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On Global Dimensions of Tree Type Finite Dimensional Algebras

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Abstract. A formula is provided to explicitly describe global dimensions of all kinds of tree type finite dimensional k-algebras for k an algebraic closed field. In particular, it is pointed out that if the underlying tree type quiver has n vertices, then the maximum global dimension is $n-1$.

1 Introduction

Let k be an algebraically closed field, and let A be a finite dimension k-algebra. By a module we mean a finite dimensional left A-module. The supremum of projective dimensions of all A-modules, or equivalently, of all simple A-modules, is called the global dimension of the algebra A, and is denoted by gl.dim.A ([We]).

It is well known that global dimensions of hereditary algebras are one, of tilting algebras as well as quasi-tilting algebras are at most two, while those of self-injective algebras except semisimple ones are infinite ([Au, As, Ha, Ri]). Some attractive issues of representation theory of algebras such as representation dimensions of finite dimensional algebras and the finitistic global dimension conjecture for Artin algebras are all closely connected with global dimensions of corresponding algebras ([Ig, Ra, Xi]). So to find global dimensions of finite dimensional k-algebras of the special type is worthwhile and interesting.

A finite dimensional k-algebra is called *tree type* if it is Morita equivalent to some k-algebra $k\Delta/\langle\rho\rangle$ with Δ a tree type quiver and $\langle\rho\rangle$ an admissible ideal of $k\Delta$. For more information on tree type finite dimensional k-algebras we refer the reader to [Bo, Br]. Although we know that the global dimension of a tree type finite dimensional k-algebra is finite, there has not been a formula to describe it until now. In this paper we provide a formula to explicitly describe global dimensions of all tree type finite dimensional k-algebras.

Denote the cardinal number of the set R_i by $|R_i|$. Then we have the following theorem.

Theorem 1.1 *Suppose a tree type finite dimensional k-algebra A is Morita equivalent to $k\Lambda/\langle\rho\rangle$, where Λ is a tree type finite quiver and $\langle\rho\rangle$ is an admissible ideal of $k\Lambda$ generated by relations on Λ. Then the global dimension of A is the maximum of*

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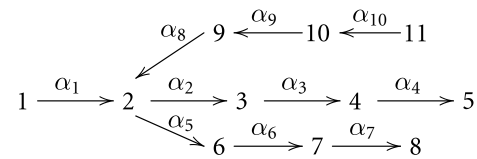
$\{|R_i| + 1 \mid i \in J\}$, where $R_i, i \in J$ are the ultimately successive relation sets on Λ deriving from the minimal relation generators of $\langle\rho\rangle$.

The technical terms in the theorem will be defined in the following section.

Example Let Λ be the following tree type quiver



Suppose $\langle\rho\rangle$ is an admissible ideal of $k\Lambda$ generated by

$$\alpha_9\alpha_{10},\ \alpha_2\alpha_8\alpha_9,\ \alpha_3\alpha_2,\ \alpha_4\alpha_3,\ \alpha_2\alpha_1,\ \alpha_6\alpha_5\alpha_1,\ \alpha_7\alpha_6.$$

Then by Theorem 1.1 the global dimension of $k\Lambda/\langle\rho\rangle$ is 5, since there are three ultimately successive relation sets on Λ in all, namely $\{\alpha_9\alpha_{10}, \alpha_2\alpha_8\alpha_9, \alpha_3\alpha_2, \alpha_4\alpha_3\}$, $\{\alpha_2\alpha_1, \alpha_3\alpha_2, \alpha_4\alpha_3\}$, and $\{\alpha_6\alpha_5\alpha_1, \alpha_7\alpha_6\}$, with the largest among them being $\{\alpha_9\alpha_{10}, \alpha_2\alpha_8\alpha_9, \alpha_3\alpha_2, \alpha_4\alpha_3\}$, with cardinal number 4.

2 Proof of Theorem 1.1

A quiver $\Delta = (\Delta_0, \Delta_1, s, e)$ is given by two sets Δ_0, Δ_1 and two maps $s, e\colon \Delta_1 \to \Delta_0$; Δ_0, Δ_1 are respectively called the set of vertices and the sets of arrows of Δ; $s(\alpha)$ and $e(\alpha)$ are respectively called the head and the tail of $\alpha \in \Delta_1$. A path p in Δ of length l means a sequence of arrows $p = \alpha_l \cdots \alpha_1$ with $e(\alpha_i) = s(\alpha_{i+1})$ for $1 \le i \le l-1$. Set $s(p) = s(\alpha_1)$, $e(p) = e(\alpha_l)$ and $l(p) = l$, which are called the head, the tail, and the length of p, respectively. Regard a vertex $i \in \Delta_0$ as a path of length 0 and denoted by e_i. For any field k and any quiver Δ, let $k\Delta$ be the k-space with basis the set of all finite length paths in Δ. For any two paths $p = \alpha_m \cdots \alpha_1$ and $q = \beta_n \cdots \beta_1$ in Δ, define the multiplication

$$qp = \begin{cases} \beta_n \cdots \beta_1 \alpha_m \cdots \alpha_1, & \text{if } e(p) = s(q), \\ 0, & \text{otherwise.} \end{cases}$$

Then $k\Delta$ becomes a k-algebra, which is called the *path algebra* of Δ. In $k\Delta$, we denote by $k\Delta^+$ the ideal generated by all arrows. Note that $(k\Delta^+)^n$ is the ideal generated by all paths of length $\ge n$.

A relation σ on a quiver Δ over a field k is a k-linear combination of paths $\sigma = a_1 p_1 + \cdots + a_n p_n$ with $a_i \in k$ and $e(p_1) = \cdots = e(p_n)$ and $s(p_1) = \cdots = s(p_n)$. We say that σ starts from $e(p_1) = \cdots = e(p_n)$ and ending in $s(p_1) = \cdots = s(p_n)$. If $\rho = \{\sigma_t\}_{t\in T}$ is a set of relations on Δ over k, the pair (Δ, ρ) is called a *quiver with relations over* k. Associated with (Δ, ρ) is the k-algebra $k(\Delta, \rho) = k\Delta/\langle\rho\rangle$, where $\langle\rho\rangle$ denotes the ideal in $k\Delta$ generated by the set of relations ρ. An ideal $\langle\rho\rangle$ of $k\Delta$ generated by the set of relations ρ in $k\Delta$ with $(k\Delta^+)^n \subseteq \langle\rho\rangle \subseteq (k\Delta^+)^2$ for some $n \ge 2$ is called an *admissible ideal* of $k\Delta$.

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A quiver is called *tree type* if its underlying graph is a tree. In graph theory, a tree is an undirected graph in which any two vertices are connected by exactly one simple path. Hereinafter, we let Λ be a tree type quiver.

Definition 2.1 If two relations ρ_1 and ρ_2 on Λ have $\rho_1 = pp_1$, $\rho_2 = p_2 p$ with p, p_1, p_2 non-trivial paths of Λ, we say that ρ_1 is *successive* to ρ_2. If ρ_1 is successive to ρ_2, we denote it by $\rho_1 \sim \rho_2$.

Definition 2.2 A set T of relations $\{\rho_i \mid i \in I\}$ on Λ is called *successive* if there is an order $\prec$ on I such that all elements of I can be listed as $i_1 \prec i_2 \prec i_3 \prec \cdots \prec i_{m-1} \prec i_m$, subject to $i_j \sim i_{j+1}$ for $j = 1, \dots, m-1$. Moreover, if ρ_{i_1} starts from vertex i, the successive set T of relations is called *starting from vertex i*.

Definition 2.3 A relation set T on Λ is called *ultimately successive* if T is a successive set on Λ that is not properly contained in other successive relation sets on Λ. A starting from vertex i ultimately successive relation set T on Λ means that T is a starting from vertex i successive set on Λ that is not properly contained in other starting from i successive relation sets on Λ.

Example 2.4 Recall the example in the introduction, and let $\rho_1, \dots, \rho_7$ respectively denote the relations $\alpha_9\alpha_{10}, \alpha_2\alpha_8\alpha_9, \alpha_3\alpha_2, \alpha_4\alpha_3, \alpha_2\alpha_1, \alpha_6\alpha_5\alpha_1, \alpha_7\alpha_6$.

Then we have

$$\rho_1 \sim \rho_2, \quad \rho_2 \sim \rho_3, \quad \rho_3 \sim \rho_4, \quad \rho_5 \sim \rho_3, \quad \rho_6 \sim \rho_7.$$

Thus $T_1 = \{\rho_1, \rho_2, \rho_3, \rho_4\}$, $T_2 = \{\rho_5, \rho_3, \rho_4\}$, $T_3 = \{\rho_6, \rho_7\}$ are ultimately successive relation sets on Λ. Then T_2 and T_3 start from vertex 1 ultimately successive relation sets on Λ. $T_4 = \{\rho_2, \rho_3, \rho_4\}$ is the only ultimately successive relation set on Λ starting from vertex 10.

Lemma 2.5 *Relations on Λ have the form $\alpha_{i_1,t_1}\alpha_{i_1,t_1-1}\cdots\alpha_{i_1,2}\alpha_{i_1,1}$.*

Proof We know that there is at most one path connecting two vertices of Λ, since the underlying graph of Λ is a tree. Therefore, the assertion is true by the definition of relation on Λ. ∎

We call $\{\rho_i \mid i \in I\}$ a set of *minimal relation generators* of an admissible ideal $\langle\rho\rangle$ of $k\Lambda$ if $\rho_i, i \in I$ are relations on Λ, $\langle\rho\rangle$ is generated by $\rho_i, i \in I$, and each ρ_i is not generated by others ρ_j with $j \neq i$, $j \in I$.

Lemma 2.6 *Let $\{\rho_j \mid j \in I\}$ be a set of minimal relation generators of an admissible ideal $\langle\rho\rangle$ of $k\Lambda$. Given any vertex i of Λ, there are only finitely many ultimately successive relation sets starting from vertex i derived from $\{\rho_j \mid j \in I\}$.*

Proof The set of all relations on Λ is finite, since any relation on the quiver Λ is of the path's form $\alpha_{i_m}\alpha_{i_{m-1}}\cdots\alpha_{i_2}\alpha_{i_1}$ by Lemma 2.5, and Λ is a tree type finite quiver. Then the assertion is clear. ∎

Lemma 2.7 *The set of minimal relation generators $\{\rho_i \mid i \in I\}$ of an admissible ideal $\langle\rho\rangle$ of $k\Lambda$ is a union of ultimately finite successive sets $R_1, R_2, \ldots, R_m$.*

Proof This can be done, since $\langle\rho\rangle$ is finitely generated by relations on Λ. ■

Given an admissible ideal of $k\Lambda$, denote by S_i the i-th simple module for algebra $k\Lambda/\langle\rho\rangle$ corresponding to the i-th vertex of the graph Λ. Denote by $pro.dim.S_i$ the projective dimension of $k\Lambda/\langle\rho\rangle$-module S_i.

A *representation* (V, f) of a quiver Δ over a field k is a set of vector spaces $\{V_i \mid i \in \Delta_0\}$ together with k-linear maps $f_\alpha \colon V_i \to V_j$ for each arrow $\alpha \colon i \to j$. If $V = (V_i, f_\alpha)$ and $W = (W_i, g_\alpha)$ are two representations, a morphism

$$\psi = (\psi_1, \ldots, \psi_n) \colon V \to W$$

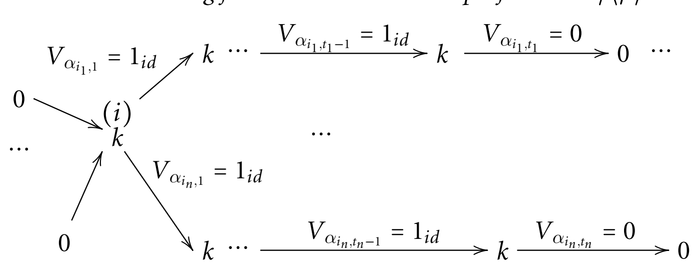
is given by $\psi_i \in \operatorname{Hom}(V_i, W_i)$ such that $\psi_t(\alpha) f_\alpha = g_\alpha \psi_s(\alpha) \colon V_s(\alpha) \to W_t(\alpha)$. This defines the category $\operatorname{rep}_k \Delta$ of representations of Δ. If $w = \alpha_l \cdots \alpha_1$ is a path in Δ, we may denote by V_w the composition $V_{\alpha_l} \cdots V_{\alpha_1}$. We say that V satisfies the relation $\rho = \sum_w c_w w$ provided $\sum_w c_w V_w = 0$.

Due to a well-known theorem of Gabriel, the category of finite dimension representation of a quiver Δ over k satisfying relations $\langle\rho\rangle$ is equivalent to the category of finite dimensional $k\Delta/\langle\rho\rangle$-modules ([Au, As]). So we usually investigate $k\Lambda/\langle\rho\rangle$-module category through corresponding quiver representation category. In the sequel we identify $k\Lambda/\langle\rho\rangle$-modules with their quiver representations.

Lemma 2.8 *Let $\langle\rho\rangle$ be an admissible ideal of $k\Lambda$. Suppose $\{\rho_j \mid j \in I\}$ is a set of minimal relation generators of the ideal $\langle\rho\rangle$. Let*

$$\alpha_{i_1,t_1} \cdots \alpha_{i_1,2}\alpha_{i_1,1}, \quad \alpha_{i_2,t_2} \cdots \alpha_{i_2,2}\alpha_{i_2,1}, \quad \ldots \quad \alpha_{i_n,t_n} \cdots \alpha_{i_n,2}\alpha_{i_n,1} \quad \in \{\rho_j \mid j \in I\}$$

all be relations on Λ starting from vertex i. Then the projective $k\Lambda/\langle\rho\rangle$-module $P(i)$ is



Proof Since $P(i) \cong k\Lambda e_i/\langle\rho\rangle$, and one basis of $k\Lambda e_i/\langle\rho\rangle$ over k is

$$\overline{e_i},\ \overline{\alpha_{i_1,1}},\ \overline{\alpha_{i_1,2}\alpha_{i_1,1}},\ \ldots,\ \overline{\alpha_{i_1,t_1-1}\cdots\alpha_{i_1,2}\alpha_{i_1,1}},\ \overline{\alpha_{i_2,1}},\ \overline{\alpha_{i_2,2}\alpha_{i_2,1}},$$
$$\ldots,\ \overline{\alpha_{i_2,t_2-1}\cdots\alpha_{i_2,2}\alpha_{i_2,1}},\ \ldots,\ \overline{\alpha_{i_n,1}},\ \overline{\alpha_{i_n,2}\alpha_{i_n,1}},\ \ldots,\ \overline{\alpha_{i_n,t_n-1}\cdots\alpha_{i_n,2}\alpha_{i_n,1}},$$

the assertion follows. ■

Lemma 2.9 *Let $\langle\rho\rangle$ be an admissible ideal of $k\Lambda$ generated by the relations on Λ. If none of the minimal relation generators of $\langle\rho\rangle$ starts from vertex i, then the projective dimension of the $k\Lambda/\langle\rho\rangle$-simple module S_i is no more than 1.*

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Proof If i is a sink vertex of Λ, then the $k\Lambda/\langle\rho\rangle$-simple module S_i is a projective $k\Lambda/\langle\rho\rangle$-module, so the projective dimension of the $k\Lambda/\langle\rho\rangle$-module S_i is 0. If i is not a sink vertex of Λ, we then denote by $i_1, i_2, \dots, i_t$ the vertices of Λ that are tails of arrows starting from i. Then

$$0 \longrightarrow \bigoplus_{\mu=1}^{t} P_{i_\mu} \longrightarrow P_i \longrightarrow S_i \longrightarrow 0$$

is a minimal projective presentation of the $k\Lambda/\langle\rho\rangle$-simple module S_i by Lemma 2.8. So the projective dimension of the $k\Lambda/\langle\rho\rangle$-simple module S_i is no more than 1. ■

Let A be a finite dimensional algebra, and let M_i, $i \in 1, 2, \dots, n$ be a family of indecomposable A-modules. We denote by $\ominus_{i=1}^{n} M_i$ the direct sum of all pairwise non-isomorphic M_i, $i \in 1, 2, \dots, n$. For example, if M_1, M_2, M_3 are indecomposable A-modules, $M_1 \ncong M_2$, and $M_2 \cong M_3$, then $\ominus_{i=1}^{3} M_i = M_1 \oplus M_2$.

Lemma 2.10 *Let $\langle\rho\rangle$ be an admissible ideal of $k\Lambda$ generated by relations on Λ. If $R_1, R_2, \dots, R_m$ are all ultimately successive relation sets starting from vertex i deriving from minimal relation generators of $\langle\rho\rangle$, then the projective dimension of the $k\Lambda/\langle\rho\rangle$-simple module S_i is the maximum of $\{|R_j| + 1 \mid j = 1, 2, \dots, m\}$.*

Proof Suppose

$$R_j = \{\alpha_{j,1,t_1}\alpha_{j,1,t_1-1}\cdots\alpha_{j,1,2}\alpha_{j,1,1},\ \alpha_{j,2,t_2}\alpha_{j,2,t_2-1}\cdots\alpha_{j,2,2}\alpha_{j,2,1}, \\ \dots,\ \alpha_{j,|R_j|,t_{|R_j|}}\alpha_{j,|R_j|,t_{|R_j|}-1}\cdots\alpha_{j,|R_j|,2}\alpha_{j,|R_j|,1}\}$$

for $j = 1, 2, \dots, m$. We denote by $e_{j,k,l}$ the vertex from which the arrow $\alpha_{j,k,l}$ starts, and by $e_{j,|R_j|,t_{|R_j|},+}$ the vertex in which the arrow $\alpha_{j,|R_j|,t_{|R_j|}}$ ends. Note that it is possible that $e_{j,k,l}$ is same as another $e_{j',k',l'}$. Denote by $P_{j,k,l}$ the projective $k\Lambda/\langle\rho\rangle$-module corresponding to the vertex $e_{j,k,l}$, and by $P_{j,|R_j|,t_{|R_j|},+}$ the projective $k\Lambda/\langle\rho\rangle$-module corresponding to the vertex $e_{j,|R_j|,t_{|R_j|},+}$. We denote the maximum of $\{|R_j| \mid j = 1, 2, \dots, m\}$ by d. Then by the structure of tree graph, we have a minimal projective presentation of S_i by Lemma 2.8:

$$0 \longrightarrow \bigoplus_{j,|R_j|=d} P_{j,|R_j|,t_{|R_j|},+} \longrightarrow \ominus_{j,|R_j|=d} P_{j,|R_j|,t_{|R_j|}} \longrightarrow \cdots \\ \longrightarrow \ominus_{j=1}^{m} P_{j,2,t_2} \longrightarrow \ominus_{j=1}^{m} P_{j,1,2} \longrightarrow P_i \longrightarrow S_i \longrightarrow 0.$$

Therefore, the projective dimension of S_i is the maximum of

$$\{|R_j| + 1 \mid j = 1, 2, \dots, m\}.$$ ■

Example 2.11 Recall the quiver Λ in Example 2.4, and let $\langle\rho\rangle$ be the admissible ideal of $k\Lambda$ generated by $\rho_1, \dots, \rho_7$. Then the projective dimension of simple $\Lambda/\langle\rho\rangle$-module S_1 is 4, since $T_2 = \{\rho_5, \rho_3, \rho_4\}$ and $T_3 = \{\rho_6, \rho_7\}$ are all ultimately successive relation sets starting from vertex 1.

Now we prove our main theorem.

***Theorem* 1.1** *Suppose a tree type finite dimensional k-algebra A is Morita equivalent to $k\Lambda/\langle\rho\rangle$, where Λ is a tree type finite quiver and $\langle\rho\rangle$ is an admissible ideal of $k\Lambda$ generated by relations on Λ. Then the global dimension of A is the maximum of $\{|R_i|+1 \mid i \in J\}$, where $R_i, i \in J$ are the ultimately successive relation sets on Λ deriving from the minimal relation generators of $\langle\rho\rangle$.*

Proof Since A is a tree type finite dimensional k-algebra, it is Morita equivalent to $k\Lambda/\langle\rho\rangle$ for a finite tree type quiver Λ and an admissible ideal $\langle\rho\rangle$ of $k\Lambda$ ([Au, As]). Therefore to determine the global dimension of k-algebra A is just to determine the global dimension of $k\Lambda/\langle\rho\rangle$. The global dimension of $k\Lambda/\langle\rho\rangle$ is the maximum of projective dimensions of all simple $k\Lambda/\langle\rho\rangle$ modules ([We]). Therefore by Lemmas 2.9 and 2.10 the global dimension of A is the maximum of $\{|R_i| + 1 \mid i \in J\}$, where $R_i, i \in J$ are all starting from vertices of Λ ultimately successive relation sets deriving from minimal relation generators of $\langle\rho\rangle$. Since it is clear that on one hand ultimately successive relation sets belong to starting from vertices of Λ ultimately successive relation sets, and on the other hand, an ultimately successive relation set starting from a vertex of Λ must be a subset of an ultimately successive relation set, the maximum of $\{|R_i| + 1 \mid i \in J\}$ where $R_i, i \in J$ are all ultimately successive relation sets starting from vertices of Λ deriving from minimal relation generators of $\langle\rho\rangle$ is the same with the maximum of $\{|R_i| + 1 \mid i \in J\}$ where $R_i, i \in J$ are all ultimately successive relation sets deriving from minimal relation generators of $\langle\rho\rangle$. So the theorem holds. ∎

Corollary 2.12 *$\{0, 1, 2, \dots, n-1\}$ are all possible global dimensions of all n-vertex tree type finite dimensional k-algebras. In particular, if $n > 1$, the maximal global dimension of all n-vertex tree type finite dimension algebras is $n-1$.*

Proof If $n = 1$, clearly the global dimension of tree type finite dimensional k-algebras is 0, since these algebras are semisimple ones. If $n > 1$, since $\{0, 1, \dots, n-2\}$ are all possible lengths of ultimately successive relation sets of an admissible ideal of the corresponding tree type path algebra $k\Lambda$ (note that the length "0" means that there are no relations), and it easy to see that any of these lengths can actually occur in suitable tree type quivers and suitable relations. So the assertion follows from Theorem 1.1. ∎

Corollary 2.13 *Suppose that Λ has n vertices. Let $\langle\rho\rangle$ be an admissible ideal of $k\Lambda$. Then the global dimension of $k\Lambda/\langle\rho\rangle$ is $n-1$ if and only if $\langle\rho\rangle$ is generated by $\alpha_2\alpha_1, \alpha_3\alpha_2, \dots, \alpha_{n-1}\alpha_{n-2}$.*

Proof On one hand, if $\langle\rho\rangle$ is generated by $\alpha_2\alpha_1, \alpha_3\alpha_2, \dots, \alpha_{n-1}\alpha_{n-2}$, then $\alpha_2\alpha_1, \alpha_3\alpha_2, \dots, \alpha_{n-1}\alpha_{n-2}$ constitutes the unique ultimately successive relation set, and its cardinal number is $n-2$. So by Theorem 1.1, the global dimension of $k\Lambda/\langle\rho\rangle$ is $n-1$.

On the other hand, if the global dimension of $k\Lambda/\langle\rho\rangle$ is $n-1$, then by Theorem 1.1 there is an ultimately successive relation set whose cardinal number is $n-2$. Suppose $\alpha_2\alpha_1, \alpha_3\alpha_2, \dots, \alpha_{n-1}\alpha_{n-2}$ is such an ultimately successive relation set. Then $\langle\rho\rangle$ has to be generated by $\alpha_2\alpha_1, \alpha_3\alpha_2, \dots, \alpha_{n-1}\alpha_{n-2}$, since Λ is a tree type quiver with n vertices. ■

Corollary 2.14 *Let B be a tree type finite dimensional k-algebra. The global dimension of B is the same as the global dimension of an A_n-type finite dimensional k-subalgebra B' of B for some $n \in \mathbb{Z}^+$.*

Proof Assume that B is equivalent to $k\Lambda/\langle\rho\rangle$. By Theorem 1.1 there is an ultimately successive relation set R such that the global dimension of B is $|R|+1$. Let $k\Lambda'+\langle\rho\rangle/\langle\rho\rangle$ be the subalgebra of $k\Lambda/\langle\rho\rangle$ where Λ' is a full sub-quiver of Λ generated by vertices and arrows occurring in R. It is easy to see that Λ' is an A_n-type quiver for some $n \in \mathbb{Z}^+$. By Theorem 1.1 the global dimension of $k\Lambda' + \langle\rho\rangle/\langle\rho\rangle$ is also $|R| + 1$. Since there exists a k-subalgebra B' of B that is Morita equivalent to $k\Lambda'+\langle\rho\rangle/\langle\rho\rangle$, the global dimension of B is the same with the global dimension of B'. ■

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