OBSERVATIONS IN NORTH GREENLAND RELATING TO THEORIES OF THE PROPERTIES OF ICE

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Abstract. To test Nye's formula relating maximum surface slope at a point on an ice sheet with the thickness at that point, observations were made of the surface slope in north Greenland. Values of the shear stress on the bed at these points are calculated. It is unlikely that variations of the shear stress can be related with rates of movement. It is suggested that the difference between values of shear stress in north Greenland and those in south Greenland is at least partly due to the presence of bottom melting in the north.

Zusammenfassung. Um Nye's Formel zu prüfen, in der die maximale Oberflächenneignung an einem Punkt einer Eisscheibe mit der Dicke an diesem Punkt in Beziehung bebracht wird, wurden in Nord-Grönland Beobachtungen in Bezug auf die Oberflächenneigung gemacht. Die Werte der Scherspannung an diesen Punkten in der Gletschersohle sind berechnet. Es ist unwahrscheinlich, dass Schwankungen in der Scherspannung mit dem Bewegungsmass in Beziehung stehen. Es wird vermutet, dass der Unterschied zwischen den Werten der Scherspannung in Nord-Grönland und Süd-Grönland zum mindesten teilweise auf Schmelzen an der Sohle des Gletschers im Norden zurückzuführen ist.

Introduction

In recent years Nye[1, 2, 3] has considered the behaviour of ice in glaciers and ice sheets as a problem in plasticity and has shown that if certain assumptions are made it is possible to derive a simple relation between the surface slope and the thickness at any point on an ice sheet. The scientific work of the British North Greenland Expedition (1952–54) included the determination of the thickness of the inland ice by seismic methods, and a traverse from Dronning Louise Land (lat. 77° 30′ N., long. 25° W.) to Nunatarssuaq (lat. 76° 30′ N., long. 68° W.) on which values of gravity were measured and the altitude of the gravity stations determined by levelling. Since theodolites were used for the levelling, the opportunity was taken to measure directly the surface slope of the inland ice, to test Nye's formula relating slope and thickness.

Measurement of Slope

Near the edges of the inland ice, where the ice is relatively thin, the local form of the upper surface is determined to some extent by the subglacial topography directly, but nearer the centre this is not the case. In this region, where local undulations do not occur, direct measurements of the slope are more accurate than values obtained from the existing contour maps.

At stations near the centre of north Greenland the slope of the inland ice surface was measured by sighting the theodolite on the horizon at azimuth intervals of 30°. The true azimuth at each station (correct to the nearest degree) was determined by sun observation. Corrections to these measured vertical angles, for dip and refraction, should be made but the values are uncertain because the distance to the horizon is not accurately known and although the refraction coefficient is known in one direction (along the line of the traverse) the assumption that refraction is the same in all directions is probably not justified. Therefore it has seemed better not to make these corrections, but to assume at each station that the maximum uphill (positive) and maximum downhill (negative) slopes are numerically equal. Hence at each station a correction has been applied to all the measured angles to equalize these maximum slopes. Ignoring the effects of very small undulations, the assumptions of uniform slope over the range of visibility from theodolite height, approximately 10 km., is justifiable for the central part of the inland ice, although at the three most westerly stations at which directly observed slopes are given, A149, A151 and A159, the applied correction may have been too large.

Complete values of the surface slopes have been published elsewhere[4] but in Table I the values and azimuths of the maximum slopes are given for 14 stations in the central part of the traverse. These maximum values have been obtained by graphical interpolation of the observed values and the azimuths of maximum slope may be in error by 5°. The accuracy of the slopes is probably better than $\pm 0{\cdot}5'$.

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The undulations occurring east of long. 36° W. make measurements of local slope virtually meaningless and in this area the following procedure has been adopted. It has been found that on the best existing maps of the inland ice (U.S.A.F. Aeronautical Charts, 1 : 1,000,000, Nos. 18, 19 and 20) the directions of the contour lines are remarkably accurate, but that the absolute altitudes ascribed to them are liable to be incorrect. Therefore in determining the surface slope at a particular station the mean slope over about 15 km. along the direction of traverse was determined from the measured altitudes of the gravity stations. From the charts the direction of maximum dip was found, together with the angle between this and the traverse direction. Hence by simple geometry, the maximum slope was determined. These maximum values are probably correct to $\pm 1'$, except at A8, A14 and A20, where the error may be larger.

Values of slope and azimuth at the first seven stations listed in Table I are based on U.S.A.F. Aeronautical Charts Nos. 18 and 19, at the remainder they are measured directly.

Measurements of Ice Thickness

As described elsewhere[5, 6], in the eastern part of north Greenland it was not possible to measure the thickness of ice by seismic reflection methods because no energy reflected to the surface from an interface at the bottom of the ice could be detected. Further west good reflection records were obtained, and the line separating the two areas has been followed for about 120 km.

Table I

Station	*Latitude* °	′	*Longitude* °	′	*Altitude* (metres)	*Maximum slope* (Minutes of arc)	*Azimuth of max. positive slope* ° True
A8	77	18·2	27	40·5	1681	45·3	220
A14	77	27·7	29	06·0	1812	17·9	230
A20	77	30·0	30	38·0	1931	10·4	250
A26	77	35·4	32	08·0	2021	12·3	250
A32	77	41·2	33	28·5	2114	7·4	260
A38	77	48·2	34	52·0	2162	6·8	265
A45	77	58·1	36	48·0	2267	6·4	265
B52	78	04·3	38	57·0	2365	9·1	240
B55	78	03·9	39	54·5	2414	7·2	230
A67	78	02·4	43	38·0	2527	6·2	190
B102	77	55·8	46	26·5	2516	6·3	145
A106	77	44·4	47	32·0	2521	6·6	140
A108	77	38·6	48	05·5	2527	8·9	145
A111	77	29·8	49	02·0	2524	7·2	145
A118	77	34·1	50	49·0	2451	7·6	130
A125	77	57·0	52	57·5	2296	9·1	145
A132	78	09·1	55	38·5	2131	10·7	155
A140	77	54·4	58	33·5	2076	10·5	130
A149	77	23·7	61	21·0	1873	20·3	90
A151	77	18·9	61	58·0	1782	21·4	90
A159	76	55·5	64	24·5	1534	29·2	110

Fortunately the thickness of ice can be estimated from the measured values of gravity, and such estimates have been used in the following, where direct results are not available. The ice thicknesses calculated from gravity values should not be in error by more than 10 per cent.

The Equilibrium Form of an Ice Sheet

Nye's considerations of the form of an ice sheet have been based on work by Prandtl[7], who dealt with the stress distribution in a plastic material squeezed between two rough plates. Laboratory investigations[8] of the deformation of polycrystalline ice show that its properties are approximately those of a perfect plastic material, that is, one in which the strain-rate is zero until a critical stress, called the yield stress, is reached, and in which the strain rate adjusts itself so that

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no stress greater than the yield stress is maintained in the material. Thus in dealing with the problem of the form of the ice sheet Nye was able to use Prandtl's solutions. Assuming (1) that the difference between the slopes of the upper and lower surfaces of the ice sheet is everywhere small, (2) the undulations in the lower surface are not large, and (3) the floor is perfectly rough (so that the shear stress is equal to the yield stress), he has shown that the following condition must apply:

$$h=\frac{k}{\rho g \sin \alpha}=h_0 \operatorname{cosec} \alpha \qquad (1)$$

where α is the maximum surface slope at any point and h is the corresponding ice thickness. With the value of the yield stress, k, which most nearly fits the laboratory results ($k=1\cdot0$ bars) and with $\rho=0\cdot90$ gm. cm.$^{-3}$ and $g=981$ cm. sec.$^{-2}$, $h_0=11\cdot3$ m.

Nye also assumes that the ice is moving laterally at all points, which is probably not justified at the highest points of the ice sheet at any time and elsewhere only if the rate of melting at the bottom is less than the rate of accumulation on the upper surface. However, at all the stations listed in Table I these assumptions are probably justified, except perhaps at the three most easterly where the bedrock surface undulates.

Nye[2] has shown that along the main west–east seismic traverse made in southern Greenland by Expéditions Polaires Françaises, the ice thicknesses calculated from equation (1) accurately fit the experimentally determined values if $h_0=10\cdot0$ m., that is, $k=0\cdot88$ bars. Similar values for k have been determined for the ice hills in Dronning Maud Land[9] and elsewhere[10].

Assuming that the same value of k applies in north Greenland, the ice thicknesses expected at the points listed in Table I are given in Table II (column 2) together with the thickness measured by seismic methods or calculated from the values of gravity (column 3). The values for the shear stress at the bottom surface, calculated from the formula, $\tau=\rho gh \sin \alpha$, are given in column 5. Since, in the central part of north Greenland, the altitude of the ice sheet lies between 2000 and 2800 m. Nye's formula predicts that bedrock should be more than 1000 m. below sea level. In fact it is approximately at sea level.

Thus it is appropriate to consider the validity of the assumptions made in arriving at equation (1). The main sources of heat in the ice sheet are the geothermal heat, and that released in

TABLE II

Station	*Calculated thickness* (metres)	*Measured thickness* (metres)	*Ratio measured: calculated thickness*	*Shear stress* (bars)
A8	(759)	880	—	—
A14	1921	850	0·43	0·38
A20	3306	720	0·22	0·19
A26	2795	1100	0·38	0·33
A32	4646	1480	0·32	0·28
A38	5056	2020	0·40	0·35
A45	5371	2200	0·41	0·36
B52	3684	2470	0·65	0·58
B55	4797	2540	0·53	0·47
A67	5574	2540	0·44	0·39
B102	5471	2650	0·49	0·43
A106	5155	2680	0·52	0·46
A108	3928	2680	0·68	0·60
A111	4796	2630	0·55	0·48
A118	4504	2450	0·54	0·48
A125	3749	2300	0·62	0·54
A132	3198	2240	0·70	0·62
A140	3248	1900	0·59	0·52
A149	1698	1500	0·88	0·78
A151	1605	1280	0·80	0·70
A159	1357	980	0·72	0·64

the body and at the bottom of the ice by differential movement within the ice and by friction at the bottom. If the ice were homogeneous and the bottom were perfectly rough the heat released by friction would be approximately $2 \cdot 4k \times l$ cals. cm.$^{-2}$ year^{-1} where l is the distance in metres moved by the ice in one year, and k the yield stress in bars, as before. Approximate calculations for the whole of the northern part of the ice sheet show that l is unlikely to average more than 25 m., so that frictional heat should not exceed 60 calories cm.$^{-2}$ year^{-1}. No measurements of geothermal heat flow in Greenland have been made but because this quantity does not vary greatly from place to place it is safe to assume that under the ice-sheet the value will not differ appreciably from the mean of the measured values, namely about 40 cals. cm.$^{-2}$ year^{-1}. Thus the total heat available for melting the bottom of the ice sheet averages less than 100 cals. cm.$^{-2}$ year^{-1}, that is, less than enough to melt 1·4 cm. of ice per year. The present rate of accumulation of snow was measured at many points along the gravity traverse and at the Expedition's central station, "Northice" (lat. 78° 04′ N., long. 38° 29′ W., altitude 2345 m.) where it was found that the mean annual accumulation has decreased from about 18 cm. water equivalent in 1880 to about 10 cm. water equivalent in 1945 [11]. The mean accumulation does not vary greatly across the inland ice along the traverse, though it is less at the eastern limit of the inland ice, due to deflation [12]. However, there seems no chance that anywhere can it be less than the rate of bottom melting.

Since the accumulation on the high ice sheet is greater than the bottom melting, there must be lateral movement if the ice sheet is to remain in equilibrium. Direct evidence of this is provided by the forward movement of glaciers in the coastal areas of north Greenland, and by the presence of new crevasses in the marginal parts of the ice sheet.

The laboratory experiments of Glen showed that, for small strain rates, the strain rate is proportional to a high power of the stress, so that serious departures from the behaviour of an ideal plastic occur. This fact has been used to explain qualitatively the observed variations of shear stress at points on part of the Barnes Ice Cap in Baffin Island [13]. In the direction of maximum strain-rate the greatest shear stress can be sustained.

It is apparent that the stress : strain-rate relationship for the ice in north Greenland is too different from that of a perfect plastic material for the results of plasticity theory to be applied directly. However, one point is noteworthy. If an analysis is made in which it is assumed that the shear stress at the bottom of the ice is less than the yield stress by a factor m, it is found that the equilibrium thickness of the ice, given by equation (1), must also be reduced by the same factor. The full analysis of the problem, taking account of the known stress : strain-rate relationship, has not yet been made.

However, independent of the plastic properties of the material, the shear stress on the bed is given by $\tau = \rho g h \sin \alpha$, so that the values given in column 5 of Table II are experimentally determined values of the shear stress at the bottom of the ice in north Greenland. A plot of the variation of τ with longitude is given in Fig. 1 (p. 71). Although there is some scatter of the points, the slope of the best-fit straight line is significantly different from zero, so that the values of τ near the centre are significantly greater than those obtained near the east coast. (The correlation coefficient of the variation of τ with longitude is 0·86, the standard error of estimate of τ from the regression equation being 0·07.)

Apparently in south Greenland the strain-rate is everywhere sufficiently large for there to be little variation in the values of shear stress, but in the north this is not so. Nor can the explanation used on the Barnes Ice Cap be used for the figures of Table II. In the eastern part the traverse crosses part of the inland ice from which a glacier system flows into Jökel Bugt, so that here the ice movement is greater than in the central part of the traverse, and the shear stress by Orvig's argument should also be greater. Similarly, the movement of ice at points in the eastern part of the traverse is greater than in the western part, which was over a lobe of the ice sheet in which there was little movement (as indicated by the virtual absence of crevasses, despite an undulating bedrock and ice thicknesses comparable with those in the crevassed area between A8 and A14).

It is probable that the differences between the shear stresses found in north and south Greenland cannot be explained solely by different rates of movement. In the western part of the traverse in south Greenland the movement may be large because of the "ice streams" extending far into the ice sheet[14], yet nearer the east coast the movement is unlikely to exceed greatly that further north.

Temperatures in the Ice Sheet

If the geothermal heat flowing into the bottom of the ice sheet is 38 cals. cm.$^{-2}$ year^{-1}, the temperature gradient which would be set up, in the absence of lateral movement or accumulation on the upper surface, is 1° C. per 44 m. Robin[15] has shown that the effect of accumulation is to reduce the temperature difference between upper and lower surfaces required to conduct away the geothermal heat, the reduction increasing with the rate of accumulation.

At "Northice" the mean annual temperature at a depth of 14 m. in the firn is −28° C. The present rate of accumulation is about 10 cm. a year, and the ice thickness is about 2500 m. Using Robin's equations, the temperature difference expected between top and bottom surfaces is about 30° C. At "Station Centrale" (lat. 70° 54·8′ N., long. 40° 38′ W., altitude 2994 m.), the mean surface temperature is also −28° C., and the ice thickness is about 3000 m., but the annual accumulation is much greater, being about 30 cm. (water equivalent), so that the temperature difference between upper and lower surfaces is only about 20° C.

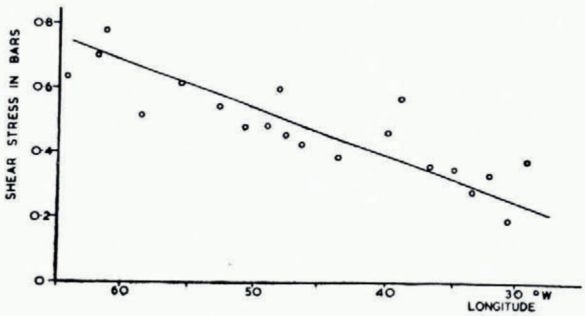

Fig. 1

No consideration has been made of the frictional heat in the ice sheet by differential movement and by slip on the bottom. The effect of this is to increase the temperature difference, but near the central parts of the ice sheet, where the lateral movement is small, the increase will be small. Therefore it is reasonable to assume that near the centre of north Greenland the bottom of the ice sheet is at the melting point (about −1·5° C. at 250 bars pressure) while in south Greenland it is well below the melting point. It is suggested that the result of the melting in the north is to provide lubrication between the ice and the underlying moraine, so that movement of the ice is possible with a smaller stress than in south Greenland.

For the centre of the ice sheet the calculations of the bottom temperatures are probably accurate, but nearer the edges more unknown factors enter. Most important of these is the amount of lateral movement, and it has been seen that a comparatively small velocity of transverse movement can produce a quantity of heat greatly exceeding the geothermal heat flow. From simple considerations of the sizes of the accumulation and ablation areas, and the mean rates of accumulation and ablation, it is possible to estimate the mean movement of the ice sheet, but the velocity may vary greatly from place to place, as with the "ice streams" in south Greenland, so that calculations of the probable basal temperatures for the edges of the ice sheet will not be of great value until more determinations have been made of the rate of movement of the ice. However, at their Camp VI near the western edge of the inland ice, the French determined the velocity of longitudinal seismic waves in the layer immediately below the ice as being 4800–5100 m. sec.$^{-1}$ [14], which corresponds closely with values determined in permafrost elsewhere. From this it can be assumed that the bottom of the ice sheet is below freezing point at this point. In north Greenland the low values of τ at points in the eastern part of the traverse suggest that the bottom is at melting point, while on the western side the values approach those obtained in south Greenland, where the bottom is frozen. It is unlikely that the differences between the western and eastern ends are due to differences in the rates of accumulation of snow, and since the rate of movement is greater in the east, larger values of τ might be expected there. However, the additional movement may provide sufficient heat for the amount of melting to be significantly greater, so that the "lubrication" is more efficient in the east.

Sufficient data are not yet available for a full analysis to be made of the properties of ice and of the form of the Greenland Ice Sheet. In particular, information is required of the rates of movement of parts of the ice sheet and of conditions existing at the bottom of the ice. It is hoped that the forthcoming International Glaciological Expedition to Greenland will be able to undertake seismic refraction experiments in north Greenland, as well as the redetermination of the positions of the beacons previously erected at many points on the ice sheet.

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PRELIMINARY RESULTS OF RESEARCH ON SNOW AND AVALANCHES IN CZECHOSLOVAKIA

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Abstract. This paper gives a brief account of the results so far obtained in research in Czechoslovakia on the crystallographic, stratigraphical and thermal properties of snow cover, and the use of these data in avalanche investigations. Avalanche danger is predicted by comparing the penetration resistance of snow layers, measured with a rammsonde, with resistance graphs of typical avalanche situations.

Zusammenfassung. Es wird ein kurzer Bericht über die Resultate gegeben, die bisher in Forschungen in der Tschechoslowakei über kristallographische, stratigraphische und thermische Eigenschaften der Schneedecke erhalten worden sind, und über die Anwendung dieser Data in Untersuchungen von Lawinen. Lawinengefahr wird durch Vergleich des Durchdringungswiderstandes von Schneeschichten, der mit einer kleinen Rammsonde gemessen wurde, mit Widerstandsdiagrammen typischer Lawinen Lagen vorausgesagt.