P 121. We shall say of two sets A, B in a topological space, that B is "peripheral" to A, if:

(a) the closure of A contains B, and

(b) the closure of B has no points in common with A.

It is easily seen that this relation is transitive. Find, in a Hausdorff space, a collection of sets which is linearly ordered by "peripheral" and has the order-type of the reals.

M. Shimrat, York University

P 122. Suppose G is a topological group, K a compact set and V a neighbourhood of the identity in G. Is there a positive integer N depending on K and V such that K contains no more than N non overlapping translates of V?

J.B. Wilker, University of Toronto

P 123. Let $u^1, u^2, \ldots$ be sequences, $u^i = \{u_n^i\}$, such that, for each i, $\sum_n |u_n^i|^p < \infty$ if and only if $p > 1$.

(Example: $\{\frac{1}{n}\}$, $\{\frac{1}{n \log n}\}$, $\{\frac{1}{n \log n \log \log n}\}$, ...)

Show that there exists a sequence x such that $\sum_n x_n u_n^i$ is convergent for each i, and $x_n \to 0$, but $\sum |x_n|^p = \infty$ for all $p \geq 1$.

A. Wilansky, Lehigh University

P 124. To every c_1 there is a c_2 so that if $a_1 < \ldots < a_k \leq n$, $k \geq c_1 n$, is any sequence of integers and $b_1 < \ldots < b_m$ is the sequence of the distinct a_i/p_i where p_i is the greatest prime factor of a_i (so $m \leq k$) then

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$$\Sigma\, 1/b_i > c_2 \log n \ .$$

P. Erdős, Israel Institute of Technology

SOLUTIONS

P 113. If $m > 4$ show that the integral part $n = [(m-1)!/m]$ is an even integer.

D.R. Rao, Secunderabad, India

Solution by A. Makowski, Warszawa, Poland.

If m is prime then by Wilson's theorem $(m-1)! = -1 + km$ where k is odd (since $(m-1)!$ is even). Thus $n = k-1$ is even. If $m = p^2$, then $(m-1)!$ contains as factors p and $2p$, hence $m \mid (m-1)!$ with even quotient n. Otherwise we may write $m = ab$, $1 < a < b$ so again $m \mid (m-1)!$ Now $(m-1)!$ contains $2.3.4$ and to have n odd we would require $a = 2$, $b = 4$; but then $m = 8$ and n is a multiple of 6.

Also solved by L. Carlitz, T.M. King and the proposer.

P 115. A set of polynomials $c_n(x)$ which appears in network theory is defined by,

$$c_{n+1}(x) = (x+2).c_n(x) - c_{n-1}(x) \qquad (n \geq 1)$$

with $c_0 = 1$ and $c_1 = (x+2)/2$.

Establish the following properties of $c_n(x)$:

(i) $c_n(x)$ satisfies the differential equation,

$$(x^2 + 4x)y'' + (x+2)y' - n^2 y = 0 \ .$$

(ii) The zeros of $c_n(x)$ are all real, negative and distinct, and these are

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