PROBLEMS FOR SOLUTION

P31. Prove that if $p > 3$ is a prime $\equiv 3 \pmod 4$ and $\zeta = e^{2\pi i/p}$, then

$$\prod_r (1 + \zeta^r) = \left(\frac{2}{p}\right)$$

where r runs through the quadratic residues of p, and $\left(\frac{2}{p}\right)$ is the Legendre symbol of quadratic residuacity.

L.J. Mordell

P32. The equation

$$(1 + 2\cos\frac{\pi}{p})(1 + 2\cos\frac{\pi}{q}) = 1$$

is obviously satisfied by $p = q = 2$. Are there any other rational solutions with $p \geq q \geq 1$?

N.W. Johnson

P33. Let

$$R_n = R_n(x) = \sum_{r=0}^{n} \binom{n+r}{n-r} x^r .$$

Show that for $n > 0$,

$$R_{n+1}R_{n-1} - R_n^2 = x.$$

L. Lorch and L. Moser

P34. Determine all Riemann surfaces with a transitive group A of automorphisms (conformal self mappings). On these surfaces a not too narrow conformal geometry can be based.

H. Helfenstein

SOLUTIONS

P25. Let H be a complex in a finite additive group and let H contain 0 and at least one other element. Does there

https://doi.org/10.1017/S0008439500025455 Published online by Cambridge University Press