MIS effects consistent with the stationary assumption. Even though the GCE produces a smaller relative bias for level estimates, the relative bias of a 6-month change appears to contain more relative bias in the right panel. If different monthly changes are important to a survey, then it's important to assess how the relative biases in the GCE's optimization impacts estimates of interest.

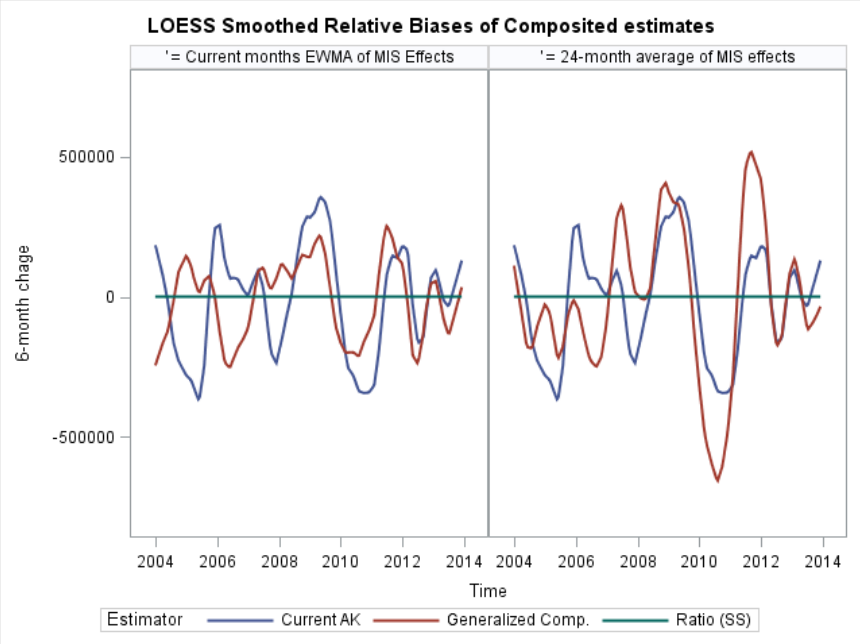

Figure 5: LOESS smoothed plots of relative biases of 6-month change estimates for Employed estimates.

5.3 Estimating MIS Variances

As shown in equation 4, month-in-sample variances are an important component of the GCE. Much previous research assumes the MIS variances and correlations in the GCE are constant. Variances for Labor Force estimates consist of the product of the binomial variance ($n^{-1}pq$) and a design effect (McIllece, 2016), so changes in the Labor Force (such as the recession in 2008) could present significant changes in the variances. There's also a significant amount of noise in the replicate variances for each MIS, which could make estimates of the variances volatile and changes over time hard to detect. We may reduce some of the volatility by using a Generalized Variance Function – a regression model relating replicate variances to characteristics of the survey design and its estimates (Wolter, 2007). GVFs often group multiple estimates to calculate model parameters, but McIllece (2016) presents a GVF for a single series that we employ for each MIS.

Figure 6 shows a time series plot of the replicate variance for MIS 1 as well as a Generalized Variance Function for the same series.

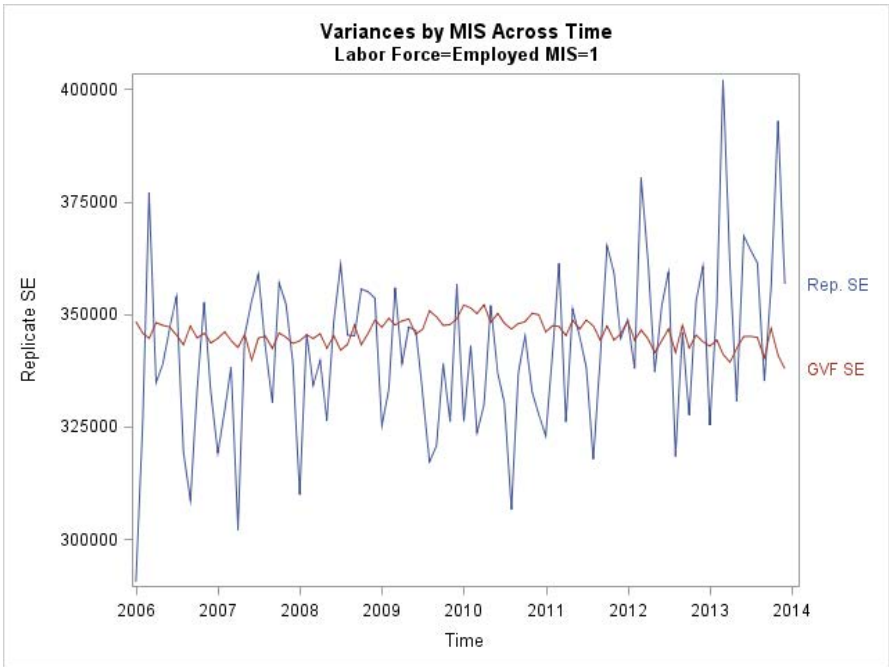

Figure 6: Time series of the replicate standard errors and GVF standard errors for Employed in MIS 1.

The GVF presents a much smoother series, and it indicates that variances may be nonstationary. The GVF series also exhibits behavior we might expect given the economic cycle.

Nonstationary variances may be incorporated into equation 4 by a linear transformation of an MIS monthly variances. Recall that the non-zero components of $diag(\sigma)$ are basically variances of the moving averages $S_t(y)$.

$$Var\left(S_t(y)\right) = Var\left((1-K)\left(\sum_{j=1}^{t}(K)^{t-j}\, y_j\right) + K^t y_0\right)$$

Assuming that any product with K^t for $t \geq 12$ makes the associated term negligible, we have:

$$Var\left(S_t(y)\right) \approx (1-K)^2\, Var\left(\sum_{j=t-12}^{t} K^{t-j} y_j\right)$$

Since all components y_j are independent we only need the $Var(y_j)$, which the GVFs provide. For an estimate of over-the-month change, we use the same general idea and isolate the individual y_j.

To test the efficacy of using GVFs in Composite estimation, we used the GVFs variances in place of averaged replicate variances in $diag(\sigma)$. Figures 7 and 8 are similar to figures 2 and 3, but figures 7 and 8 are based on Employed. Once again, we estimate parameters to minimize the variance of the OTMC and the relative bias of monthly levels.

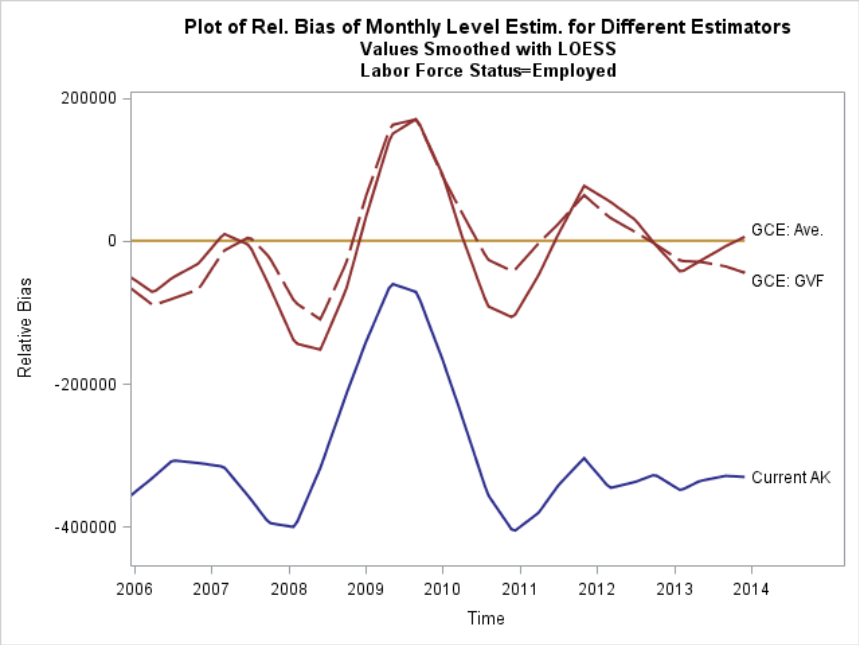

Figure 7: Relative biases for different Composited level estimates of Employed.

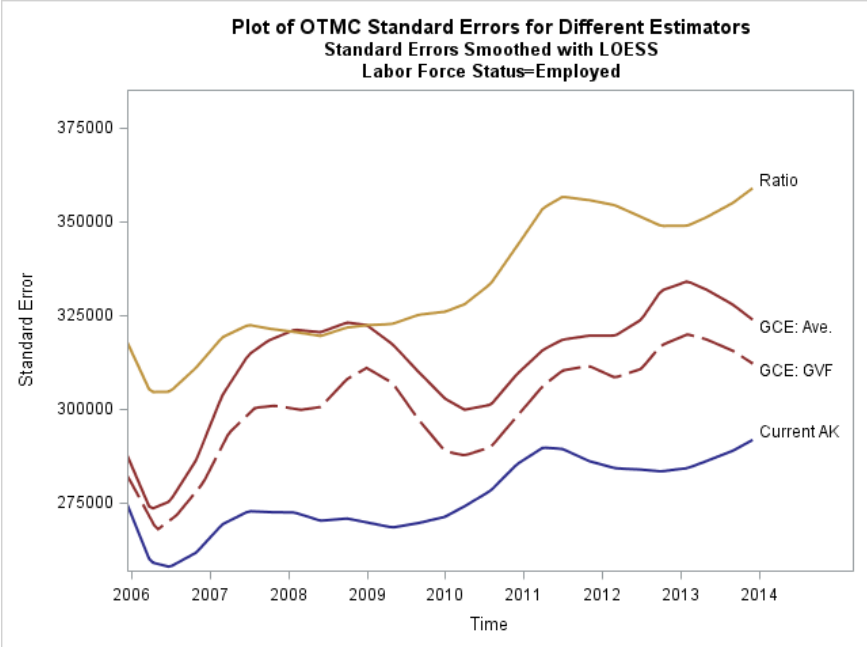

Figure 8: Standard errors of different Composite estimate for Employed.

Figure 7 shows that the relative biases with and without the GVFs are comparable, but figure 8 shows that the resulting standard errors are about 5% lower when using the GVF variances for inputs, which indicates that the additional smoothing provides some benefits for the time period analyzed here.

5.4 Optimizing for multiple estimates

Some of the primary estimates from the CPS are estimates of Labor Force changes, but analysts use a variety of Labor Force estimates to better understand general trends and the economy's current status. Some of the estimates include: levels, 3-month changes, 6-month changes, 12-month changes, and annual averages. If considering each estimate of change in optimization, each estimates' components compete with each other. In figure 8 we plotted standard errors of OTMC estimates after optimizing for the OTMC variance and the bias of level estimates. We could improve the variance if we eliminate the bias component from optimization, but the estimate's bias would be considerably larger.

The same tradeoffs occur for any estimate – optimization for one component affects other components – so we need to understand the tradeoffs and interactions of optimizing components for each estimate of interest. To balance between different estimates we may write the objective function in terms of the variance and bias terms in equation 4:

$$f^*(a,b) = \sum_{c=1}^{m} w_{\text{var},c}\, f_{\text{var},c}(a,b) + w_{bias,c}\, f_{bias,c}(a,b) \qquad (5)$$

where f^* is the objective function used in the quadratic minimization, and $w_{\text{Var},c}$ and $w_{\text{Bias},c}$ are weights attached to the variance and bias components of equation 4 for estimate c. The higher a component's weight the more influence that component has in the optimization.

The AK composite estimator could be optimized by looking at each estimates' bias-variance tradeoffs on a grid of A and K parameter values, and selecting values that meet the desired outcomes. The GCE contains 17 parameters, so the same approach becomes much more complicated. To overcome this difficulty, we could borrow from the literature on large-scale simulation experiments.

Sanchez and Wan (2015) describe the role of experimental designs in military simulation experiments. Military simulations often use a specified model and simulate multiple replications of that model to test a variable's impact. Instead of varying one component at a time, experimental designs are employed to reduce the number of simulations and isolate the impact of each component. The impact may be measured by modeling the outcome (i.e., variance or bias) dependent on a component's weight.

To explore the impact of an experimental design, we used a Nearly Orthogonal Latin Hypercube (NOLH) design provided by the Naval Post Graduate School's SEED center for data farming (Sanchez, 2011). These designs provide the following benefits:

- Nearly orthogonal quantitative factors (absolute correlations won't exceed .03)
- Near orthogonality holds for all factors as well as first-order interactions.
- Space filling properties that uniformly cover the range of interesting parameter values (Sanchez and Wan, 2015; Cioppa and Lucas, 2007)
- Ability to add experiments by a simple permutation of the columns (Cioppa and Lucas, p. 52)

Sanchez (2011) provides an Excel workbook with NOLH designs for a large number of quantitative factors. To test the design we used the bias and variance of three different estimates using six of the 7 columns in a 7-factor design. Weights $w_{\text{Var},c}$ and $w_{\text{Bias},c}$ are taken from this design with a range [.01,1], and optimal parameters for the GCE calculated based on those weights. The left section of table 2 shows the first 6 rows of the design matrix from Sanchez for four factors. We used these values as objective function weights in equation 5. The right section of the table shows the weights in which a single factor is varied, which we use to test the NOLH design.

	Weights for simulation with Designed Experiment					**Weights for simulation for single varied factor**			
Run	$w_{\text{Var},otmc}$	$w_{\text{Bias},otmc}$	$w_{\text{Var},Level}$	$w_{\text{Bias},Level}$		$w_{\text{Var},otmc}$	$w_{\text{Bias},otmc}$	$w_{\text{Var},Level}$	$w_{\text{Bias},Level}$
1	0.01	0.69	0.63	0.2		0.01	0.5	0.5	0.5
2	0.07	0.26	0.88	0.57		0.07	0.5	0.5	0.5
3	0.13	0.44	0.07	0.26		0.13	0.5	0.5	0.5
4	0.2	0.63	0.32	1.00		0.2	0.5	0.5	0.5
5	0.26	0.07	0.57	0.88		0.26	0.5	0.5	0.5
6	0.32	1.00	0.81	0.38		0.32	0.5	0.5	0.5

Table 2: First six rows of a experimental design matrix for an NOLH design (left) and a design with one factor varied.

We calculated optimal GCE parameters and the resulting SEs for OTMC estimates using the two sets of weights in equation 5. Note that table 2 only shows the first six rows of the design matrix, while a total of 17 rows exist.

To measure the impact of changing an input's weight, we look at each simulations' standard errors relative to the standard errors from optimizing the OTMC variance. All simulation are based on the Employed Labor Force. Table 3 shows regression parameters where the dependent variable is the relative SE and the independent variable is the input weight for the variance of the OTMC ($w_{\text{Var},otmc}$). The regression is run with and without the NOLH design.

	Parameter Estimates with and without a NOLH Design			
	Variable	**Parameter Estimate**	**Standard Error**	**t Value**
Without NOLH	*Intercept*	1.10831	**0.00329**	337.32
	$w_{Var,otmc}$	-0.03202	**0.00558**	-5.74
With NOLH	*Intercept*	1.10474	**0.00325**	340.12
	$w_{Var,otmc}$	-0.03425	**0.00551**	-6.21

Table 3: Univariate regression parameters for $w_{\text{Var},otmc}$ in simulations without and with the NOLH design.

The regression parameters and their standard errors are very similar with and without the NOLH design. This test seems to indicate that the experimental design provides a reasonable method to explore relationships between the objective function's input weights and different estimates' characteristics. While experimental designs seem common in large-scale simulations, there use in survey estimation seems unexplored, so further exploration is prudent to verify that relationships and interactions are well-measured. We also want to note that simulations provide an initial estimate of relationships that must be measured and verified with actual data.

6. Summary

Composite estimation is useful method to improve the efficiency of Labor Force estimates, but it may be biased if the MIS Labor Force estimates differ consistently from each other. The main inputs into the GCE are MIS effects, panel correlations, and panel variances. The MIS effects also evolve over time, and even short-term variations may cause changes to different measurements of Labor Force changes. Variances are also volatile, so measurement of each input impacts the efficiency of a Composite estimate.

This paper looked at several different practical considerations with respect to inputs for the GCE. While practical, each one contains a theoretical consideration. The GCE shows that the impact of MIS effects are front-loaded to the most recent month. Replicate variances are very noisy and possibly non-stationary. For each input, it's important to consider how to best measure that input to provide more reasonable measurements for a Composite estimator. We looked at several scenarios and alterations with real data, and can demonstrate a positive impact for the time period in our study. The final consideration looks at how we might properly balance between multiple estimates. While the CPS' purpose is to make Labor Force estimates, it's important to consider a Composite estimator's impact of the many other estimates created by the CPS.

Appendix

Consider a *t*-term exponential moving average:

$$S_t(y) = \sum_{j=1}^{t} \alpha(1-\alpha)^{t-j} y_j + (1-\alpha)^t y_0$$

$$= \sum_{j=1}^{t} \alpha(1-\alpha)^{t-j} y_j + (1-\alpha)^t y_0 - (1-\alpha)^{t+1} y_0 + (1-\alpha)^{t+1} y_0$$

$$= \alpha \sum_{j=0}^{t} (1-\alpha)^{t-j} y_j + (1-\alpha)^{t+1} y_0$$

Substituting $(1-\alpha) = \mathrm{K}$, the leftmost summand is identical to the weighted sums of *t* terms in the GCE, and those sums may be expressed as:

$$\sum_{j=0}^{t} K^{t-j} y_{i,j} = \frac{S_t(y_i) - K^{t+1} y_{i,0}}{(1-K)} \approx \frac{S_t(y_i)}{(1-K)}$$ for large values of *t* since 0 < K < 1.

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