IMPLICATIONS OF MASS BALANCE STUDIES

THE next symposium paper was read by Dr. J. F. Nye on "Implications of mass balance studies". The paper was essentially a version of the paper "Theory of glacier variations" presented by Dr. Nye to the Endicott House Conference on Engineering Glaciology, 12–16 February 1962, the Proceedings of which are being printed by the M.I.T. Technology Press, and only an abbreviated version is reported here.

In this paper Dr. Nye considers how glacier fluctuations can be related to changes in net budget or net mass flux. Starting from the kinematic equations for glacier flow, the theory seeks (i) to find the effect on glacier thickness and on the position of the snout of a given change in net mass flux for the various points on the glacier, and (ii) given observations on the variations of thickness or of the position of the snout to deduce the changes in budget which caused them. The mathematical analysis starts by considering a *datum state*, which is the state the glacier would reach if the net mass flux remained constant at all points for a long time, and then calculates the departures from this datum state caused by the actual, varying, net mass flux. The calculations can be simplified by assuming that the net mass flux plotted against distance down glacier $a(x)$ remains the same shape, the time variation merely consisting of a displacement of the curve up or down—a result that Meier (1962) has shown to be approximately true for the annual budget—though the theory can deal with more complicated cases.

The theory has two assumptions, that we can neglect density changes and that the volume of ice passing down-glacier per unit time Q is a function of position, glacier depth, and surface slope. The variation of Q with depth leads to the propagation of kinematic waves down the glacier; the variation with slope leads to a diffusion of these waves. Using these concepts it is possible to make deductions about the thickness and length of the glacier given the changes in net mass flux (Nye, 1960, 1961); however, this in general involves the solution of a diffusion equation where the diffusion constant varies with position—a rather formidable problem.

The problem of working back from the observed variations of glaciers to the budget changes which caused them is in many ways more interesting, particularly as it is possible to give a general solution. If we have information about the changes of thickness of a glacier measured normal to the surface near the snout—we call this h_1 and it will be varying with time—then what we wish to discover is the change of net mass flux a_1 (assumed as a first approximation to be the same at all points on the glacier) which has produced this history of thickness changes. If we know h_1 as a function of time over a sufficiently long period around some time t_0, then we can expect this information to be sufficient to enable us to determine a_1 at this time. We can use this information about h_1 in the form of its time derivatives, $\dot{h}_1$, $\ddot{h}_1$, $\dddot{h}_1$, . . . Thus, since the whole theory is linear, we may tentatively write

$$a_1 = \lambda_0 h_1 + \lambda_0 \dot{h}_1 + \lambda_2 \ddot{h}_1 + \ldots,$$

and the problem reduces to finding the values of λ_0, λ_1, λ_2, . . . which turns out to involve a series of first-order differential equations, one for each λ, in which one needs to know the width of the glacier at all points, the forward velocity in the datum state, the slope at all points down the glacier in the datum state, and the net mass flux at all points down the glacier in the datum state.

These data are available from Meier's work on the South Cascade Glacier from 1957–60, and using these to calculate the coefficients, we get

$$a_1 = 0 \cdot 0034\, h_1 + 0 \cdot 15\, \dot{h}_1 + 1 \cdot 5\, \ddot{h}_1 + 2\, \dddot{h}_1 + \ldots$$

the times being measured in years. For convergence the true function $h_1(t)$ must be smoothed before taking its derivatives, Fourier components with periods less than 40 yr. being removed.

This equation can be used to transfer advance and retreat data into changes of budget; however, snout variations have, unfortunately, not been studied for South Cascade Glacier for a long enough period, so no direct check is possible there. For most of the glaciers where there are such records, we do not have the detailed study of the glacier and its flow covering the accumulation as well as the ablation area.

A physical interpretation can be given for these coefficients λ. Consider first λ_0; suppose the glacier has been stationary for a long time, so that $\dot{h}_1$, $\ddot{h}_1$, . . . can be taken as zero; then

$$a_1 = \lambda_0 h_1,$$

i.e. λ_0 is the ratio of the change in net mass flux to the steady change of thickness it would produce. For South Cascade Glacier, for example, $\lambda_0 = 0{\cdot}0034$ yr.$^{-1}$ implies that if h_1 changes permanently at the snout by 10 m. (which corresponds to an advance of 50 m.), the net mass flux must have changed by an amount $a_1 = 0{\cdot}0034 \times 10 = 0{\cdot}034$ m. yr.$^{-1}$ $= 3{\cdot}4$ cm. yr.$^{-1}$, i.e., there are $3{\cdot}4$ cm. more accumulation per year in the accumulation area and $3{\cdot}4$ cm. less ablation per year in the ablation area.

λ_1 is a dimensionless constant. If we suppose the snout to have been advancing at a constant rate for many years and to be just passing through the datum position, then

$$a_1 = \lambda_1 \dot{h}_1,$$

i.e. λ_1 measures the ratio of the excess mass flux to the rate of change of thickness when the glacier is just passing through the datum state. Similar interpretations can be given for λ_2, λ_3, and so on.

These parameters λ are therefore fundamental indicators of the way in which a glacier responds to changes in budget, and so could be used to classify the glaciers in a way which would be related to their theoretical response to climatic change; for example λ_2/λ_1 measures approximately the delay between a zero value of net mass flux and the corresponding maximum advance (about 10 yr. for the South Cascade Glacier). When more field observations of the kind obtained by Dr. Meier on South Cascade Glacier are available, we will be able to classify glaciers much more rationally with respect to their response to climatic change.

REFERENCES

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DISCUSSION OF DR. J. F. NYE'S PAPER

Dr. J. W. Glen: How do you determine your datum state velocities? You need to know your velocities when the glacier is in a steady state, but no glaciers that I know are ever in a steady state for long enough for one to measure these quantities, how then do you determine them?

Dr. Nye: This is quite tricky, though not as subjective as you might imagine. You have the curve for the net mass flux in the datum state, $a_0(x)$, at your disposal. What I have done is to take as the datum state for South Cascade Glacier a glacier that would be in equilibrium for the existing length of South Cascade Glacier under a regime that is a certain curve parallel to those found by Meier, i.e. I have chosen the particular curve of this family that will make the South Cascade Glacier come to an end at the place where it actually does. That gives a_0, and hence Q_0, the steady-state value of Q, which is, roughly speaking, the velocity times the thickness. Then the problem is to separate these two things, and you can do this if you have a theory as to how velocity changes with thickness. There is a good deal more to this, but I cannot go into it in the discussion.

Dr. G. de Q. Robin (Chairman): I think Dr. Nye hopes that someone will put him on to the right facts and figures so that the theory can be tested out rather more exhaustively before being handed over to someone like Professor Manley as a technique which he would use for his climatic extrapolations.